Can you calculate Pi using a Solver?

9th December, 2019, 15:12 **Post: #1**

EdS2
Senior Member
Posts: 525
Joined: Apr 2014

Can you calculate Pi using a Solver?

I'm wondering what approaches there might be for computing Pi using one of the Solvers on the various HP calculators. Of course, no trig functions allowed!

Let's call this a mini-challenge, unless it turns out to be so easy and obvious that it's a nano-challenge.

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9th December, 2019, 15:44 **Post: #2**

SlideRule
Senior Member
Posts: 1,328
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

Reference:
Using A Minicalculator to Find An Approximate Value for *Π*, E. J. Bolduc (University of Florida)

"One of the many ways to use a minicalculator in a classroom is in the calculation of an approximate value for n using a variation of the method of Archimedes … if a circle of radius 1 is chosen, then the area of any inscribed polygon is less than *Π* and the area of any circumscribed polygon is greater than n. He finally arrived at the fact that 3(10/17) < *Π* < 3(1/7). We can use the idea that, as the numbers of sides of an inscribed polygon increases, the perimeter of the polygon approaches the circumference of the circle and the ratio of the perimeter to the diameter of the circle is an approximation for *Π* …
We now have our iterative formula, **$S' = \sqrt{2r^2 - r\sqrt{4r^2 - S^2}}$**"

I leave the recursion and subsequent calculation to the reader, but after nine iterations, the equation yields 3.141592…

BEST!
SlideRule

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9th December, 2019, 15:50 **Post: #3**

jthole
Member
Posts: 60
Joined: Nov 2017

RE: Can you calculate Pi using a Solver?

This does not answer your question directly, but on the back of my 12C I have a sticker with "355 / 113".

For my purposes, that is good enough 🙂

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9th December, 2019, 15:56 (This post was last modified: 9th December, 2019 15:57 by toml_12953.) **Post: #4**

toml_12953
Senior Member
Posts: 1,990
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

> **SlideRule Wrote:** (9th December, 2019 15:44)

Reference:
Using A Minicalculator to Find An Approximate Value for *Π*, E. J. Bolduc (University of Florida)

"One of the many ways to use a minicalculator in a classroom is in the calculation of an approximate value for n using a variation of the method of Archimedes … if a circle of radius 1 is chosen, then the area of any inscribed polygon is less than *Π* and the area of any circumscribed polygon is greater than n. He finally arrived at the fact that 3(10/17) < *Π* < 3(1/7). We can use the idea that, as the numbers of sides of an inscribed polygon increases, the perimeter of the polygon approaches the circumference of the circle and the ratio of the perimeter to the diameter of the circle is an approximation for *Π* …
We now have our iterative formula, **S′ = √2r² - r√4r² - S²**"

I leave the recursion and subsequent calculation to the reader, but after nine iterations, the equation yields 3.141592…

BEST!
SlideRule

Here's a simple BASIC version from Problems for Computer Solution by Stephen J. Rogowski:

Code:
```
50 PRINT "ARCHIMEDEAN DETERMINATION OF PI!"
60 PRINT
70 PRINT "NO. OF SIDES","INSCR PER","CIRCUM PER"
80 PRINT
100 FOR X=2 TO 15
105   LET N=2^X
110   LET D=360/N
120   LET T=3.1415927*(D/180)
130   LET A=2*N*SIN(T/2)
140   LET B=2*N*TAN(T/2)
150   PRINT N,A/2,B/2
```

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9th December, 2019, 17:35 **Post: #5**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Can you calculate Pi using a Solver?

EdS2 Wrote: (9th December, 2019 15:12)

I'm wondering what approaches there might be for computing Pi using one of the Solvers on the various HP calculators. Of course, no trig functions allowed!

Let's call this a mini-challenge, unless it turns out to be so easy and obvious that it's a nano-challenge.

If your Solver has Square Root and Factorial, this one from Ramanujan converges very fast and it's quite simple, few bytes to program it and only a few terms to add up for full precision...

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103+26390k)}{k!^4 \left(396^{4k}\right)}.$$

... and no trigs used, as requested, just basic arithmetic. Once you get 1/Pi just invert it with 1/x

Regards.
V.

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9th December, 2019, 17:53 **Post: #6**

Csaba Tizedes
Senior Member

Posts: 581
Joined: May 2014

RE: Can you calculate Pi using a Solver?

Code:
```
120   LET T=    3 . 1 4 1 5 9 2 7      *(D/180)
```

WHAT?

Code:
```
120   LET T=3.1415927*(D/180)
130   LET A=2*N*SIN(T/2)
140   LET B=2*N*TAN(T/2)

155   IF A=B THEN GOTO 170

170 END
```

Err... this looks like a checking how precise the arithmetics of that computer...

BTW: HP32SII - a fraction finder

Code:
```
---------------------
LBL P          // Increase P
  1
  STO + P
  GTO Q
---------------------

Checksums:
==========
LBL Q: CK=9676 / 22.5 byte
LBL P: CK=C1C6 / 6.0 byte
```

Csaba

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9th December, 2019, 20:38 (This post was last modified: 9th December, 2019 20:51 by jthole.) **Post: #7**

jthole
Member

Posts: 60
Joined: Nov 2017

RE: Can you calculate Pi using a Solver?

> **Valentin Albillo Wrote:** (9th December, 2019 17:35)
>
> If your Solver has Square Root and Factorial, this one from Ramanujan converges very fast and it's quite simple, few bytes to program it and only a few terms to add up for full precision...
>
> $$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{k!^4 \left(396^{4k}\right)}.$$
>
> ... and no trigs used, as requested, just basic arithmetic. Once you get 1/Pi just invert it with 1/x
>
> Regards.
> V.

Very nice! Here is the equation for the 17Bii:

1 ÷ PIVAL = ((2 x SQRT(2)) ÷ 9801) x sigma (I : 0 : N : 1 : (FACT(4 x I) x (1103 + 26390 x I) ÷ ((FACT(I) ^ 4) x (396 ^ (4 x I))))

I only tested it on the iPhone emulator, but it goes to full precision very quickly (N=2) indeed.

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10th December, 2019, 04:48 **Post: #8**

mfleming
Senior Member

Posts: 881
Joined: Jul 2015

RE: Can you calculate Pi using a Solver?

Needs one more closed parenthesis at the end. Works on the 27S!

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10th December, 2019, 21:45 **Post: #9**

JurgenRo
Member

Posts: 192
Joined: Jul 2015

RE: Can you calculate Pi using a Solver?

> **Valentin Albillo Wrote:** (9th December, 2019 17:35)
>
> > **EdS2 Wrote:** (9th December, 2019 15:12)
> >
> > I'm wondering what approaches there might be for computing Pi using one of the Solvers on the various HP calculators. Of course, no trig functions allowed!
> >
> > Let's call this a mini-challenge, unless it turns out to be so easy and obvious that it's a nano-challenge.
>
> If your Solver has Square Root and Factorial, this one from Ramanujan converges very fast and it's quite simple, few bytes to program it and only a few terms to add up for full precision...
>
> $$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{k!^4\left(396^{4k}\right)}.$$
>
> ... and no trigs used, as requested, just basic arithmetic. Once you get 1/Pi just invert it with 1/x
>
> Regards.
> V.

(Pi^2)/6=sum_{n=1}^{infinity}1/n^2 might fall into the same category.
Juergen

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10th December, 2019, 23:07 (This post was last modified: 10th December, 2019 23:36 by jthole.) **Post: #10**

jthole
Member

Posts: 60
Joined: Nov 2017

RE: Can you calculate Pi using a Solver?

> **JurgenRo Wrote:** (10th December, 2019 21:45)
>
> > **Valentin Albillo Wrote:** (9th December, 2019 17:35)
> >
> > If your Solver has Square Root and Factorial, this one from Ramanujan converges very fast and it's quite simple, few bytes to program it and only a few terms to add up for full precision...
> >
> > $$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{k!^4\left(396^{4k}\right)}.$$
> >
> > ... and no trigs used, as requested, just basic arithmetic. Once you get 1/Pi just invert it with 1/x
> >
> > Regards.
> > V.
>
> (Pi^2)/6=sum_{n=1}^{infinity}1/n^2 might fall into the same category.
> Juergen

Converges much more slowly though. I just let it run from n=0 to 1 million (on the 17Bii simulator on my iPhone), and the difference with the Pi constant is still -9,5x10E-7. In contrast, the one suggested by Valentin already equals the Pi constant for n=2.

I'll test it on the 17Bii later, to see how much slower than the iPhone 7 it is 🙂

Edit: in order not to pollute this thread, I posted the results here: https://www.hpmuseum.org/forum/thread-14101.html

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10th December, 2019, 23:35 (This post was last modified: 15th April, 2023 16:21 by Albert Chan.) **Post: #11**

Albert Chan
Senior Member

Posts: 2,148
Joined: Jul 2018

RE: Can you calculate Pi using a Solver?

Pi via AGM, Emu71 HP-71B basic

Code:
```
10 A=1 @ B=SQRT(.5) @ S=1 @ P=1
20 C=A-B @ B=SQRT(A*B) @ A=A-C/2 @ S=S-P*C*C @ P=P+P
30 DISP A*B/(.25*S)
40 IF A<>B THEN 20
```

>RUN
3.14040401902
3.1415926456
3.1415926536
3.14159265359

Comment: Apr 14, 2023

Above code started with S=1, and lowered it bit by bit.
It may be more accurate to avoid this cancellation errors.
Just click the green arrow for Free42 AGM code.

> **Albert Chan Wrote:** (20th June, 2020 18:23)
>
> To improve agm2 accuracy, I redefined agm2 returns:
>
> x, y = agm2(a, b)
> → x = converged GM of agm(a, b)
> → y = $-\Sigma(2^k (½gap_k)^2$, k = 1 .. n), n = number of iterations to converge GM
>
> With this new setup, ellipse_perimeter(a,b) = 4 a E(1-(b/a)²) = pi (y + b² + a²)/x

With above setup, x, y = agm2(1, sqrt(0.5)) --> S = 1+2*y --> pi = 2*x^2 / (y+1/2)

.5 SQRT 1 XEQ "AGM"

X = 8.4721308479397908660649912348219l6e-1
Y = -4.3053418955536374625033377452316 69e-2

X^2 2 * SWAP .5 + /

3.1415926535897932384626433832795 03

Or, less keystrokes, x, y = agm2(1, sqrt(2)) --> pi = 2*x^2 / (y+1)

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11th December, 2019, 12:29 (This post was last modified: 11th December, 2019 12:30 by EdS2.) **Post: #12**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

Ah, always interesting to get some unexpected results - thanks everyone!

In some, then, we see a 'sigma' function being used to do most of the work, and the solver is only needing to cope with a simple transformation like reciprocal or square root. I hadn't realised a 'sigma' function might be to hand - of course these offerings are valid, for the machines which offer 'sigma'.

For the first submission, the iterative one posted by sliderule, is there a way to render this as a Solver problem?

> **Quote:**
>
> We now have our iterative formula, S′ = √2r² - r√4r² - S²

Likewise, for the AGM method posted by Albert Chan, is there a way to get a Solver to do the work, rather than a program?

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11th December, 2019, 13:50 **Post: #13**

toml_12953
Senior Member

Posts: 1,990
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

> **Csaba Tizedes Wrote:** (9th December, 2019 17:53)
>
> **Code:**
> ```
> 120   LET T=   3 . 1 4 1 5 9 2 7     *(D/180)
> ```
>
> WHAT?
>
> **Code:**
> ```
> 120   LET T=3.1415927*(D/180)
> 130   LET A=2*N*SIN(T/2)
> 140   LET B=2*N*TAN(T/2)
>
> 155   IF A=B THEN GOTO 170
>
> 170 END
> ```
>
> Err... this looks like a checking how precise the arithmetics of that computer...

It shows that both an inscribed and circumscribed polygon approach PI as the number of sides increases. I agree it was poorly named.

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11th December, 2019, 15:09 (This post was last modified: 12th December, 2019 00:14 by Albert Chan.) **Post: #14**

Albert Chan
Senior Member

Posts: 2,148
Joined: Jul 2018

RE: Can you calculate Pi using a Solver?

> **toml_12953 Wrote:** (11th December, 2019 13:50)
>
> It shows that both an inscribed and circumscribed polygon approach PI as the number of sides increases.

We can do this with right triangles, starting from a hexagon ("radius" = side = 2S = 1)

10 N=6 @ S=.5
20 H=SQRT(1-S*S) @ A=N*S @ B=A/H
30 DISP N,A,B
40 N=N+N @ S=.5*SQRT(S^2+(1-H)^2)
50 IF A<B THEN 20

Code:
```
>RUN
 6                    3                    3.46410161514
 12                   3.10582854122        3.21539030916
 24                   3.13262861328        3.1596599421
 48                   3.13935020304        3.14608621512
 96                   3.14103195089        3.14271459965
 192                  3.14145247229        3.14187304999
 384                  3.14155760792        3.14166274706
 768                  3.14158389215        3.1416101766
 1536                 3.14159046322        3.14159703431
 3072                 3.141592106          3.14159374877
```

Edit: line 40 may be replaced with simpler formula (approx. same accuracy)
40 N=N+N @ S=S/SQRT(H+H+2)

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11th December, 2019, 19:49 **Post: #15**

Roland57
Junior Member

Posts: 47
Joined: Nov 2018

RE: Can you calculate Pi using a Solver?

Hi,

May be it is too easy, but what about this

pi = 2 * integral(1 / sqr(1-x*x), x, 0, 1)

It gives 3.141592(48348) on the iOS prime-emulator

Roland

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11th December, 2019, 22:00 (This post was last modified: 12th December, 2019 18:46 by Albert Chan.) **Post: #16**

Albert Chan
Senior Member

Posts: 2,148
Joined: Jul 2018

RE: Can you calculate Pi using a Solver?

> **Roland57 Wrote:** (11th December, 2019 19:49)
> pi = 2 * integral(1 / sqr(1-x*x), x, 0, 1)
>
> It gives 3.141592(48348) on the iOS prime-emulator

Some calculators have problem with singularity at the limits.
We can do this instead:

pi = 6 asin(.5) = integrate(6 / sqrt(1-x*x), x = 0 .. 0.5)

or, to speed it up a bit ...

pi = 4 atan(1) = integrate(4 / (1+x*x), 0, 1)

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12th December, 2019, 10:18 **Post: #17**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

ah yes, like summation, integration certainly has ways to reach pi.

I'm still curious about the solver though. If any HP calculator's solver worked in the complex domain (and I imagine none do) then Euler's identity would give a route to pi.

(By the solver, I mean the facility which takes a function of one variable and finds a value of that variable which makes the function zero.)

As we know that pi isn't algebraic, it might be that we know there's no way to do this. But finding a fixed point of a recurrence feels a bit like finding a zero of a function, so I wonder if that could be a way.

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12th December, 2019, 13:24 **Post: #18**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

I found a couple of not-great examples of the sort of thing I might hope exists...

There's a good approximation for pi as a root of an 8th order polynomial:

0=1−1635840576x−343853312x^2 +60576043008x^3 +1865242664960x^4 −16779556159488x^5 +37529045696512x^6 −29726424956928x^7 +6181548457984x^8

And there's a bad approximation as a solution to
x=(1+1/x)^(x+1)

Here's a quartic:
0=16-160x+168600x^2-841000x^3+250625x^4

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12th December, 2019, 16:12 **Post: #19**

SlideRule
Senior Member

Posts: 1,328
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

> **EdS2 Wrote:** (12th December, 2019 13:24)
> I found a couple of not-great examples of the sort of thing I might hope exists...

An excerpt from PI a source book, Springer (2e), 96-40196, © 2000
[attachment=7919]

The PDF title says it all.

BEST!
SlideRule

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12th December, 2019, 17:02 (This post was last modified: 12th December, 2019 17:05 by EdS2.) **Post: #20**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

Thanks! Also of interest (and relevant) is The Quest for Pi, a 16 page paper by BBP.

(I'm still not sure how to get the solver to make use of this kind of iterative approach. Merely evaluating an expression doesn't feel like it's making proper use of a solver.)

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12th December, 2019, 19:49 **Post: #21**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Can you calculate Pi using a Solver?

.
Hi again, EdS2:

> **EdS2 Wrote:** (12th December, 2019 17:02)
> Merely evaluating an expression **doesn't feel like it's making proper use of a solver**.)

Frankly, I don't understand why you're so fixed in *solving* something (as in finding the root of some equation, say) and think evaluating expressions isn't *"proper use of a solver"*.

Matter of fact, what distinguishes the best solvers is their capability to do much more than finding roots, and being in fact able to essentially do some kind of programming with them increases their power and usefulness an order of magnitude. **That** is making proper use of a solver, the same way that doing *synthetic programming* was a proper use of RPN in the *HP-41C*.

Anyway, I'll indulge you. You want to use the *Solver* to compute a 10 or 12 digit approximation to *Pi* by *"using the Solver to Solve Some Equation"* ? Try this one:

Find the root between 3 and 4 of: $e^{\frac{x\sqrt{163}}{3}} - 640320 = 0$.

If the *Solver* works as it should it will find the root:

x* = *3.141592653589793*...** which agrees with ***Pi to all *16 digits* shown.

Regards.
V.

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12th December, 2019, 20:03 (This post was last modified: 12th December, 2019 20:05 by EdS2.) **Post: #22**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

That's a very nice one, thanks Valentin!

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12th December, 2019, 23:40 **Post: #23**

SlideRule
Senior Member

Posts: 1,328
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

Excerpt from Π Unleashed, Springer, ISBN 3-540-66572-2, © 2000

Chapter 16
[attachment=7920]

154 algorithms.

BEST!
SlideRule

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13th December, 2019, 00:07 (This post was last modified: 13th December, 2019 00:12 by EdS2.) **Post: #24**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

Here's a nice one by Ramanujan:
22x^4-2143=0
x≈3.14159265258...

and another from the same page
x^4+x^5-e^6=0
x≈3.14159268...

(Although it has to be said, the answers here are not pi but something rather close to pi. And so, not exactly what I was hoping for.)

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13th December, 2019, 20:35 (This post was last modified: 13th December, 2019 20:35 by EdS2.) **Post: #25**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

> **Valentin Albillo Wrote:** (12th December, 2019 19:49)
>
> > **EdS2 Wrote:** (12th December, 2019 17:02)
> >
> > Merely evaluating an expression **doesn't feel like it's making proper use of a solver**.)
>
> Frankly, I don't understand why you're so fixed in *solving* something (as in finding the root of some equation, say) and think evaluating expressions isn't *"proper use of a solver"*.

It's a good question, and I haven't managed to be clear, for which I apologise. I wonder if I can do better...

For me, this challenge, like many, is about constrained programming of a kind. If I had a programmable calculator, I might want a challenge which was to write a program. If it was not programmable, but had an Integrate command, I might want an interesting challenge which involved integration. But if it was programmable, I might still be interested in a challenge to use the Integrate command in an interesting way. Or indeed, I might be interested in a challenge to write a program for numerical integration, which doesn't use the Integrate command.

So it is with this challenge. I'm thinking of a solver as a complex feature which finds roots to an equation - preferably not an easy equation like x^2-3=0 or 1/x-4=0 but an equation which we couldn't readily solve ourselves, like x^4+x^5-e^6=0. But I want the root found to be pi, not a specific number that's close to pi but not equal to pi. And I think, as yet, I haven't seen such an equation - maybe there isn't one.

(Of course, an equation like sin(x)=0 could suffice, but it's trivial, which is why I wanted to avoid use of trig functions.)

It's always interesting to see various ways to compute pi: sums, products, nested surds, iterative algorithms, even spigots. And this, I'd hoped, is another possible way.

Hope this makes things a bit clearer. (And I'm still glad to see all the various contributions and references!)

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14th December, 2019, 01:30 **Post: #26**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Can you calculate Pi using a Solver?

.
G'night, EdS2 (Dec 14th 0:30 am here):

> **EdS2 Wrote:** (13th December, 2019 20:35)
> It's a good question, and I haven't managed to be clear, for which I apologise. I wonder if I can do better...

Thanks but no need to apologize, it's just that I think that if a *Solver* somehow manages to return an answer to some problem, it doesn't matter if it does it finding a root of some equation or adding up terms of a series or anything, it's still *"solving"* the problem.

Remember, it isn't called a ***"root finder"*** but a ***"solver"***.

> **Quote:**
> But I want the root found to be pi, not a specific number that's close to pi but not equal to pi. And I think, as yet, I haven't seen such an equation - maybe there isn't one.

Unless the *Solver* can work with complex numbers I don't think there's such an equation using just real numbers and no trigs. Equations whose roots are arbitrarily accurate approximations to Pi (say 34 digits) can be produced but ones returning *exactly* Pi (in theory, limited to 10-12 digits in practice) is a no-go IMHO.

> **Quote:**
> It's always interesting to see various ways to compute pi: sums, products, nested surds, iterative algorithms, even spigots.

The comprehensive list given in a previous post doesn't include *Monte-Carlo* methods to compute Pi if I'm not mistaken (cursory read), and there are some really pretty, though very slow-converging (typically like the square root, i.e.: 100 tries give 2 digits, 10,000 tries give 4, a million tries give 6, and so on.)

As for "spigots", have a look at this 6-line program o'mine for an HP calc which produces an arbitrary number of digits of Pi one at a time using a spigot algorithm. The sample run in the PDF document produces 1,000 digits.

Producing Digits of Pi one at a time

Thanks for your comments and have a nice weekend.
V.

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14th December, 2019, 02:34 **Post: #27**

SlideRule
Senior Member

Posts: 1,328
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

> **Valentin Albillo Wrote:** (14th December, 2019 01:30)

> The comprehensive list given in a previous post doesn't include *Monte-Carlo* methods to compute Pi if I'm not mistaken (cursory read), and there are some really pretty, though very slow-converging (typically like the square root, i.e.: 100 tries give 2 digits, 10,000 tries give 4, a million tries give 6, and so on.)

Π Unleashed, Springer, © 2000, ISBN 3-540-66572-2, page 39

3.4 *Π* and chance (Monte Carlo methods)

The needle problem of the Comte de Buffon

During the American Civil War, Captain C.0. Fox was recovering from a wound in a military hospital. To pass the time, he threw a number of identical needles *in random fashion* onto a board on which he had previously drawn a series of parallel lines each a needle's length apart. He counted the number of throws and the number of hits, i.e. instances in which a needle touched or intersected a line.
After 1100 throws, the Captain had determined *Π* to two decimal places. How come?
First of all it seems to have been the Comte de Buffon (1707-1788), who examined this kind of experiment and in whose honour it is now known as the *Buffon needle problem*. In 1777, Buffon showed that the ratio of hits to throws was 2:*Π*, or, stated otherwise, that the **probability** of a needle thrown at random onto the area coming to rest across one of the lines was, $^{2}/\Pi \approx 63.7\%$. With this knowledge, Fox was able to calculate *Π* by doubling the number of throws and dividing by the number of hits.
The interesting thing about the needle problem is that it forges a link between the "geometric" *Π* and the quite different area of probabilities.
There are other similar relationships between *Π* and chance, from which other methods of calculating *Π* are derived. They are informatively called ***Monte Carlo methods***.

emphasis mine

BEST!
SlideRule

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14th December, 2019, 02:56 **Post: #28**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Can you calculate Pi using a Solver?

.
Self-quoting a little:

> **Valentin Albillo Wrote:** (14th December, 2019 01:30)
> Unless the *Solver* can work with complex numbers **I don't think there's such an equation using just real numbers and no trigs**. Equations whose roots are arbitrarily accurate approximations to Pi (say 34 digits) can be produced but ones returning *exactly* Pi (in theory, limited to 10-12 digits in practice) is a no-go IMHO.

Well, to make myself clearer, the previous quotation refers to polynomials or rational functions, which indeed can't produce Pi as an exact root because Pi is transcendental.

But if we involve other functions but polynomials, rational functions and trigs, there are many ways to concoct an equation whose root is exactly Pi. For instance, if your *Solver* admits nesting and can use the ***Gamma*** function, then this will do:

Code for the **HP-71B**, where **FNROOT** *(Find Root)* is the *"official" HP-71B's "Solver"*:

```
DEF FNF(X) = FNROOT(1, 1, GAMMA(FVAR) - X)
DISP FNROOT(3, 4, FNF(SQR(FVAR)) - 1/2)
```

>RUN

3.14159265359

This produces as a root the theoretically *exact* (not an approximation) value of Pi.

V.

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14th December, 2019, 12:51 **Post: #29**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

(I'm sometimes surprised that I don't already own *Pi Unleashed* and indeed *Pi a source book*. Oh, and I see a reviewer mentions Beckman's *A History of Pi* and Blatner's *The Joy of Pi*)

Yes, the Gamma function is a nice way in!

I had hopes that Euler's equation would serve - if we could take the magnitude, or the real value, of e^(ix)+1, then we have a (complex) function which takes a real to a real, and the job would be done. But the HP-35s lacks the capability. Can the 15c manage it??

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14th December, 2019, 23:03 **Post: #30**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Can you calculate Pi using a Solver?

> **EdS2 Wrote:** (14th December, 2019 12:51)
> I had hopes that Euler's equation would serve - if we could take the magnitude, or the real value, of e^(ix)+1, then we have a (complex) function which takes a real to a real, and the job would be done. But the HP-35s lacks the capability. **Can the 15c manage it??**

You're kidding, right ?

... and two question marks, no less ...

Regards.
V.

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15th December, 2019, 12:39 **Post: #31**

J-F Garnier
Senior Member

Posts: 820
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

> **EdS2 Wrote:** (14th December, 2019 12:51)
> I had hopes that Euler's equation would serve - if we could take the magnitude, or the real value, of e^(ix)+1, then we have a (complex) function which takes a real to a real, and the job would be done. But the HP-35s lacks the capability.

Following your suggestion, here is a solution of Euler's equation on the HP32S (I like this little machine:-):

```
LBL E
INPUT X
RCL X
0
CMPLXe^X
1
+
x^2
x<>y
x^2
+
RTN
```

Then:
FN= E
SOLVE X

The 32S displays "NO ROOT FND" but the best guess 3.14159265359 is in X.

Anyhow, for me it's a complicated way to just solve COS(X)=-1.

J-F

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15th December, 2019, 13:28 **Post: #32**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

Very nice - well done!

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16th December, 2019, 13:44 (This post was last modified: 16th December, 2019 13:51 by EdS2.) **Post: #33**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

So, indeed, I got my trusty Limited Edition HP15C out, set flag 8 (surprised to see it not documented on the back plate) and indeed it does a very solid job of being a complex-number calculator, and is willing to have a go at solving Euler's identity for a value of pi.

It doesn't do terribly well, but it is willing.

Using the Real() method:
Solver input 1,4: result is 3.141591597
Solver input 2,3: result is 3.141583326

Using the ABS() method:
Solver input 1,4: result is Error 8, aka 3.139172931
Solver input 2,3: result is Error 8, aka 3.141225122

(Edit: I see now that using Re<>Im or the I function puts the calculator into complex mode without any flag manipulation. I wonder if the unaware user might then be stuck in complex mode forever? I must read more manual.)

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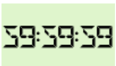
16th December, 2019, 14:31 **Post: #34**



Werner
Senior Member

Posts: 777
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

> **Valentin Albillo Wrote:** (14th December, 2019 02:56)
>
> Code for the **HP-71B**, where **FNROOT** *(Find Root)* is the *"official" HP-71B's "Solver"*:
>
> 1 DEF FNF(X) = **FNROOT**(1, 1, **GAMMA**(FVAR) - X)
> 2 DISP **FNROOT**(3, 4, FNF(SQR(FVAR)) - 1/2)
>
> >RUN
>
> **3.14159265359**
>
> This produces as a root the theoretically *exact* (not an approximation) value of Pi.
>
> V.

Hi, Valentin!
Why not just use GAMMA(0.5) - SQRT(X) = 0 as the generating equation?
I don't own a 71, but then it ought to be something like
DISP **FNROOT**(3, 4, GAMMA(1/2) - SQR(FVAR))

Cheers, Werner

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16th December, 2019, 23:54 **Post: #35**

Moggul
Member

Posts: 68
Joined: Jun 2019

RE: Can you calculate Pi using a Solver?

I tried this method on the 32S and worked perfectly. However when I tried it on the 32SII the result is way off. Is the programming that different on both machines?

> **J-F Garnier Wrote:** (15th December, 2019 12:39)
>
> > **EdS2 Wrote:** (14th December, 2019 12:51)
> >
> > I had hopes that Euler's equation would serve - if we could take the magnitude, or the real value, of e^(ix)+1, then we have a (complex) function which takes a real to a real, and the job would be done. But the HP-35s lacks the capability.
>
> Following your suggestion, here is a solution of Euler's equation on the HP32S (I like this little machine:-):
>
> LBL E
> INPUT X
> RCL X
> 0
> CMPLXe^X
> 1
> +
> x^2
> x<>y
> x^2
> +
> RTN
>
> Then:
> FN= E
> SOLVE X
>
> The 32S displays "NO ROOT FND" but the best guess 3.14159265359 is in X.
>
> Anyhow, for me it's a complicated way to just solve COS(X)=-1.
>
> J-F

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17th December, 2019, 03:22 **Post: #36**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Can you calculate Pi using a Solver?

.
Hi, **Werner**:

> **Werner Wrote:** (16th December, 2019 14:31)
>
> > **Valentin Albillo Wrote:** (14th December, 2019 02:56)
> >
> > Code for the **HP-71B**, where **FNROOT** *(Find Root)* is the *"official" HP-71B's "Solver"*:
> >
> > ```
> > DEF FNF(X) = FNROOT(1, 1, GAMMA(FVAR) - X)
> > DISP FNROOT(3, 4, FNF(SQR(FVAR)) - 1/2)
> > ```
> >
> > >RUN
> >
> > **3.14159265359**
> >
> > This produces as a root the theoretically *exact* (not an approximation) value of Pi.
> >
> > V.
>
> Hi, Valentin!

> **Why not just use GAMMA(0.5) - SQRT(X) = 0 as the generating equation?**
> I don't own a 71, but then it ought to be something like
> DISP **FNROOT**(3, 4, GAMMA(1/2) - SQR(FVAR))

Because that's not much use of a Solver as the OP intended it, i.e.: to solve some equation which isn't *explicitly* solvable (i.e: the variable can't be isolated, e.g.: cos(x)-x=0), doesn't involve trigs, and of course doesn't feature Pi explicitly in the equation.

Your proposed equation fails in those regards. For starters, *Gamma(1/2)* is a ***constant***, namely **√Pi**. You can call it *"Gamma(1/2)"* or you can call it *"Pepe"* but it's still just *√Pi*.

So, your equation becomes:

√Pi - SQRT(X) = 0 → √x = √Pi → x = Pi

and not only does it have the variable x *isolated* and Pi *included* in the equation but it's also as utterly trivial as it gets, and probably this is not what the OP was asking for.

Anyway, thanks for your question and most of all, for your interest.

And if you don't own an *HP-71B* but would like to, there are at least two excellent, free *Emu71* out there (running in *MS-DOS*, *DOS* console in *Win*/16/32, or full-GUI *Win*/16/32/64 or under *DOSEMU* in any operating system, *Android* included and even some electronic-ink eBook readers) which you can download. For free.

Best regards.
V.

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17th December, 2019, 09:42 (This post was last modified: 17th December, 2019 10:12 by Stevetuc.) **Post: #37**

Stevetuc
Senior Member

Posts: 311
Joined: Jan 2014

RE: Can you calculate Pi using a Solver?

> **Valentin Albillo Wrote:** (14th December, 2019 02:56)
>
> .
> Self-quoting a little:
>
> > **Valentin Albillo Wrote:** (14th December, 2019 01:30)
> > Unless the *Solver* can work with complex numbers **I don't think there's such an equation using just real numbers and no trigs**. Equations whose roots are arbitrarily accurate approximations to Pi (say 34 digits) can be produced but ones returning *exactly* Pi (in theory, limited to 10-12 digits in practice) is a no-go IMHO.
>
> Well, to make myself clearer, the previous quotation refers to polynomials or rational functions, which indeed can't produce Pi as an exact root because Pi is transcendental.
>
> But if we involve other functions but polynomials, rational functions and trigs, there are many ways to concoct an equation whose root is exactly Pi. For instance, if your *Solver* admits nesting and can use the ***Gamma*** function, then this will do:
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> Code for the **HP-71B**, where **FNROOT** *(Find Root)* is the *"official" HP-71B's "Solver"*:
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> 1  DEF FNF(X) = FNROOT(1, 1, GAMMA(FVAR) - X)
> 2  DISP FNROOT(3, 4, FNF(SQR(FVAR)) - 1/2)
> ```
>
> >RUN
>
> **3.14159265359**
>
> This produces as a root the theoretically *exact* (not an approximation) value of Pi.
>
> V.

Tried this on the prime but it appears that the solver doesnt support recursion.
I created a thread on prime forum to followup on this https://www.hpmuseum.org/forum/thread-14184.html

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17th December, 2019, 10:01 (This post was last modified: 17th December, 2019 10:15 by J-F Garnier.) **Post: #38**

J-F Garnier
Senior Member

Posts: 820
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

> **Moggul Wrote:** (16th December, 2019 23:54)
> I tried this method on the 32S and worked perfectly. However when I tried it on the 32SII the result is way off. Is the programming that different on both machines?

The solver will find one of the many solutions pi+(2*pi)*n, depending on the initial guess. Try setting the X variable (and x register) to zero first.

I'm pretty sure that there is no difference in the 32S and 32SII solvers, if there are I'm very interested to know them!

J-F

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17th December, 2019, 11:36 (This post was last modified: 17th December, 2019 12:19 by EdS2.) **Post: #39**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

Interesting - my 15C did a lot better finding 3pi given a starting range of 9 to 12. (Edit: compared to finding pi)

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7th January, 2020, 20:37 (This post was last modified: 7th January, 2020 20:40 by EdS2.) **Post: #40**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

> **EdS2 Wrote:** (16th December, 2019 13:44)
> ... Euler's identity...

As a slight digression, in related explorations on the web I landed on a marvellous article by Alon Amit explaining nicely the nature of and the linkage between pi and e:

“So here, again, is the exponential function, stemming from its unique property as the fixed point of the operator d/dx. And here, again, is our friend pi, always accompanying its master of which it is the period (times 2i). And this, finally, is where pi comes from, and what it is, and there are no circles to be found.”

What is pi? also available here and in archived form here.

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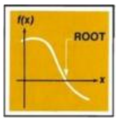
9th January, 2020, 14:10 (This post was last modified: 9th January, 2020 14:14 by Gamo.) **Post: #41**



Gamo
Senior Member

Posts: 719
Joined: Dec 2016

RE: Can you calculate Pi using a Solver?

HP-15C have a SOLVE function but I find the better way
to find the estimated value of Pi just by using the GAMMA function.

Formula: [Γ(1/2)]^2 = Pi

To find the estimated value of Pi follow this keystroke steps:

[.] 5 [ENTER] 1 [-] display -0.5 [x!] display 1.772453851 [x^2]

display 3.141592654

Answer is the same from the built-in Pi function.

Gamo

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10th January, 2020, 19:25 (This post was last modified: 10th January, 2020 23:19 by Namir.) **Post: #42**

Namir
Senior Member

Posts: 887
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

Using the series developed by the Chudnovsky formula published in 1987, I can calculate pi to 14 digits using just ONE TERM!:

1/pi=12/640320^1.5*13591409
pi = 640320^1.5/(12*13591409)

Namir

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11th January, 2020, 02:21 **Post: #43**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Can you calculate Pi using a Solver?

> **Namir Wrote:** (10th January, 2020 19:25)
>
> Using the series developed by the Chudnovsky formula published in 1987, I can calculate pi to 14 digits using just ONE TERM!: [...]
> **pi = 640320^1.5/(12*13591409)**
>
> Namir

Your expression includes **640320**, **1.5**, **12** and **13591409**, which means you're using **18** digits to get just **14** digits of *Pi*.

That's *highly inefficient*: using 18 digits to output 14 digits => 18-14 = **-4 digits** "gained" (lost, more like).

Far better woould be:

3*Ln(**640320**)/√**163** = *3,1415926535897930+*

which gives **17** digits (save 2 ulps) while using just **10** digits, i.e., 17-10 = **+7 digits** gained.

Besides, nothing of this has anything to do with getting *Pi* using a *Solver*, as the OP requested.

V.

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11th January, 2020, 02:34 (This post was last modified: 11th January, 2020 02:39 by Namir.) **Post: #44**

Namir
Senior Member

Posts: 887
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

> **Valentin Albillo Wrote:** (11th January, 2020 02:21)
>
> > **Namir Wrote:** (10th January, 2020 19:25)
> >
> > Using the series developed by the Chudnovsky formula published in 1987, I can calculate pi to 14 digits using just ONE TERM!: [...]
> > **pi = 640320^1.5/(12*13591409)**
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> > Namir
>
> Your expression includes **640320**, **1.5**, **12** and **13591409**, which means you're using **18** digits to get just **14** digits of *Pi*.
>
> That's *highly inefficient*: using 18 digits to output 14 digits => 18-14 = **-4 digits** "gained" (lost, more like).
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> Far better woould be:

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>
> Besides, nothing of this has anything to do with getting *Pi* using a *Solver*, as the OP requested.
>
> V.

Thanks for your version. I don't see the logic in using Solve to calculate pi. Curiosity to use Solve? Maybe? Using recursive formulas or even integrals comes across as more sensical.

Your version, based on a single-term of the the Chudnovsky formula, leaves 355/113 in the dust!!!

Namir

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11th January, 2020, 03:01 **Post: #45**

Valentin Albillo
Senior Member

Posts: 970
Joined: Feb 2015
Warning Level: 0%

RE: Can you calculate Pi using a Solver?

> **Namir Wrote:** (11th January, 2020 02:34)
>
> > **Valentin Albillo Wrote:** (11th January, 2020 02:21)
> >
> > Far better woould be:
> >
> > **3***Ln(**640320**)/√**163** = *3,1415926535897930+*
> >
> > which gives **17** digits (save 2 ulps) while using just **10** digits, i.e., 17-10 = **+7 digits** gained.
> >
> > Besides, nothing of this has anything to do with getting *Pi* using a *Solver*, as the OP requested.
>
> Thanks for your version. **I don't see the logic in using Solve to calculate pi**. Curiosity to use Solve? Maybe? Using recursive formulas or even integrals comes across as more sensical.

It's not a question of *"logic"* or of being *"more sensical"*. The OP simply was curious to know if it could be done using a Solver so posted it as a kind of "challenge", nothing else.

> **Quote:**
>
> **Your version, based on a single-term of the the Chudnovsky formula**, leaves 355/113 in the dust!!!

My version **isn't** *"based on a single-term of the Chudnovsky formula"* as yours is; actually it's based on the ***Ramanujan's constant***, i.e.: cf. *Wikipedia*:

*"Ramanujan's constant is the transcendental number e^(Pi*sqrt(163)), which is an almost integer, in that it is very close to an integer: 262,537,412,640,768,743.99999999999925... , approximately equal to 640,320^3+744. [...] This coincidence is explained by complex multiplication and the q-expansion of the j-invariant."*

V.

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11th January, 2020, 11:11 (This post was last modified: 11th January, 2020 11:17 by EdS2.) **Post: #46**

EdS2
Senior Member

Posts: 525
Joined: Apr 2014

RE: Can you calculate Pi using a Solver?

> **Valentin Albillo Wrote:** (11th January, 2020 02:21)
>
> **3***Ln(**640320**)/√**163** = *3,1415926535897930+*
>
> which gives **17** digits (save 2 ulps) while using just **10** digits, i.e., 17-10 = **+7 digits** gained.

Also good value, also by Ramanujan:
√√(9^2 + 19^2/22) = 3.14159265258...
(via a commenter on an article by David Bau)

Also can be written as
√√(2143/22) = 3.14159265258...
for slightly better value.

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11th January, 2020, 11:49 **Post: #47**

Paul Dale
Senior Member

Posts: 1,787
Joined: Dec 2013

RE: Can you calculate Pi using a Solver?

The Wikipedia article for Ramanujan's constant claims transcendental with a reference to the Mathsworld article with only claims irrationality.

Which is correct?

Irrationality is easy to believe. Transcendence is not much harder (albeit *much* harder to prove).

Pauli

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