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Hyperbolics with the Advantage

Message #1 Posted by [Ángel Martin](#) on 28 Nov 2003, 8:55 a.m.

Who said the Advantage ROM doesn't have the Hyperbolic functions?

I suspected this couldn't be true, as it does have programs to calculate SINZ and COSZ, the complex SIN and COS. These can be expressed as:

$$\text{SINZ} = \text{SIN}x \text{COSH}y + i \text{COS}x \text{SINH}y$$

$$\text{COSZ} = \text{COS}x \text{COSH}y + i \text{SIN}x \text{SINH}y$$

And as a matter of fact, it sure does have them, all it's lacking are their global labels! Try this:

SINH - program line 129 of "Z^N", starts w/ "LBL 10"

COSH - program line 140 of "Z^N", starts w/ "LBL 11"

TANH - well, just do SINH/COSH

I have no idea why they omitted the global labels, as there's still available entries in the FAT. IT's not MCODE (like the ones in the AECROM), but certainly well worth the free ride!!

Now, is this a sad story or what?

Best, ÁM.

Re: Hyperbolics with the Advantage

Message #2 Posted by [Karl Schneider](#) on 30 Nov 2003, 5:33 p.m.,
in response to message #1 by [Ángel Martin](#)

Angel --

Insightful finding! I'm sure it has something to do with the new ROM you are apparently developing. I would be interested in that one.

You pointed out that SINH, COSH, and TANH (for real arguments only) were not included in the Advantage ROM even though most of the code, by necessity for SINZ and COSZ, was already in place.

The reason for the exclusion, I suspect, is that a "complete" implementation of hyperbolics would have also included ASINH, ACOSH, and ATANH. That may have been a bit too much, especially considering that the Advantage ROM is already 12 kB.

In fact, comparing the "Z^N" routines of the Advantage with the Math ROM, we find that lines 1-128 are identical except for the following:

	Math ROM	Advantage ROM
lines 81, 96, 119	XROM 'SINH	XEQ 10
lines 86, 102, 116	XROM 'COSH	XEQ 11

This is consistent with what you discovered.

It should be pointed out that the added transcendental-function mathematics of either ROM is by no means complete. Inverse (arc) functions of the trig/hyperbolics for complex-valued arguments are quite a bit more involved; these were not provided by HP for the 41 series. Such functions are available in HP calculators as m-coded routines in the 15C, 42S, and RPL-based models. (The 71B Math ROM might also have these; I don't know about the 38G/39G and other more-recent models).

To illustrate, here is a table for trigonometric and hyperbolic sine (the same applies for cosine and tangent):

Function	HP-41	Adv ROM	Math ROM	15C/42S/RPL
SIN	x			x
ASIN	x			x
SINZ		x	x	x
ASINZ				x
SINH		-	x	x
ASINH			x	x
SINHZ				x
ASINHZ				x

"x" = available; "-" = code is present

Makes you appreciate the sophistication of the 1982 HP-15C, doesn't it?

Best,

-- Karl S.

Edited: 30 Nov 2003, 5:41 p.m.

Re: Hyperbolics with the Advantage

Message #3 Posted by **Valentin Albillo** on 30 Nov 2003, 7:35 p.m.,
in response to message #2 by Karl Schneider

Hi, Karl:

Karl posted:

"It should be pointed out that the added transcendental-function mathematics of either ROM is by no means complete. Inverse (arc) functions of the trig/hyperbolics for complex-valued arguments [...] were not provided by HP for the 41 series [...] (The 71B Math ROM might also have these)"

No. Unfortunately, the HP-71B Math ROM does provide a lot of complex number handling functionality, but inverse trigonometric or hyperbolic functions aren't extended to complex arguments, which is a real pity as it leaves the 'complex' function set sorely incomplete.

Of course these can be expressed if necessary as user-defined functions, in terms of exponentials and logarithms (which do admit complex arguments) but this is most inconvenient. The Gamma function isn't extended to complex arguments as well and it's not so easy to implement that yourself.

These are some of the very few shortcomings this wonderful ROM suffers. Another (IMHO) is the fact that PROOT, the awesome all-roots-at-once polynomial root finder, does not work for polynomials with complex coefficients, and it's a real pity, too. Doing it yourself is a bit more complicated in this case.

Best regards from V.

Re: Hyperbolics with the Advantage

Message #4 Posted by **Gene** on 30 Nov 2003, 9:36 p.m.,
in response to message #3 by Valentin Albillo

Have to ask...

give me an example of something that uses complex hyperbolics. I'm just exposing my lack of knowledge on this one, but I'm curious! :-)

Thanks Valentin!

Gene

Re: Hyperbolics with the Advantage

Message #5 Posted by [Valentin Albillo](#) on 1 Dec 2003, 4:34 a.m.,
in response to message #4 by Gene

Hi, Gene:

Gene posted:

"give me an example of something that uses complex hyperbolics."

They're used all the time in structural engineering, Gene, since the formula for the shape (called "catenary") of a weighty chain freely hanging from two separate points happens to be an hyperbolic cosine, and most suspension bridges and other such superstructures feature catenaries all the time. See first link below.

This also applies not only to macro-scale, plain-vanilla bridges, but also to nano-scale bridges in superconducting materials, which also happen to be hyperbolic cosines, see second link below.

Also, some cultures use shapes defined by hyperbolic functions in their civil engineering not only for practical reasons, but for spiritual and artistic reasons as well, specially for shrines and temples. The exponential growth of an hyperbolic cosine is felt as much more uprising and tending towards "higher spirituality" than a simple, quadratic parabola, so Japanese temples feature spikes and elevations with hyperbolic shapes. Have a look at the very revealing third link, which makes it clear that people can easily notice that something's not right with a mere parabola, despite initial appearances.

Finally, the fourth link will tell you why you need to apply an "hyperbolic cosine" coating to critical parts of supersonic fighter aircrafts at Boeing.

[Superstructure Engineering](#)

[Superconducting fluctuations ...](#)

[Curves in traditional Japanese architecture and Civil Engineering](#)

[Surface coating for supersonic fighter aircraft](#)

In case you're wondering about the use of hyperbolic functions with **complex** (i.e. "imaginary") arguments, the applications are the same, see these references for instance (unfortunately they don't seem to be available in the web, only as printed publications):

Nsugbe EAO & Williams CJK (1999). The Use of the Inverse Hyperbolic Cosine Function of a Complex Variable in Defining a Bridge Geometry. In Astudillo R & Madrid AJ (eds.) Proc. IASS 40th Aniversary Congress, 2: H21-28. CEDEX Sección de Edición, Madrid.

The generation of bone-like forms using analytic functions of a complex variable. E.A.O. Nsugbe, C.J.K. Williams, Journal of Computers and Structures, Jan 1998

[they demonstrated their ideas by actually building a bridge in bone-like shape, the shape being generated by using inverse hyperbolic cosine functions with complex arguments]

There are many more applications, but I hope the above ones will serve you well.

Best regards from V.

Edited: 1 Dec 2003, 6:06 a.m.

Re: Hyperbolics with the Advantage

*Message #6 Posted by **Gene** on 1 Dec 2003, 7:35 a.m.,
in response to message #5 by Valentin Albillo*

Thanks. I'll look them over.

I knew what hyperbolics were used for (I wrote the 49G+ Using Hyperbolics training aid), but had no idea about an application using hyperbolics with complex arguments - that was really my question.

Thanks again! Gene

Analyticals w/complex: Math ROM & 32S/ii

*Message #7 Posted by **Karl Schneider** on 1 Dec 2003, 9:38 p.m.,
in response to message #5 by Valentin Albillo*

Hello, Valentin --

Thanks for answering Gene's question; I was wondering the same thing, having never encountered a problem requiring evaluation of trigs or hyperbolics (or their inverses) with complex-valued arguments.

Yet another gem for a future compilation of the MoHPC "AlbilloPost" archives!

A more common example of analytic functions with complex variables is $y(t) = A_0 * \exp(a + j\omega)t$ -- an expression of a damped waveform for signal processing and power system analysis.

There is a close similarity between the m-coded complex-value functionality of the 32S/ii and the RPN routines of the 41 Math ROM. Both lack support for the "arc" functions. In the case of the Math ROM, I suspect that the complexity of the equations precluded RPN routines called, e.g., "ASINZ" and "ASINHZ". However, for the 32S/ii, lack of common practical use for such functions must have been the primary reason for their exclusion.

Or, maybe it was the unpalatable

[lshift][cplx][lshift][hyp][lshift][asin]

; -)

-- Karl

Re: Analyticals w/complex: Math ROM & 32S/ii

*Message #8 Posted by [Angel Martin](#) on 2 Dec 2003, 1:30 a.m.,
in response to message #7 by Karl Schneider*

Karl posted:

"Or, maybe it was the unpalatable

[lshift][cplx][lshift][hyp][lshift][asin]"

There it is! The living proof that the user interface for the 32/42/etc is faulty! Compare that to:

XEQ "ASINHZ", or just "ASINHZ" once assigned to a key.

Which one is easier/better/faster/simpler to rememeber?

Now we only need the function underneath :-)

Best, AM

"Leave the 42s out of it"

*Message #9 Posted by **Ron Ross** on 2 Dec 2003, 9:15 a.m.,
in response to message #8 by Angel Martin*

With custom soft menu keys, the 42s can just put these functions on the top row always, if they happen to be used often. Result, single button push, just like your 41.

Re: "Leave the 42s out of it"

*Message #10 Posted by **Angel Martin** on 2 Dec 2003, 3:32 p.m.,
in response to message #9 by Ron Ross*

That almost sounds like a warning, but I will take it as a friendly advise, thanks.

Best, AM

"No, I'm pyschotic."

*Message #11 Posted by **Ron Ross** on 2 Dec 2003, 4:07 p.m.,
in response to message #10 by Angel Martin*

I just have to remind myself, they are just calculators. "NO! they are NOT!" "YES, they ARE!" "NO!" "YES!", Well you get the picture.

8o)

Sorry for the terse statement.

Re: "No, I'm pyschotic."

*Message #12 Posted by **Angel Martin** on 3 Dec 2003, 3:59 a.m.,
in response to message #11 by Ron Ross*

Ron, I do get the picture... and BTW I also own a 42S that I enjoy too, as a good machine with clear advantages and disadvantages, like any other model.

However, having spent a previous life designing user interfaces in graphical environments, I'm naturally biased against soft-keys. Call it my personal problem if you want :-)

"Better" is after all, a matter of taste, costums and what we get proficient at in the first place.

Best, ÁM

Re: Analytics w/complex on ROM for HP-41

Message #13 Posted by [Karl Schneider](#) on 3 Dec 2003, 12:20 a.m.,
in response to message #8 by Angel Martin

Angel Martin posted:

Quote:

Now we only need the function underneath :-)

Seriously, implementing inverse functions for trigonometrics and hyperbolics with complex-valued arguments is non-trivial. The basic equations can be found in Schaum's Outline "Mathematical Handbook" or other reference. However, a proper implementation would require much exception coding, for the many rules and special handling to determine which answer to provide.

Also, Ron is right that the 42S user interface is better, despite the tedious entering of non-numeric characters. (The idea is that manually entering names should be infrequently necessary.)

Re: Analyticals w/complex: Math ROM & 32S/ii

Message #14 Posted by [Valentin Albillo](#) on 3 Dec 2003, 9:50 a.m.,
in response to message #7 by Karl Schneider

Hi, Karl:

Karl posted:

"Yet another gem for a future compilation of the MoHPC "AlbilloPost" archives!"

Thak you very much for your extremely kind (if undeserved) words. If you like my posts, I would humbly and heartily recommend that you don't miss this one:

[SHARP vs. HP comparisons](#)

It's certainly one of the 'juiciest' ones, and full of interesting links to materials and pictures. Hope you enjoy it.

"A more common example of analytic functions with complex variables is $y(t) = A_0 * \exp(a + jw)t$ -- an expression of a damped waveform for signal processing and power system analysis."

Actually, as you're probably well aware, trigonometric and hyperbolic functions are nothing else but combinations of complex exponentials similar to your function, and their inverses are nothing but inverse exponentials, i.e., logarithms. I remember seeing some graphs of iterated sines for complex arguments in Mathematica, and they were awesome, featuring fractal-like regions and such, due to their essentially 'iterated exponential' nature (despite their being nominally 'sines').

"Or, maybe it was the unpalatable [lshift][cplx][lshift][hyp][lshift][asin]"

The HP-15C version is simply [g][HYP-1][SIN], i.e. three less keystrokes, which is certainly more palatable.

Best regards from V.

Re: Analyticals w/complex: Math ROM & 32S/ii

*Message #15 Posted by **Nelson M. Sicuro (Brazil)** on 3 Dec 2003, 10:03 a.m.,
in response to message #14 by Valentin Albillo*

Hi, Valentin

Is there a source of this formulas? I'm interested in learn how do you calculate all those functions with complex numbers (and the non-complex too). I'm trying to gather as much information as I can get to build a good calculator.

Best regards,

Nelson

(P.S. I'm NOT a mathematician, I'm a computer programmer that likes mathematics (and know a little about it) but I just love calculators... My dream is to build a calculator of the size of the 15C with the functions of the 42S and some I/O like the Sharp ones... I'm creating an emulated CPU that have some facilities to work with math)

Re: Analyticals w/complex: Math ROM & 32S/ii

*Message #16 Posted by **Valentin Albillo** on 3 Dec 2003, 1:17 p.m.,
in response to message #15 by Nelson M. Sicuro (Brazil)*

Hi, Nelson:

Nelson posted:

"Is there a source of this formulas? I'm interested in learn how do you calculate all those functions with complex numbers (and the non-complex too)."

Have a look at these:

[Hyperbolics \(i\)](#)

[Hyperbolics \(ii\)](#)

[Hyperbolics \(iii\)](#)

[Hyperbolics \(iv\)](#)

[Hyperbolics \(v\)](#)

[Hyperbolics \(vi\)](#)

[Hyperbolics \(vii\)](#)

Hopefully, this will be what you wanted. Else, let me know. Best of lucks with your interesting ongoing project, and

Best regards from V.

Re: Analyticals w/complex: Math ROM & 32S/ii

*Message #17 Posted by [Nelson M. Sicuro \(Brazil\)](#) on 3 Dec 2003, 4:57 p.m.,
in response to message #16 by Valentin Albillo*

Thanks! It'll keep me busy for some time...

Best regards,

Nelson

Re: Hyperbolics with the Advantage

Message #18 Posted by [Ángel Martin](#) on 1 Dec 2003, 3:40 a.m.,

in response to message #2 by Karl Schneider

Karl,

Yes, it really makes you appreciate the 15C's virtues, way ahead of its time indeed in the Math department...

You're correct suspecting this all has to do with my work on the SANDBOX-II, an 8k extension to the original one. I have extended the Math functions section, incorporating the Real Hyperbolics functions (present in the AECROM), plus Complex Arithmetic ($Z+$, $Z-$, Z^* , $Z/$, $1/Z$), plus a Quadratic Equation Roots Solver (QROOT) also posted on this forum). They are fast (MCODE) and simple enough to use!

Space limitation prevents me from adding more of those, although the project isn't finished yet (almost there). I could trade a few housekeeping routines for a 'real' math function, but which one(s)??? All suggestions are welcome.

Best, ÁM

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