Estimation quiz!

Threaded Mode | Linear Mode

30th July, 2020, 12:54 **Post: #1**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

Estimation quiz!

This is from an old computer magazine: which is larger, the number of millimetres in a mile or the number of square yards in a square kilometre?

Any other quick numeric or numeracy quizzes to share?

(Of course I liked this one because I got it right...)

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30th July, 2020, 13:27 (This post was last modified: 30th July, 2020 13:30 by Paul Dale.) **Post: #2**

Paul Dale
Senior Member

Posts: 1,787
Joined: Dec 2013

RE: Estimation quiz!

Interesting problem. Being confident in both systems of units makes it easier.

Which is larger e^{π} or π^{e}?

Pauli

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30th July, 2020, 15:00 **Post: #3**

cruff
Member

Posts: 238
Joined: Dec 2013

RE: Estimation quiz!

> **Paul Dale Wrote:** (30th July, 2020 13:27)
>
> Which is larger e^{π} or π^{e}?

Neither? It appears, to 9 significant digits, these are the same number. Too early in the morning to work out why at the moment.

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30th July, 2020, 15:04 **Post: #4**

Paul Dale
Senior Member

Posts: 1,787
Joined: Dec 2013

RE: Estimation quiz!

They differ in their second digit...

Might the arguments to y^x have been backwards?

Pauli

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30th July, 2020, 15:56 **Post: #5**

KeithB
Senior Member

Posts: 382
Joined: Jan 2017

RE: Estimation quiz!

Which is larger eπ or πe?

isn't an estimation problem. 22 or 23 are the same for an estimate.

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30th July, 2020, 16:13 **Post: #6**

Albert Chan
Senior Member

Posts: 2,148
Joined: Jul 2018

RE: Estimation quiz!

> **EdS2 Wrote:** (30th July, 2020 12:54)
> which is larger, the number of millimetres in a mile or the number of square yards in a square kilometre?

We can estimate relative size without multiplying

1 yard = (36 in) (25.4 mm/in) = 914.4 mm → **1 yard ≈ 1 m**

1 mile = 1760 yard ≈ 1.760e6 mm // over-estimated 10%
1 km^2 = (1e3 m)^2 ≈ 1e6 yard^2 // under-estimated 20%

Since 1.76 > 1.30, we have mile/mm > km^2/yard^2

> **Paul Dale Wrote:** (30th July, 2020 13:27)
> Which is larger e^π or π^e?

e^x = 1 + x + x^2/2! + x^3/3! + ... ≥ 1+x
x ≥ ln(1+x)

→ if x≠0, x > ln(1+x)

ln(e^y) = y
ln(y^e) = e*(1 + ln(1 + (y/e-1))) ≤ e*(1 + (y/e-1)) = y

→ if y≠e, ln(y^e) < y ⇒ e^y > y^e
→ e^pi > pi^e

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30th July, 2020, 16:24 **Post: #7**

Werner
Senior Member

Posts: 777
Joined: Dec 2013

RE: Estimation quiz!

Or, even simpler:

Since e^x = 1 + x + x^2/2 +..., e^x > 1 + x if x>0
Take x = pi/e - 1, x>0 because pi>e
e^(pi/e-1) > 1 + pi/e - 1
e^(pi/e) / e > pi/e
e^(pi/e) > pi
(e^(pi/e))^(e) > pi^(e)
e^pi > pi^e

Cheers, Werner

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30th July, 2020, 16:34 **Post: #8**

Massimo Gnerucci
Senior Member

Posts: 2,589
Joined: Dec 2013

RE: Estimation quiz!

> **Werner Wrote:** (30th July, 2020 16:24)
>
> Or, even simpler:
>
> Since e^x = 1 + x + x^2/2 +..., e^x > 1 + x if x>0
> Take x = pi/e - 1, x>0 because pi>e
> e^(pi/e-1) > 1 + pi/e - 1
> e^(pi/e) / e > pi/e
> e^(pi/e) > pi
> (e^(pi/e))^(e) > pi^(e)
> e^pi > pi^e
>
> Cheers, Werner

Clever! ;)

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30th July, 2020, 17:10 **Post: #9**

Albert Chan
Senior Member

Posts: 2,148
Joined: Jul 2018

RE: Estimation quiz!

Another way, let f = ln(e^x/x^e) = x - e*ln(x)

f' = 1 - e/x = 0 ⇒ x=e // locate extremum
f'' = e/x^2 > 0 // 2nd derivative test, f(e) is minimum

if f x ≠ e: f(x) > f(e) = 0 ⇒ e^x > x^e

→ e^pi > pi^e

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30th July, 2020, 18:20 **Post: #10**

rprosperi
Super Moderator

Posts: 5,748
Joined: Dec 2013

RE: Estimation quiz!

Albert and Werner: It's at times like this that I like to remind people this is exactly why we have calculators - when presented with a problem like this, we can carefully select the weapon of choice, perhaps an HP model not recently used, or an unusual competitive product, and bang it out.

With all due respect to your ever-obvious extreme math skills, I simply say: "Calculators gentlemen! The tool of discerning geeks everywhere" 😃

As for the expected question of how to reply if someone puts a gun to my head and demands I solve the problem without a calculator; my reply: "shoot"

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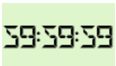
30th July, 2020, 19:37 **Post: #11**



Werner
Senior Member

Posts: 777
Joined: Dec 2013

RE: Estimation quiz!

Two remarks: the proof I presented is not mine, I found it somewhere.
And: you win, Albert!
Cheers, Werner

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30th July, 2020, 20:10 **Post: #12**

Albert Chan
Senior Member

Posts: 2,148
Joined: Jul 2018

RE: Estimation quiz!

Third way, using the fact that continuous compouding maximize capital growth.
For finite periods n, r>0: (1 + r/n)^n < e^r

Let r = pi - e, n = e:

pi^e = (e + r)^e = e^e * (1 + r/n)^n < e^e * e^r = e^pi

→ e^pi > pi^e

30th July, 2020, 22:03 **Post: #13**

rprosperi
Super Moderator

Posts: 5,748
Joined: Dec 2013

RE: Estimation quiz!

> **Albert Chan Wrote:** (30th July, 2020 20:10)
> Third way, using the fact that continuous compouding maximize capital growth.

Thanks Albert, that page is actually quite good. It's a clear and easy to follow derivation of the various interest formulae.
[bookmarked]

31st July, 2020, 00:25 **Post: #14**

Paul Dale
Senior Member

Posts: 1,787
Joined: Dec 2013

RE: Estimation quiz!

> **KeithB Wrote:** (30th July, 2020 15:56)
> isn't an estimation problem. 22 or 23 are the same for an estimate.

They differ in the second significant digit just like the original problem.
Why would it be an estimation problem?

Pauli

31st July, 2020, 02:01 **Post: #15**

cruff
Member

Posts: 238
Joined: Dec 2013

RE: Estimation quiz!

> **Paul Dale Wrote:** (30th July, 2020 15:04)
> They differ in their second digit...
> Might the arguments to y^x have been backwards?

Indeed, pilot error. That's what I get for not looking at the calculator key legend closely.

31st July, 2020, 15:26 (This post was last modified: 31st July, 2020 15:28 by EdS2.) **Post: #16**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Estimation quiz!

Very nice view of the e^pi question!

For the headline (metric and imperial) I got it first by reckoning that both numbers were the same order of magnitude, and then by reckoning that miles are a bit on the large side for metric units, as are square yards, and both of those push in the same direction.

The puzzle is from PCW magazine, in a puzzle feature called Leisure Lines. There's a corresponding book, but almost all the puzzles are not estimations. Here's the one I spotted which is:

> **Quote:**
> If you could fold a sheet of rice paper, a thousandth of an inch thick, successively doubling the thickness, 50 times over, how thick would the result be?

[Content warning for that book: many cultural references have not dated well. *Math and Logic Puzzles for PC Enthusiasts*, by J. J. Clessa]

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31st July, 2020, 15:40 **Post: #17**

Paul Dale
Senior Member

Posts: 1,787
Joined: Dec 2013

RE: Estimation quiz!

$1000 \approx 2^{10}$, so $\approx 2^{40}$ inches thick -- thicker than the sun's diameter by a factor of 5 or 6.

Pauli

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31st July, 2020, 16:33 **Post: #18**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Estimation quiz!

> **Paul Dale Wrote:** (31st July, 2020 15:40)
> $1000 \approx 2^{10}$, so $\approx 2^{40}$ inches thick -- thicker than the sun's diameter by a **factor of 5 or 6**.

Well, no.

2^40 inches are some 27.9275 million kilometers, and the sun's diameter is about 1.3927 million kilometers, so **the factor is ~20.**

Your estimation was wrong by some **300-400%**.

V.

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1st August, 2020, 03:09 **Post: #19**

Paul Dale
Senior Member

Posts: 1,787
Joined: Dec 2013

RE: Estimation quiz!

Thanks for the correction.
Regardless, it is a huge number.

Pauli

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1st August, 2020, 08:48 (This post was last modified: 1st August, 2020 08:49 by EdS2.) **Post: #20**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Estimation quiz!

Very good. I thought Pauli's estimate interesting - evidently each of us have our favourite yardsticks. For me, the distance to the Moon is the biggest one(*) in miles, at a quarter of a million. I don't (didn't) even have a figure in mind for the size of the Sun. I think in future I'll count it as nearly a million miles. (I'm from the UK but old enough to be mostly unmetricated.)

A bit of mental arithmetic, reckoning 30 inches to the yard and 1500 yards to the mile, and 30*30 about 1000, got me to the answer of 20 million miles, which isn't too far off.

(I'd like to make a joke comparing a Pauli estimate to a Fermi estimate, but I can't do it.)

(*) Oh, no, I do have the figure 93 million miles for the distance to the Sun. Perhaps the earliest large number I came across. And two digits of precision!

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1st August, 2020, 09:30 **Post: #21**

Thomas Okken
Senior Member

Posts: 1,828
Joined: Feb 2014

RE: Estimation quiz!

The distance between the sun and Earth is my favorite almost-round number, at 499 light-seconds.

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1st August, 2020, 12:09 **Post: #22**

Massimo Gnerucci
Senior Member

Posts: 2,589
Joined: Dec 2013

RE: Estimation quiz!

> **Thomas Okken Wrote:** (1st August, 2020 09:30)
> The distance between the sun and Earth is my favorite almost-round number, at 499 light-seconds.

Or almost exactly 150Gm.

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1st August, 2020, 19:24 **Post: #23**

ijabbott
Senior Member

Posts: 1,243
Joined: Jul 2015

RE: Estimation quiz!

A useful approximation for electronics: speed of light ≈ 1 foot per nanosecond.

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1st August, 2020, 22:41 (This post was last modified: 1st August, 2020 22:41 by johanw.) **Post: #24**

johanw
Member

Posts: 222
Joined: Nov 2019

RE: Estimation quiz!

> **ijabbott Wrote:** (1st August, 2020 19:24)
> A useful approximation for electronics: speed of light ≈ 1 foot per nanosecond.

Are all these ancient units still used in papers? I still remember the scolding I got when I entered wavelengths in Ångström (1Å = 10^{-10}m) in my masters thesis. I had to change it to nm before I could get a passsing grade, the faculty had a strict SI-only policy.

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2nd August, 2020, 01:16 **Post: #25**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Estimation quiz!

> **Thomas Okken Wrote:** (1st August, 2020 09:30)

The distance between the sun and Earth is my favorite almost-round number, at **499** light-seconds.

I've always taken that value at exactly **500** seconds (i.e.: 8 min 20 sec.) as the distance between the sun and the Earth varies by as much as 5 MKm between aphelion (~152.1 Mkm) and perihelion (147.1 Mkm), and that's plus or minus ~8 light-seconds, give or take a second.

So **500** it is for me. Easier to remember and way *"rounder"* (in base 10; for a round value in base 2, the value 512 ($1{,}000{,}000{,}000_2$) would do

Regards.
V.

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2nd August, 2020, 09:56 **Post: #26**

J-F Garnier
Senior Member

Posts: 820
Joined: Dec 2013

RE: Estimation quiz!

> **ijabbott Wrote:** (1st August, 2020 19:24)
> A useful approximation for electronics: speed of light ≈ 1 foot per nanosecond.

As a continental european EE, my reference is 30cm for 1ns ...

> **johanw Wrote:** (1st August, 2020 22:41)
> Are all these ancient units still used in papers? I still remember the scolding I got when I entered wavelengths in Ångström (1Å = 10^{-10}m) in my masters thesis. I had to change it to nm before I could get a passsing grade, the faculty had a strict SI-only policy.

Non-SI units are very common in astronomy, think of the old parsec (distance where a length of 1AU is seen under an angle of 1 arc-second , with 1AU being the average Earth-Sun distance :-)
The parsec seems now less used and often replaced by the light-year (1 pc is about 3 ly), but is still used for instance for the Hubble constant (cosmological expansion) Ho ≈ 70 km/s/Mpc.

Also masses of stars are generally expressed not in kg but in solar mass, and so on.

A funny coincidence: there are (about) as many km in one light-year than Ångström in one km.

J-F

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2nd August, 2020, 14:23 **Post: #27**

SlideRule
Senior Member

Posts: 1,328
Joined: Dec 2013

RE: Estimation quiz!

from Science Unit Conversion Humor

This is a list of funny, made-up scientific unit conversions.

453.6 graham crackers = 1 pound cake
There are 453.6 grams in 1 pound.
Ratio of an igloo's circumference to its diameter = Eskimo Pi
Pi is the ratio of a circle's circumference to diameter, while there is a stereotype that Eskimos dwell in igloos.
2000 pounds of Chinese soup = Won ton
A wonton is a type of Chinese dumpling. There are 2000 pounds in 1 ton.
Time between slipping on a peel and smacking the pavement = 1 bananosecond
Instead of expressing the unit in terms of nanoseconds, it's bananoseconds because a banana caused the fall.
1 millionth of a mouthwash = 1 microscope
This refers to the popular mouthwash, Scope. The metric prefix "micro" means one millionth.
1 million bicycles = 1 megacycles
The metric prefix "mega" means 106 or one million.
Weight an evangelist carries with God = 1 billigram
This refers to the American evangelist Billy Graham.
Time it takes to sail 220 yards at 1 nautical mile per hour = Knotfurlong
365.25 days of drinking low-calorie beer = 1 Lite year
16.5 feet in the Twilight Zone = 1 Rod Serling
The rod is a unit of length equal to 16.5 feet. Rod Serling is the American tv producer, screenwriter, and narrator responsible for "The Twilight Zone."

Basic unit of laryngitis - 1 hoarsepower
One symptom of laryngitis is hoarseness.
Shortest distance between two jokes - a straight line
To deliver a joke as a straight line means it's a short joke delivered with a straight face (like it's not a joke at all).
1 million microphones = 1 megaphone
365.25 days = 1 unicycle
365.25 days is one year or one cycle of the Earth around the Sun. It's especially clever because unicycle has another meaning. It's a bike with one wheel.
Half a large intestine = 1 semicolon
the large intestine is also called the colon. Since it's only half a colon, it's a semicolon, much like half a circle is a semicircle.
2000 mockingbirds = two kilomockingbirds
"To Kill a Mockingbird" is a famous novel by author Harper Lee published in 1960. The kilo is the prefix for a thousand. So, 2000 is two kilo.
10 cards = 1 decacard
Deca is the prefix for 10.
52 cards = 1 deckacard
There are 52 cards in a deck of playing cards.
1,000,000 aches = 1 megahurtz
There are one million (106) hertz in 1 megahertz. This is a play on words, substituting hurtz (like pain, but with a "z") for hertz.
1 millionth of a fish = 1 microfiche
The word "microfiche" is pronounced like micro-fish. The prefix micro means one millionth.
2.4 statute miles of intravenous surgical tubing at Yale University Hospital = 1 I.V.League
Intravenous tubing is also called IV tubing. Yale is one of the Ivy League school, plus 2.4 statute miles is a length equal to 1 league.
1 kilogram of falling figs = 1 fig newton
The newton is a unit force, which is mass under acceleration (such as you'd get from falling figs). This play on words refers to the Nabisco cookie, the fig newton.
1000 grams of wet socks = 1 literhosen
Lederhosen are short breeches (not actually socks). There are 1000 grams of water (more or less) in one liter. The liter is a unit of volume used for liquids, so wet socks are literhosen.
1 trillion pins = 1 terrapin
The prefix terra means a trillion.
10 rations = 1 decaration
The prefix deca means 10.
100 rations = 1 C-ration
C is the Roman numeral for 100.
2 monograms = 1 diagram
Mono is the prefix for one, while dia means two.
2 new dimes = new paradigms
Two dimes is a pair of dimes. A paradigm is a model or pattern.

These and many more available for your perusal somewhere out there on the net.
An implied **rhetorical** question, does *SCIENCE* own unit conversions?

BEST!
SlideRule

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2nd August, 2020, 15:10 **Post: #28**

Paul Dale
Senior Member

Posts: 1,787
Joined: Dec 2013

RE: Estimation quiz!

The micro-fortnight is a surprisingly useful unit.

Pauli

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2nd August, 2020, 17:13 **Post: #29**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Estimation quiz!

Here's one that might tax the most mathematical, and also needs some not very general knowledge: how many Jupiters could fit into the (volume of the) Sun?

(The readily searchable answer for this assumes we just compare volumes, which is as wrong as anyone with a jar of marbles can tell you.)

Or, something which is actually within the bounds of present mathematical knowledge, and closer to everyday experience: how many Moons could fit into the Earth?

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2nd August, 2020, 18:02 **Post: #30**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Estimation quiz!

> **EdS2 Wrote:** (2nd August, 2020 17:13)
> how many Moons could fit into the Earth?

I'd estimate about **23**, give or take a Moon.

V.

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2nd August, 2020, 19:04 (This post was last modified: 2nd August, 2020 19:04 by Maximilian Hohmann.) **Post: #31**

Maximilian Hohmann
Senior Member

Posts: 1,124
Joined: Dec 2013

RE: Estimation quiz!

Hello!

> **Valentin Albillo Wrote:** (2nd August, 2020 18:02)
> I'd estimate about **23**, give or take a Moon.

23 is always a good answer :-) But this one really got me interested and surprisingly (using finite time for my google search) I only found one paper online with an equation for this problem: Nelson M- Blachman: The Closest Packing of Equal Spheres in a Larger Sphere (1963). It can be freely accessed from here (registration required):
https://www.jstor.org/stable/2312064?rea...b_contents

If I entered the numbers correctly, there is place for just 11.8 moond (or 11 full moons) insdide our earth.

Regards
Max

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2nd August, 2020, 19:41 **Post: #32**

Albert Chan
Senior Member

Posts: 2,148
Joined: Jul 2018

RE: Estimation quiz!

> **EdS2 Wrote:** (2nd August, 2020 17:13)
> (The readily searchable answer for this assumes we just compare volumes,
> which is as wrong as anyone with a jar of marbles can tell you.)

Planet sized spheres might not act like marbles.
Gravity may squash them into a giant sphere, possibly even denser.

Volume ratios might actually be a good estimate. From Wolfram Alpha:

(earth volume) / (moon volume) = 49.31
(sun volume) / (jupiter volume) = 986.7

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2nd August, 2020, 19:54 **Post: #33**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Estimation quiz!

> **Maximilian Hohmann Wrote:** (2nd August, 2020 19:04)
> Hello!

> **Valentin Albillo Wrote:** (2nd August, 2020 18:02)
> I'd estimate about **23**, give or take a Moon.
>
> 23 is always a good answer :-)

You bet.

> **Quote:**
> **If I entered the numbers correctly**, there is place for just **11.8** moond (or 11 full moons) insdide our earth.

It seem's you didn't or you used the wrong formula, as it's trivial to see your estimation is wrong.

Consider N spheres of unit diameter touching a central one (unit diameter too). Now it's well known that the maximum number N of non-overlapping spheres touching a central one is **12** (a 13th almost fits but ultimately doesn't). This makes **13** equal spheres in all of unit diamter, 12 of them in contact with the central one.

But as it's trivial to see, they all fit in a sphere of diameter exactly **3**: O O O

And as the ratio of Moon's to Earth's diameter is about 0.27 this means that if the Moon's sphere has diameter **1** then Earth is a sphere of diameter ~**3.7**, and thus if 13 spheres could be easily contained within a sphere of diameter just 3, then a sphere of diameter 3.7 will hold way more, certainly more than your "11.8", let's say **23** ...

Regards.
V.

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2nd August, 2020, 23:01 **Post: #34**

Maximilian Hohmann

Senior Member

Posts: 1,124
Joined: Dec 2013

RE: Estimation quiz!

Hello!

> **Valentin Albillo Wrote:** (2nd August, 2020 19:54)
> And as the ratio of Moon's to Earth's diameter is about 0.27 this means that if the Moon's sphere has diameter **1** then Earth is a sphere of diameter ~**3.7**, and thus if 13 spheres could be easily contained within a sphere of diameter just 3, then a sphere of diameter 3.7 will hold way more, certainly more than your "11.8", let's say **23** ...

I am not so sure about that. After your 13 moons (the central one and the next layer of 12 adjacent to it) there is not enough space for another full layer of moons within the Earth's radius. The central moon has a radius of 0,27 to this you have to add the diameter of the next layer of 0,54, so the envelope of that lfirst ayer reaches out to 0,81). If at all, the extra moons need to fit into the gaps between the 12 moons of the first layer and the radius of the Earth. My guess is that there is no symmetrical solution for a maximum number of moons (as it seems to be the case with the old equation I found) but the 10 extra moons have to be added to one side only. But how can that be calculated?

Regards
Max

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3rd August, 2020, 01:04 (This post was last modified: 6th August, 2020 00:07 by SlideRule.) **Post: #35**

SlideRule

Senior Member

Posts: 1,328
Joined: Dec 2013

RE: Estimation quiz!

> **Maximilian Hohmann Wrote:** (2nd August, 2020 19:04)
> ... I only found one paper online with an equation for this problem: Nelson M. Blachman: The Closest Packing of Equal Spheres in a Larger Sphere (1963) ...

An interesting read:
Grundlehren def mathematischen Wissenschaften 290
Sphere Packings, Lattices and Groups
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