♥MoHPC♠ *The Museum of HP Calculators*

HP Forum Archive 13

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New HP Calculators

*Message #1 Posted by **Richard Garner** on 9 July 2003, 7:42 p.m.*

This was posted on com.sys.hp48 today. It is an excel file with new calulators for 2003 for several producers including HP.

http://www.dstewart.com/newdsc/c2es_03_orderbook/excel/Calculators_2003.XLS

Any comments?

HP33S, HP39G+, HP48GII and HP49G+ ...

*Message #2 Posted by **Vieira, Luiz C. (Brazil)** on 9 July 2003, 11:41 p.m.,*
in response to message #1 by Richard Garner

Hi, Richard;

thank you for sharing the information.

I saw these two references that called my attetnion: the HP33S, as a new release, and the three vage references: HP39G+, HP49G+ (more memory?) and what actualy called my attention: the HP48GII.

What would these be?

Weird...

Luiz (Brazil)

Re: New HP Calculators

*Message #3 Posted by **Patrick** on 10 July 2003, 12:10 a.m.,*
in response to message #1 by Richard Garner

You try to stay firm, you try not to hope too much, and then you see *this*! How are we good folk supposed to maintain our skepticism when such a credible looking document is presented??

The supposed MSRP of the 33S suggests that this is no Kinpo special.

See? I'm doing it again... you can see real *hope* in that last sentence. Darn, darn, darn.

What does MSRP stand for?

Message #4 Posted by ***Vieira, Luiz C. (Brazil)*** *on 10 July 2003, 1:19 a.m.,*
in response to message #3 by Patrick

Hi, Patrick;

actualy, I do not know. Brand reference?

Cheers.

Luiz (Brazil)

Manufacturer's Suggested Retail Price <nt>

Message #5 Posted by ***Patrick*** *on 10 July 2003, 3:01 a.m.,*
in response to message #4 by Vieira, Luiz C. (Brazil)

Re: New HP Calculators

Message #6 Posted by ***Scuba Diver*** *on 10 July 2003, 8:46 a.m.,*
in response to message #3 by Patrick

Well, the 17BII + and 19BII + caught my eye.

I wonder what features they will posess? I find it hard to believe that they could improve on either of those calcs, save adding trig to the 17...

Maybe they added some newer bond calculations or something...more memory (not that they needed it...most 19BII users never even use the memory outside of the registers!)

B.

HP 19BII Memory

Message #7 Posted by ***Thibaut.be*** *on 10 July 2003, 8:54 a.m.,*
in response to message #6 by Scuba Diver

Well, key in your contact list and your RAM will not support it...

A 128K RAM would be more convenient UMHO

Re: HP 19BII Memory

Message #8 Posted by ***Scuba Diver*** *on 10 July 2003, 12:04 p.m.,*
in response to message #7 by Thibaut.be

True, although with Palms and Blackberries that sync up to your Outlook/Lotus Notes calendar, very few people use it for appointments anymore.

The fact that this calculator was a PDA yeaars before Palm shows just how visionary HP was!!!

B.

Re: New HP Calculators

Message #9 Posted by ***Justin*** *on 11 July 2003, 6:25 a.m.,*
in response to message #6 by Scuba Diver

I wonder what features they will posess? I find it hard to believe that they could improve on either of those calcs, save adding trig to the 17...

Not that I think they'll do it, but my dream would be PC connectivity so Solver equations could be typed in/proofed on a computer and then d/l'd to the calc.

To 33S beta testers

Message #10 Posted by ***Patrick*** *on 10 July 2003, 4:23 p.m.,*
in response to message #1 by Richard Garner

No, I don't expect you to identify yourselves (if you're even out there) nor break your confidentiality agreement. I just would like you to check something for me...

What is the calculator's answer to 0^0?

It would be great if the answer was 1 and not "Error", like it is on the 11C. I mean, 0! is calculated correctly as 1, why not 0^0?

Re: 0! vs. 0^0

Message #11 Posted by ***Paul Brogger*** *on 10 July 2003, 7:25 p.m.,*
in response to message #10 by Patrick

If I remember right, 0! is not *calculated as* 1, but is *agreed to be* 1. Didn't our mathematical forefathers decide it should evaluate to 1 just to keep the curve smooth?

However, 1^0=1.00, -1^0=1.00, but yes, also on my 34c, 0^0 = "Error 0".

And, 0^1=**0.00**?!?!? In a politician, such inconsistency would provoke outrage!

Calculated vs Agreed

Message #12 Posted by ***Patrick*** *on 10 July 2003, 7:58 p.m.,*
in response to message #11 by Paul Brogger

In my professional opinion, I would have to challenge you on your distinction of *calculated* vs *agreed*. The definition of any function at any point is something that is *agreed* upon by the scientific community. Whether or not there is an arithmetic or other *calculation* involved is just a particular kind of evaluation algorithm.

So, one *definition* of the factorial function consistent with the scientific literature is:

```
0! = 1
n! = n * (n-1)! for n an integer >= 1.
```

Likewise, the definition of y^x includes certain special cases:

```
0^x = 0 for any x != 0
x^0 = 1 for all x (including zero)
```

There are good reasons for defining 0^0 to be 1 which I believe have to do with the continuity of the exponental function on the complex plane.

I'm surprised Mr. Kahan, with his careful attention to detail in the complexities of the SOLVE and INTEGRATE functions of the 34C, and in all of its algorithms in general, let this one sneak by.

Re: 0! vs. 0^0

Message #13 Posted by ***Definition of n! for 0! = 1*** *on 11 July 2003, 4:05 p.m.,*
in response to message #11 by Paul Brogger

A definition of the factorial function

n! = product(k) for k =1 to k greater than or equal to n

(My symbols did not survive the posting process.)

Note that if n = 3, we start the product with k = 1 and go up to k =3. n! = 6

If n = 0, we start the product with k = 1. At this point k is greater than n so we stop with the product equal to one. Thus 0! = 1 without treating 0 as a special case.

It is a natural definition of the function as it is usually used to determine the number of possible ways to order a set of n objects. If n = 0 there is one way to order the set. This assumes that whether 0 is considered to be a number or not, it is an object in that it represents the empty set or at least a place holder.

Re: To 33S beta testers

Message #14 Posted by ***Ernie Malaga*** *on 10 July 2003, 9:36 p.m.,*
in response to message #10 by Patrick

Patrick:

0^0 is undefined by definition -- like 0/0 and 1 raised to infinity. In general, x^0 is 1 because 0 = n - n, so x^(n - n) = x^n/x^n = 1. But if x is 0, the denominator would be 0^n, which is 0, and you cannot divide by zero.

I am not certain if this is the correct explanation for the undefinability of 0^0, but it's a good one anyway. If your calculator reports "Error" when you attempt 0 [ENTER] 0 [y^x], the calculator is only giving you the mathematically correct answer.

Other undefined expressions are ln(x) for x <= 0, tan(90), etc.

-Ernie

0^0

Message #15 Posted by ***Patrick*** *on 10 July 2003, 11:26 p.m.,*
in response to message #14 by Ernie Malaga

I have to disagree with you, Ernie. The mathematical definition of y^x for y>0 is exp(x ln(y)). The ln(x) function is defined as the integral from 1 to x of 1/t dt and the exp(x) function is defined as the inverse of ln(x).

Since the definition does not provide a value when y is zero, we are free to define it as we like. The natural definition of 0^x for x non-zero is zero. This leaves the hole at x=0, y=0. To fill that hole, one might try to take the limit as either x or y approaches zero with the other one equal to zero. The result is that there is no unique limit, meaning there is no definition of y^x which makes the function continuous. However, there are good reasons to *define* 0^0 as equal to 1, as it makes a number of other theorems easy to state without having to have an exception at that point. This is what actually has been done. In higher mathematics, we define the value of 0^0 to be equal to 1. The function does not end up to be continuous, of course, it cannot be, but it is the *best* value we can choose for a host of other reasons.

As a reference, I direct you to this article written by the mathematician William H. Kahan, the numerical analyst who consulted with HP on many of their algorithms, including SOLVE and INTEGRATE. You can see on page 10 of that document a definition for the complex power function z^w, which is defined to be equal to 1 whenever w = 0, for all complex z.

Re: 0^0

Message #16 Posted by ***Ernie Malaga*** *on 11 July 2003, 1:22 a.m.,*
in response to message #15 by Patrick

Patrick:

I'll defer to your obviously greater expertise on the matter. From my long-ago college days, however, I vividly remember that my textbooks specified 0^0 as undefined -- every bit as much as 0/0.

I'll try to read the article you mentioned, but I doubt that I'll understand much of it. I flunked integral calculus, barely learning to integrate e^x. 8^)

-Ernie

Re: 0^0

Message #17 Posted by ***Ernie Malaga*** *on 11 July 2003, 1:36 a.m.,*
in response to message #15 by Patrick

Patrick:

I checked page 10 of the document. Unless I'm missing something, it seems to confirm what I said:

Quote:

z^w = exp(w*ln(z)) except z^0 := 1 for all z, and 0^w := 0 if Re(w)>0

Observation 1: There is really no need to state as an exception the case when w=0, since w*ln(z) will be 0 whenever w=0 (no matter what value z may have), and exp(0) = 1 anyway.

Observation 2: "and 0^w :=0 if Re(w)>0" Notice that w is non-zero (otherwise it couldn't have a real portion greater than 0), so the definition given for z^w does _not_ cover the case when both z and w are 0.

If you still disagree with me I'll take your word for it, but that's what I'm reading here.

-Ernie

Re: 0^0

Message #18 Posted by ***Patrick*** *on 11 July 2003, 2:45 a.m.,*
in response to message #17 by Ernie Malaga

Observation 1: You can't use the w*ln(z) argument because ln(z) is undefined for z=0. You can't argue that a multiplication involving zero has to be zero when the other part of the multiplication is undefined!

Observation 2: The definition *does* cover the case when both are zero since it says that z^0 = 1 **for all z**. Surely, the number z=0 would have to be included in *all*, would it not?

0^0 != 0^0 ????? (edited)

Message #19 Posted by ***Vieira, Luiz C. (Brazil)*** *on 11 July 2003, 2:54 a.m.,*
in response to message #17 by Ernie Malaga

Hi Ernie, guys;

There is no reason to discuss this fact. 0^0 is not defined, period. And what Patrick wrote is easily identified as conflicting.

Quote:

Likewise, the definition of y^x includes certain special cases:

```
0^x = 0 for any x != 0
x^0 = 1 for all x (including zero)
```

Unfortunately, *x^0 = 1 for all x (including zero)* is wrong. Zero is not included. Suppose you have y^x and x^y.

If y == x, then x^y == y^x, right?

Based on what you wrote:

0^0 is not defined when you analise 0^x when x = 0; and

0^0 == 1 when you analise x^0 when x = 0.

So, 0^0 != 0^0. Not consistent, isn't it? And that's enough reason to make it undefined.

My 2¢.

Luiz (Brazil)

Edited: 11 July 2003, 3:01 a.m.

Are you sure ?

*Message #20 Posted by **Valentin Albillo** on 11 July 2003, 5:18 a.m.,*
in response to message #19 by Vieira, Luiz C. (Brazil)

Luiz wrote:*"There is no reason to discuss this fact. 0^0 is not defined, period."*

Are you sure ? It's certainly rare to see such a nice, *suave* person like you uttering such a bold, intransigent statement. I usually think that absolutely everything can be discussed, and that denying that possibility is akin to fundamentalism (i.e: "There is no reason to discuss the existence (or non-existence) of God (say). God (say) exists (or doesn't), period." Get the point ?

Also, I tend to think that mathematics aren't invented, but *discovered*, have an existence of their own, independent of human beings or a physical universe existing or nor, and thus tend to be rather impervious to anyone's attempt to "define" or not what they should do. For the case in discussion, x^x when x is 0, look at these results:

```
     x            x^x
--------------------------
0.1000000000  0.7943282347
0.0100000000  0.9549925860
0.0010000000  0.9931160484
0.0001000000  0.9990793900
0.0000100000  0.9998848774
```

```
0.0000010000  0.9999861846
0.0000001000  0.9999983882
0.0000000100  0.9999998158
0.0000000010  0.9999999793
0.0000000001  0.9999999977
```

Does that ring a bell ? Of course, one can argue that when x is *exactly* zero, an indefinition occurs. That's debatable but if it were true, so what ? Any calculation will show the values merrily going to 1 as x goes to 0, and if some mathematician's committee were to *define* that 0^0 = Pi, it would do them no good, for it would generate contradictions at once. If you are *forced* to accept a definite value to avoid contradictions, you're not *defining* anything, you're merely trying to save your face on account of the evidence.

As a further example, consider a function defined thus:

```
f(x) = x / x
```

Now, you evaluate it for all finite arguments, and find that f(x) is always 1. Then you try to evaluate it for x=0 and you claim that 0/0 is undefined, and so assigning the value of "1" to f(0) is a matter of *definition* ... Come on !! Common sense shouldn't be incompatible with mathematics !!

Best regards from V.

O.K.: I started my arguments wrongly...

Message #21 Posted by ***Vieira, Luiz C. (Brazil)*** *on 11 July 2003, 6:21 a.m.,*
in response to message #20 by Valentin Albillo

Thank you , Valentin;

I'd remove the first two sentences in my post and convert it to an open discussion and turn it into my view of the situation: I see as not defined based on what I wrote.

Back to the arena: B^}

Math approaches are (always?) a matter of discussion and definition. And I teach that...

Thanks.

Luiz (Brazil)

Another reference

Message #22 Posted by ***Vieira, Luiz C. (Brazil)*** *on 11 July 2003, 8:25 a.m.,*
in response to message #21 by Vieira, Luiz C. (Brazil)

I'm not sure this can be done, but I simply extracted part of what's written in the below mentiond site. Particualr attention to the first sentence I set as italic.

> Quote:
>
> ---
>
> Other than the times when we want it to be indeterminate, *0^0 = 1* ***seems to be the most useful choice*** *for 0^0* . This convention allows us to extend definitions in different areas of mathematics that would otherwise require treating 0 as a special case. Notice that 0^0 is a discontinuity of the function f(x,y) = x^y, because no matter what number you assign to 0^0, you can't make x^y continuous at (0,0), since the limit along the line x=0 is 0, and the limit along the line y=0 is 1.
>
> ---

Extracted from:

The Math Forum under 0^0 (zero to zero power)[/link]. This e-address was suggested by Juergen(CH)

Please, if this cannot be posted here, remove password is 12345.

Thanks.

Luiz (Brazil)

Edited: 11 July 2003, 8:27 a.m.

Re: Are you sure ?

Message #23 Posted by ***Ernie Malaga*** *on 11 July 2003, 7:03 a.m.,*
in response to message #20 by Valentin Albillo

Valentin:

Your table of X and X^X values only proves that the _limit_ (not the function itself) is 1 as x approaches zero.

> Quote:
>
> ---
>
> As a further example, consider a function defined thus: f(x) = x / x

> Now, you evaluate it for all finite arguments, and find that f(x) is always 1. Then you try to evaluate it for x=0 and you claim that 0/0 is undefined, and so assigning the value of "1" to f(0) is a matter of definition ... Come on !! Common sense shouldn't be incompatible with mathematics !!

And mathematics shouldn't be incompatible with common sense, either. You can't ignore singularities and discontinuities with special-case definitions and leave it at that. The function f(x) = x/x is a straight line, yes, but it gets interrupted for x=0. "Defining" f(0) as 1 goes against common sense.

-Ernie

[OT] Re: Are you sure ?

*Message #24 Posted by **Valentin Albillo** on 11 July 2003, 8:54 a.m.,*
in response to message #23 by Ernie Malaga

Hi again, Ernesto:

Obviously this discussion could go on a on, and we'll probably end it "agreeing to disagree", but there's one final remark I'd like to bring to your attention: stating that

x / x => 1

for all x, just means (passing the denominator to the right side) that:

x = x

which I'm sure you'll agree it's valid for every conceivable x, be it whatever it may, zero included. That's what the equation and the function are telling us, and trying to see "discontinuities" where there are none, and using "definitions" where none are necessary is just plain silly, a kinf of bureaucratic formalism, that goes against common sense. Also, "defining" that some function has such and such value where no other value is possible at all without inconsistency, is absurd, because if you have no choice, if your are forced, issuing a "definition" is a meaningless act on your part.

That said, don't think I'm blinded to the subleties of mathematical formalism, is just that sometimes I think some of them are pretty stupid, and 0^0 or x/x for that matter just trigger that feeling for me.

On the other hand, I know all too well that you can never prove anything my common sense or intuition alone; logically consistent, formal proofs are always necessary. For instance, assume that you are calculating this infinite sum:

Sum(n = 1, n -> inf, INT(n * tanh(Pi))/10^n)

and the answer you get coincides with 1/81 to 12 decimal places. "How nice!" you think, "I didn't expect this hyperbolic tangent thing to add up to a simple rational fraction as 1/81. Maybe it's just a coincidence ?" and so you duly proceed to compute it to greater accuracy, say 30 decimals.

It still agrees with 1/81 to all 30 decimals. "It's wonderful, let's try that multiprecision package I've got for my 48/49/71 !", you say, and proceed to compute the sum to 100 decimal places. It agress with 1/81. Then to 200 decimal places. It still agrees.

Obviously, common sense would dictate by now that the sum equals exactly 1/81, no doubt about it, right ? 200 decimal places in agreement is more than enough evidence to settle the matter, isn't it ?

Yet common sense would be wrong this time, as the sum does NOT equal 1/81. So much for common sense in mathematics ... :-)

Best regards from V.

Re: [OT] Re: Are you sure ?

Message #25 Posted by ***Ernie Malaga*** *on 11 July 2003, 2:02 p.m.,*
in response to message #24 by Valentin Albillo

Valentin:

> Quote:
>
> Obviously this discussion could go on a on, and we'll probably end it "agreeing to disagree"

Yes, the discussion could go on forever. Which is another way of saying "it's undetermined." If instead of "0^0 = x" it were "2 + 2 = 4" no one would question it and there would be no discussion at all.

It seems, therefore, that we need to agree to disagree.

> Quote:
>
> x / x => 1
>
> for all x, just means (passing the denominator to the right side) that:

x = x

Except for x=0, of course, since you cannot divide by zero.

-Ernie

Re: Are you sure ?

*Message #26 Posted by **WigglePig** on 11 July 2003, 3:39 p.m.,*
in response to message #20 by Valentin Albillo

Luiz wrote:"There is no reason to discuss this fact. 0^0 is not defined, period."

Then Valentin responded:"Are you sure ?"

Right, here we go...

0^0 is undefined. Period. Simple as that. In fact, we can even go so far as to say that 0^0 is singular. Sometimes it is called an 'indeterminate form', but the outcome is the same.

Now, in certain applications we would like the expression x^x to be continuous. So, we define 0^0 to be 1, which is quite reasonable since lim(x->0) x^x is 1 whether x approaches zero from above or below.

Soooo...the point x=0 where f(x)=x^x is a 'removeable singularity'. Quite simple really.

Can we stop the argument now?

PS: 0.9999...=1

ttfn WigglePig

Sooo... if you were on the programming team...

*Message #27 Posted by **glynn** on 13 July 2003, 1:02 p.m.,*
in response to message #26 by WigglePig

Okay, What would all of you do?

"Error (0)" _or_ "1" ?

It would probably confuse heck out of the average user to offer a CHOICE, though this probably could be done, say, via some flag setting. The default might be "error (0)", but you could flip a bit and make the calc default to 1.

Or, maybe we should have the calculator give predefined answers, but light up a "YMMV" ("Your Mileage May Vary") indicator: "I'm saying this is 1, but it's really not".

I have heard some describe mathematics as "the perfect science". I believe it probably is, but the subtleties and special cases here and there seem to indicate our mathematical grammar has had a hard time perfectly expressing it in toto.

So, what's a calculator to do? ;-)

1/3 × 3 = 1? 1/3 × 3 = .99999999999...?

Message #28 Posted by ***Vieira, Luiz C. (Brazil)*** *on 13 July 2003, 1:26 p.m.,*
in response to message #27 by glynn

Hi, Glynn;

good reading your posts again.

The title says much of what I want to express, but I'll take the benefit of doubt: 0^0 is not defined.

After reading The Math Forum comments about the limits for x^y when each one tend to zero, what shows it is discontinuous because both limits tend to different values, I embrace the cause.

About programming: I'd set flag 25 in an HP41/42 and test it later, or I'd use an IFERR structure if using an HP-RPL model or any error-trap structure available in other languages to detect this sort of fact, but I'd keep the original math definitions (unless others appear) if I'm developing a new language or compiler. I'd not let the user interfere at this point because it would lead to the development of "conditional programs", that would behave in different ways without specific pre-conditioning. I think the HP49G math approach is already outstanding when allowing the use of many flags to predefine the way the user want an answer to be treated by the O.S. and how it must be shown. The HP48G already gives much control, but after reading some (far from a complete reference) of CAS resources, I can see some interesting possibilities. Nothing I can convert into words right now, but surely new perspectives. About a year ago I posted a message where I criticized CAS without knowing what was it able to do. One of the contributors called my attention and mentioned Urroz' books. I bought four of them and read parts of them in brief. Enough to change my mind about CAS and related stuff.

I still keep math principles as valid and applicable, although many math enhancements have occurred and I'm not aware af them as a whole. For as long as there are math situations with doubtful answers, I'll take them as undefined. As you say, math must be precise all the times, mostly when treating its own lacks.

Luiz (Brazil)

Re: Sooo... if you were on the programming team...

*Message #29 Posted by **Patrick** on 13 July 2003, 1:55 p.m.,*
in response to message #27 by glynn

Well, I plead guilty to starting all of this brou-ha-ha. I never anticipated this kind of response. But here is what I think would happen...

If you have a company driven by engineering principles (e.g., the old HP) they might consider defining the function to be equal to 1. They would see the mathematical arguments for this, such as the rather important statement that the binomial theorem would extend naturally to the case of n=0.

If you have a company driven by marketing principles (e.g., the new HP) they would no doubt leave the function undefined at (0,0), and to hell with the binomial theorem and its ilk. A marketeer would need no further justification than this thread to convince them that is the right thing to do. You never want to confuse your customers. Not with that 50% rollback in customer support coming next month.

On the other hand, a company that *was* driven by engineering principles (i.e., the old HP) did in fact leave 0^0 undefined in their calculator logic. Earlier, I said that I was surprised that Dr. Kahan let this one slip through. I now believe him to be much wiser than I had ever thought. I now believe this was a conscious decision on his part. He was intimately aware of the *good mathematical logic* behind the reason to set the value to 1 -- he teaches this very fact in his complex analysis course for crying out loud -- so he must have had a very good reason not to do it. I think I might agree with him now.

Re: Sooo... if you were on the programming team...

*Message #30 Posted by **Rodger Rosenbaum** on 13 July 2003, 2:42 p.m.,*
in response to message #29 by Patrick

If you look at the sci.math reference: http://www.faqs.org/faqs/sci-math-faq/specialnumbers/0to0/ there you will see: "Kahan has argued that 0.0^(0.0) should be 1, because if f(x), g(x) --> 0 as x approaches some limit, and f(x) and g(x) are analytic functions, then f(x)^g(x) --> 1" As you know, Prof. Kahan was involved in the mathematical behavior of the later HP calcs, reaching its pinnacle, in my opinion, with the HP71. The HP71, when given 0^0, beeps, returns ERRN=6, and returns a value of 1. I guess this is the 71's way of saying "there is disagreement about what 0^0 should return, but we have chosen to return 1, and also return an error message." I'm pretty sure this didn't happen by accident; Prof. Kanan knew what he was doing. My HP48 returns 0^0=1 without error even if I set flags 20 and 21 and clear flag 22 to give it the greatest possibility to generate an error. I prefer the way the HP71 does it.

Re: Sooo... if you were on the programming team...

*Message #31 Posted by **Massimo Gnerucci (Italy)** on 13 July 2003, 6:29 p.m.,*
in response to message #30 by Rodger Rosenbaum

The 49G gives **1** in approximation mode and **?** in exact mode.

Massimo

Re: Sooo... if you were on the programming team...

*Message #32 Posted by **Werner Huysegoms** on 14 July 2003, 1:58 a.m.,*
in response to message #31 by Massimo Gnerucci (Italy)

Indeed. 0^0 (exact) is undefined. It is not because there is a large class of analytic functions for which the *limit* for x approaching zero of f(x)^g(x) equals 1 , that its value should be. Take sin(x)/x. The limit for x approaching zero is 1, but the *value* at x=0 is undefined. The 49G will thus return '?' for 0^0 - but will return 1. for 0.^0. - because of compatibility reasons with the 48, where it was probably chosen because it is a very convenient value to return.

Werner

Re: Sooo... if you were on the programming team...

*Message #33 Posted by **Massimo Gnerucci (Italy)** on 14 July 2003, 3:18 a.m.,*
in response to message #32 by Werner Huysegoms

Absolutely right Werner,
I pointed out the 49G behaviour to show how you can choose the result you like more depending on your taste/application.
I think that such a choice would be welcome in new calculators from HP...

Massimo

Re: Sooo... if you were on the programming team...

Message #34 Posted by ***Dave Shaffer*** *on 13 July 2003, 6:00 p.m.,*
in response to message #27 by glynn

Didn't the IEEE allow for similar situations: I think you can get NAN ("not a number") as the official result from illegal (questionable?) operations such as divide by zero or exponentiations giving results that are outside the range (e.g. 10^+/-99) handled by various computers and/or compilers?

Maybe 0^0 deserves something similar.

Re: Sooo... if you were on the programming team...

Message #35 Posted by ***Rodger Rosenbaum*** *on 13 July 2003, 11:29 p.m.,*
in response to message #34 by Dave Shaffer

IEEE 854 (IEEE Standard for Radix-Independent Floating-Point Arithmetic) only talks about +,-,/,* and remainder as required Operations. Most of the rest of it is about rounding, infinity arithmetic, normalization, NaN's, etc. The other functions such as trig, exponential, log, are left up to the implementor of a particular computer. Our topic here, 0^0 isn't mentioned at all.

Just for grins, I checked what Maple and Mathematica do. Maple returns 1 without any beeps or error indications. Mathematica returns "Indeterminate".

Re: Sooo... if you were on the programming team...

Message #36 Posted by ***unspellable*** *on 14 July 2003, 11:41 a.m.,*
in response to message #35 by Rodger Rosenbaum

leaving the theoretical aspects aside, 0^0 is abitrarily defined as 0 in some applications and as 1 in some others. So I'd vote for a three valued flag so you can set the calculator to return 0,1, or undefined as required.

Re: Sooo... if you were on the programming team...

Message #37 Posted by ***Rodger Rosenbaum*** *on 15 July 2003, 12:36 a.m.,*
in response to message #36 by unspellable

Which applications do you know of that define it as zero?

Re: 0^0

*Message #38 Posted by **Rodger Rosenbaum** on 11 July 2003, 5:59 a.m.,*
in response to message #15 by Patrick

See http://www.faqs.org/faqs/sci-math-faq/specialnumbers/0to0/ for a discussion the problem, which has been discussed to death.

Ask Dr. Math

*Message #39 Posted by **Juergen (CH)** on 11 July 2003, 5:45 a.m.,*
in response to message #14 by Ernie Malaga

Seems to be an old discussion: http://mathforum.org/dr.math/faq/faq.0.to.0.power.html

Re: definitions

*Message #40 Posted by **unspellable** on 11 July 2003, 8:42 a.m.,*
in response to message #39 by Juergen (CH)

Some place I have seen a defininiton of the fatorial function, n!, that includes 0! = 1 as a general case rather than a special case.

I'll try to dig it up.

As for 0^0 I don't have any answers just now, but 0 is often viewed as not being a number at all. Of course the thing about that view that bothers me is what do we put on the number line to plug the space between the smallest positive number and the smallest negative number? On the other hand the number line graphs like a function and lots of functions have discontinuities.

Now I have to go read the article on Z^W.

Re: definitions

*Message #41 Posted by **Ellis Easley** on 11 July 2003, 8:33 p.m.,*
in response to message #40 by unspellable

> Quote:
>
> ... but 0 is often viewed as not being a number at all.

Would this one case: on digital computers, zero represents not one number but all the numbers between the positive and negative numbers with the smallest magnitudes that can be represented?

[OT] Re: Ask Dr. Math

Message #42 Posted by ***Valentin Albillo*** *on 11 July 2003, 9:20 a.m.,*
in response to message #39 by Juergen (CH)

Ask Dr. Math is a valuable service and 99.99% of the time their answers are accurate and useful. But not always ...

Have a look for instance at the answer they gave to this question:

Why is Pi Everywhere?

quoting from it:

" ... when pi appears, it is likely that some circle is present dictating the overall structure of the problem. If you think you can find a place where pi really does appear without circles around (not made up!), please send it along and I'll try to show you where the circle is."

Needless to say, most math-oriented members of this Forum can name *scores* of places where Pi does appear with *no* circle in sight, myself included.

Best regards from V.

Re: [OT] Re: Ask Dr. Math

Message #43 Posted by ***Thibaut.be*** *on 11 July 2003, 11:17 a.m.,*
in response to message #42 by Valentin Albillo

Valentino,

you wrote :

> Quote:
>
> Needless to say, most math-oriented members of this Forum can name scores of places where Pi does appear with no circle in sight, myself included

Well, I'm not a Maths specialist, so if you could give some examples, I'd be delighted !

Circles and PI

Message #44 Posted by ***Andrés C. Rodríguez (Argentina)*** *on 11 July 2003, 12:18 p.m.,* *in response to message #43 by Thibaut.be*

At least the PI and/or SQRT(PI) which appears on normal distribution formulas (gaussian distribution) is not obviously related to any circle, but I am prepared to find someone with great math skills who may show the relation.

On the other hand, I do remember a curved surface we studied in my Math II course in Electronics Engineering, which was defined by many arcs (it was kind of part of a hemispheric surface, but with a S-shaped cut). While all its features looked unavoidably very "circular", PI was remarkably absent of it's area formula, obtained as an integration exercise.

I vaguely remember a possible name for such surface, which could be translated as Viviani's Vault (Bóveda de Viviani), but I may be confusing similar exercises from some 27 years ago, and I have no time now to look for more references. Undoubtedly some of the more math knowledgeable people here can offer enlightment and/or corrections to this comment.

Or, I can just wait for a couple of months, because my first daughter is about to take the same course on her first Industrial Engineering year. (She is taking many of the "same" courses than I did, but her calculator is a 32Sii, while mine was a 25 at the time)

Re: [OT] Re: Ask Dr. Math

Message #45 Posted by ***Valentin Albillo*** *on 11 July 2003, 12:40 p.m.,*
in response to message #43 by Thibaut.be

Hi, Thibauto:

Thibauto posted:

"Valentino, [...] Well, I'm not a Maths specialist, so if you could give some examples, I'd be delighted !"

I'm glad you're so interested in non-circle pi ramblings, but instead of using up bandwith with this very interesting off-topic, I suggest you have a look at this:

The Joy of Pi

by David Blatner. Read it, as I did, and afterwards, if you still want more examples, we can discuss it, ok ?

Best regards from V.

Re: [OT] Re: Ask Dr. Math

Message #46 Posted by ***Trent Moseley*** *on 11 July 2003, 2:44 p.m.,* *in response to message #45 by Valentin Albillo*

Valentin,

Has anyone published "The Joy of 'e'"?

tm

Re: [OT] Re: Ask Dr. Math

Message #47 Posted by ***Dave Shaffer*** *on 11 July 2003, 5:26 p.m.,*
in response to message #46 by Trent Moseley

re: Has anyone published "The Joy of 'e'"?

That's not quite the name, but yes. I have it - somewhere! If I find it, I'll post a proper reference. Actually, it was by the bed: "e The Story of a Number" by Eli Maor.

There is also a book about zero (which was a very strange idea all by itself to folks like the ancient Greeks - let alone the above discussion about 0^0): "ZERO The Biography of a Dangerous Idea" by Charles Seife.

Re: [OT] Re: Ask Dr. Math

Message #48 Posted by ***Trent Moseley*** *on 11 July 2003, 7:45 p.m.,*
in response to message #47 by Dave Shaffer

Thanks Dave.

tm

Re: [OT] From my bookshelf...

Message #49 Posted by ***Massimo Gnerucci (Italy)*** *on 12 July 2003, 6:28 a.m.,*
in response to message #47 by Dave Shaffer

> Quote:
>
> ---
>
> There is also a book about zero (which was a very strange idea all by itself to folks like the ancient Greeks - let alone the above discussion about 0^0): "ZERO The Biography of a Dangerous Idea" by Charles Seife.
>
> ---

Another one from Robert Kaplan: ***The nothing that is: a natural history of Zero***

Massimo

Re: [OT] Re: Ask Dr. Math

*Message #50 Posted by **Werner Huysegoms** on 11 July 2003, 3:16 p.m.,*
in response to message #43 by Thibaut.be

The probability that two random integers are relatively prime (ie contain no common factors) is 6/PI^2.

Werner

Stochastic statistic? (edited)

*Message #51 Posted by **Vieira, Luiz C. (Brazil)** on 11 July 2003, 5:52 p.m.,*
in response to message #50 by Werner Huysegoms

Hi, Werner;

such a well defined value, although with as many digits as a powerful computer can compute, remembered me Introduction to Stochastic Statistics. Just a feeling of mine? Or it is obtained by common statistics analysis?

Just curiosity.

Luiz (Brazil)

(This thread is heating up everytime I touch it to sense temperature... Wow! Brainstorm, for sure... Healthy... brainstorm)

Edited: 11 July 2003, 9:44 p.m.

Re: Stochastic statistic? (edited)

*Message #52 Posted by **Werner Huysegoms** on 14 July 2003, 8:17 a.m.,*
in response to message #51 by Vieira, Luiz C. (Brazil)

Hi Luiz. It was the first example in my Probability course, long ago.. Werner

Re: Stochastic statistic? (edited)

*Message #53 Posted by **Patrick** on 14 July 2003, 12:54 p.m.,*
in response to message #52 by Werner Huysegoms

The term "random integer" is, of course, not well defined. There is no such thing -- at least none which captures the implied uniformity of the distribution.

However, I believe the statistic of which you speak is obtained as the limit as N->infinity of the probability, P_N, of two random integers in the interval [1,N] being relatively prime. Picking two integers "at random" from a finite interval is, of course, very well defined.

What is the definition for PI?

*Message #54 Posted by **Vieira, Luiz C. (Brazil)** on 11 July 2003, 11:19 a.m.,*
in response to message #42 by Valentin Albillo

Hi Valentim, guys;

this is not a math forum, of course, but computing is always a related subject.

If the definition for PI goes beyond the relation between the radius (or diameter) of a circle and the lenght of a complete arc (360º, 400 grads, 2PI radians), then I see what you mean.

If not, if PI is only related to this particular relation, I think that what Dr. Math states is that wherever PI is used, that radius<->lenght relation is there, too. What he is proposing, in my understanding, is that: found the relation, a circle, as base defintion, is there, too.

My 2¢.

Luiz (Brazil)

Re: What is the definition for PI?

*Message #55 Posted by **unspellable** on 11 July 2003, 2:38 p.m.,*
in response to message #54 by Vieira, Luiz C. (Brazil)

Dr. Math is not always the best source for definitions and the like. For one thing it tends to written for those who have not been exposed to much math theory or higher math.

It's a bit like high school when I was told than negative numbers do not have logs. Utter nonsense, but they did not have the time to explain it to high school level kids.

My calculus text says that e^(x^2) has no integral. Also utter nonsense, but they probably didn't want to spend the time on the question. Made me mad, so I went home and solved the integral myself. Of course this is the sort of exercise all students should be doing. You learn far more this way than doing things by rote.

Once past calculus, the defintions angles (excepting circular angles and ordinary solid angles), trig functions, and the like do not involve circles at all. There is probably some such defintion of pi. After all, the ratio of diameter to circumferance of a circle is only pi if you are talking about plane geometry, a concept that does not exist in the real world. (At least given the thereotical physics models currewntly in vogue.)

For comittee decided defintions I have always heard a story to the effect that some state legislature created a legal defintion of pi as 3.14. I find this quite believable after lookign at some of the other lawas around.

Re: What is the definition for PI?

*Message #56 Posted by **Ernie Malaga** on 11 July 2003, 5:20 p.m.,*
in response to message #55 by unspellable

> Quote:
>
> ---
>
> For comittee decided defintions I have always heard a story to the effect that some state legislature created a legal defintion of pi as 3.14. I find this quite believable after lookign at some of the other lawas around.
>
> ---

That's nothing. According to _Asimov on Numbers_ by Isaac Asimov, "There is always the danger that some individuals, too wedded to the literal words of the Bible, may consider 3 to be the divinely ordained value of pi in consequence. I wonder if this may not have been the motive of the simple soul in some state legislature who some years back, introduced a bill which would have made pi legally equal to 3 inside the bounds of the state. Fortunately, the bill did not pass or all the wheels in that state (which would, of course, have respected the laws of the state's august legislators) would have turned hexagonal."

-Ernie

Re: What is the definition for PI?

*Message #57 Posted by **Karl Schneider** on 12 July 2003, 1:17 a.m.,*
in response to message #56 by Ernie Malaga

> Quote:

> According to _Asimov on Numbers_ by Isaac Asimov, "There is always the danger that some individuals, too wedded to the literal words of the Bible, may consider 3 to be the divinely ordained value of pi in consequence. I wonder if this may not have been the motive of the simple soul in some state legislature who some years back, introduced a bill which would have made pi legally equal to 3 inside the bounds of the state.

I've read about that story: the state was Indiana, I believe. Regarding approximations as "definitions", 22/7 wouldn't have been *too* far off the mark, but the HP-35 manual gives the amazingly-good example of 355/113.

Re: What is the definition for PI?

Message #58 Posted by ***Massimo Gnerucci (Italy)*** *on 12 July 2003, 6:56 a.m.,*
in response to message #57 by Karl Schneider

> Quote:
>
> the HP-35 manual gives the amazingly-good example of 355/113

Actually this approximation predates the HP-35 manual a little... ;-)

Tsu Ch'ung-chih (430-501) proposed this value versus 22/7. This was the best approximation until XV century even if the same Tsu Ch'ung-chih computed 3.1415927 as an "excess" approx. and 3.1415926 as a "defect" approx.

[source: ***A History of Mathemathics*** *by Carl B. Boyer*]

Quite impressive, isn't it?

Massimo

Re: What is the definition for PI?

Message #59 Posted by ***Howard*** *on 13 July 2003, 5:24 p.m.,*
in response to message #57 by Karl Schneider

In 1896 a bill was passed in the Indiana legislature 67-0 to make the value of pi = 3.2. The senate after representation from a Purdue University professor caused the senate to defer the bill "to a later date". It is still deferred.

Re: What is the definition for PI?

Message #60 Posted by ***Ed Martin*** *on 13 July 2003, 8:16 p.m.,*
in response to message #59 by Howard

The worst part about the Indiana legislature proposing that pi be made equal to 3.2 is that 3.1 would be a better approximation.

Maybe they decided 3.2, being larger than 3.1, was "better". Perhaps it's the same logic that leads modern politicians to spend more than they take in ....

- Ed

Re: What is the definition for PI?

Message #61 Posted by ***John (Norway)*** *on 12 July 2003, 11:54 a.m.,*
in response to message #54 by Vieira, Luiz C. (Brazil)

Since 1 radian = arctan(b/a), for e^i written in the form a+ib, and a complete circle is 2pi radians, then it looks like pi can be related to 'e' and 'i'. It is interesting to observe that ln(-n) also has the form a+ib, where a=ln(n) and b=pi.

Definition of "pi"

Message #62 Posted by ***Norm*** *on 13 July 2003, 10:24 p.m.,*
in response to message #61 by John (Norway)

Its not about 3.1 or 3.2

The definition of pi is which flavor U like the best.

I think w/o question that chocolate pudding pi with whipped cream topping is absolutely unbeatable, but it needs some chocolate sprinkles to decorate the top.

All together

Message #63 Posted by ***Nenad (Croatia)*** *on 14 July 2003, 4:12 a.m.,*
in response to message #61 by John (Norway)

1 + e^(i * pi)= 0

In a single equation you related geometry (pi), mathemathical analysis (e), complex numbers (i), 1 and 0 (whatever the last two may mean).

Value × Definition

Message #64 Posted by ***Vieira, Luiz C. (Brazil)*** *on 13 July 2003, 11:20 p.m.,*
in response to message #54 by Vieira, Luiz C. (Brazil)

Hi, guys;

I read many posts related to PI as a number and its value. Unspellable posted:

> Quote:
>
> *After all, the ratio of diameter to circumference of a circle is only pi if you are talking about plane geometry, a concept that does not exist in the real world.*

I still wonder about its "origin" or "definition" because, as far as I remember, PI is considered an irrational number, together with "e" (:(I can't remember any other irrational constant). As an irrational number cannot be expressed by a relation, PI must be expressed in another way (maybe as a movie...).

I'm trying to keep only "plain" (number) set theory and definitions. I know many of you will discuss what is an irrational number, that all numbers have an infinite set of digits, and all numbers are contained in R-set (real), if Euclid is right/wrong... I think it's a valuable brainstorm, and I'll read all posts. But I still wonder if PI was defined as a relation based on circle elements (plane geometry) or in some other math relation, what leads us to the fact that PI is NOT an irrational number if it is defined as a relation. By definition, of course.

And concepts are mostly related to the "conceptual" world, not the real one, as unspellable stated so well. Algebra and math, in a conceptual, pure number theory, also deal with a conceptual world as well.

I know it's going a bit far from what we are used to, but this is a teasing subject and I'd love going further.

Thanks.

Comments?

Luiz (Brazil)

Re: Value × Definition

Message #65 Posted by ***unspellable*** *on 14 July 2003, 8:02 a.m.,*
in response to message #64 by Vieira, Luiz C. (Brazil)

The square root of two is an irrational number. pi (A capital PI is the symbol for a series product.) and e are transcendental numbers.

Transcendental numbers, among other things, cannot be the roots of a polynomial equation. This is connected with the fact they are represented as an infinite series when evaluated.

Irrational Numbers (Was: Value × Definition)

Message #66 Posted by ***Mark Hardman (LED)*** *on 14 July 2003, 7:02 p.m.,*
in response to message #65 by unspellable

Let's not forget the golden ratio which is the ratio of height to width of a golden rectangle. It is frequently referred to as phi and is derived with the following series:

phi = sqrt(1+sqrt(1+sqrt(1+sqrt(1+sqrt(1+...)))))

phi = 1.6180339...

Mark Hardman

Re: Irrational Numbers (Was: Value × Definition)

Message #67 Posted by ***Patrick*** *on 14 July 2003, 9:36 p.m.,*
in response to message #66 by Mark Hardman (LED)

Odd, I've always known the derivation of phi as

```
phi = (1+sqrt(5))/2
```

It, and its reciprocal, are the solutions to the quadratic $x^2=x+1$. It is also the limit of f_{n+1}/f_n, where f_n is the n'th Fibonnacci number.

Re: Irrational Numbers (Was: Value × Definition)

Message #68 Posted by ***Mark Hardman (LED)*** *on 14 July 2003, 11:50 p.m.,*
in response to message #67 by Patrick

But its so much more fun to derive phi as an infinite series.

Here is another one for phi:

```
                  1
phi = 1 + ------------------
                    1
           1 + -------------
                       1
                1 + --------
                     1 + ...
```

One interesting property of phi is that **any** positive integer can be represented as the sum of nonconsecutive powers of phi. For example:

$234 = phi^{11} + phi^{7} + phi^{3} + phi + phi^{-5} + phi^{-9} + phi^{-12}$

You can visit The Phi Numbering System page to try this out on any integer you want.

Mark Hardman (LED)

Edited: 14 July 2003, 11:58 p.m.

The Golden Ratio

*Message #69 Posted by **Valentin Albillo** on 15 July 2003, 5:24 a.m.,*
in response to message #68 by Mark Hardman (LED)

Phi's always been a favourite constant of mine, though when I was young and fresh, it was called 'tau' (a greek letter), not 'phi'. This URL has a lot of interesting information on phi:

The Golden Ratio

It seems that the Indian mathematical Genius (with a capital 'G'), Srinivasa Ramanujan was also very fond of it, as he discovered a number of out-of-this-worldish identities and infinite series featuring it. No wonder world-class mathematicians Hardy and Littlewood were so impressed with him:

"In 1913, the English mathematician G. H. Hardy received a strange letter from an unknown clerk in Madras, India [...]Every prominent mathematician gets letters from cranks, and at first glance Hardy no doubt put this letter in that class. But something about the formulas made him take a second look, and show it to his collaborator J. E. Littlewood. After a few hours, they concluded that the results "must be true because, if they were not true, no one would have had the imagination to invent them".

(Source: Srinivasa Ramanujan)

An interesting phi fact rarely mentioned is that the increasing powers of phi are closer and closer to being integer values, something I noticed for the first time when programming a trivial loop on my recently acquired (1976 or so) HP-25, which simply filled up the stack with phi and kept on multiplying it by itself in an endless loop. I noticed at once that the results approached integer values, from above and from below alternately. Look at this 'evidence':

```
phi    =          1.6180339887498948482045...
phi^5  =         11.09 0169943749474241022...
phi^10 =        122.99 1869381244216651252...
phi^11 =        199.00 5024998740641490208...
phi^20 =      15126.9999 33893038648104029...
phi^21 =      24476.0000 40856349008447407...
phi^30 =    1860497.999999 462509500144429...
phi^31 =    3010349.000000 332187397540912...
phi^40 =   228826126.99999999 5629869660818...
phi^41 =   370248451.00000000 2700889084881...
phi^50 = 28143753122.9999999999 64468136299...
phi^51 = 45537549124.0000000000 21959899450...
...
phi^100 =  792070839848372253126.99999999999999999999 873...
phi^101 = 1281597540372340914251.00000000000000000000 78...
```

Speaking of near integers, Ramanujan knew about the following striking result, where a simple looking exponential expression gives a totally unexpected, very near integer value (less than 1E-12 from being an integer), called 'Ramanujan's constant':

e^(pi*sqrt(163)) = 262537412640768743.999999999999 25...

Certainly the number of amazing mathematical curiosities is inexhaustible, indeed !

Best regards from V.

Edited: 15 July 2003, 5:26 a.m.

Re: Irrational Numbers (Was: Value × Definition)

Message #70 Posted by ***Thibaut.be*** *on 15 July 2003, 3:53 a.m.,*
in response to message #66 by Mark Hardman (LED)

...89

Infinite Series and Transcendentals

Message #71 Posted by ***Patrick*** *on 14 July 2003, 9:52 p.m.,*
in response to message #65 by unspellable

There is no link between transcendental numbers and infinite series. While it is true that transcendental numbers can often be expressed as the sum of an infinite series, it is possible to get even rational numbers through non-trivial infinite series.

For those who have never seen it, the proof that sqrt(2) is irrational is quite short. We simply suppose the opposite is true and reach a contradition. Here goes...

If sqrt(2) is rational, then it must be expressed as a fraction of integers. Let a/b be such a fraction *reduced to lowest terms* (i.e., there are no common factors between a and b):

```
sqrt(2) = a/b
```

Square both sides of this equation and clear denominators to get:

$2b^2 = a^2$

Clearly, this says that the square of a is an *even* number. Well, if you square any *odd* number you get an *odd* number, so a cannot be odd. Therefore *a must be even*! This means we can write it as equal to twice some integer, k:

```
a = 2k
```

Substitute into the earlier equation:

$2b^2 = (2k)^2 = 4k^2$

Divide both sides by 2:

$b^2 = 2k^2$

Well now, lookie here... the square of b is *also* even. Like before, we must reason that *b is even*!

Hold on, now. *Both a and b are even*, which means they have a common factor!! But we assumed right at the start that this was not the case. This *contradiction* proves that our starting assumption, that the square root of 2 was rational, is in error.

QED

Re: Infinite Series and Transcendentals

Message #72 Posted by ***unspellable*** *on 15 July 2003, 8:40 a.m.,*
in response to message #71 by Patrick

Transcendental numbers are not "often" expressed as an infinite series, they are always expressed as an infinite series. This a property stemming from the fact that they cannot be the root of a polynomial.

They are called transcendental numbers because they are always expressed as an infinite series.

Re: Infinite Series and Transcendentals

Message #73 Posted by ***Patrick*** *on 15 July 2003, 1:24 p.m.,*
in response to message #72 by unspellable

Every number is the sum of an infinite series whose elments are rational numbers. If you take a number, x, between 0 and 1, say, and let its decimal representation be $0.d_1d_2d_3...$ then what you are really saying is that x can be represented as the following infinite sum of rationals:

$x = \mathrm{Sum}_{(n=1,2,3,...)}(d_n/10^n)$

By adding an extra term to the above sum to represent the integer portion of x, you can easily see how to remove the restriction that x is between 0 and 1.

Of course, the sum isn't infinite if the decimal representation of x terminates, but that is not the point.

The point is that, since *every* number has such a representation, this property hardly distinguishes the ones which are transcendental. The thing about transcendentals is that there are usually *very few other ways* to represent them, such as by giving a rational polynomial for which they are its roots.

The motive for the original mathematician to call that class of numbers *transcendental* was that such numbers *transcend* normal arithmetic (at least, this is how it was explained to me). You cannot get such numbers from integers through addition, subtraction, multiplication, division and the taking of roots.

Edited: 15 July 2003, 1:29 p.m.