Program Description II

Program Title FACTORS OF A NUMBER: THE FASTEST PROGRAM ON THE SUBJECT

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Program Description, Equations, Variables

≡ THIS PROGRAM WILL FIND ALL PRIME FACTORS OF A POSITIVE INTEGER N. THEY WILL BE PAUSED / PRINTED PIECEMEAL, AND THE LAST ONE WILL BE DISPLAYED WITH A MINUS SIGN. (THE SIGN INDICATES THAT IS THE LAST ONE. IF YOU ARE TO USE THE PROGRAM AS A SUBROUTINE, STEP 135 CHS MAY BE DELETED)

≡ THIS PROGRAM IS SPECIALLY INTENDED TO RUN THE FASTEST. THE MOST EFFICIENT METHOD TO TEST IF A NUMBER IS A PRIME (OR TO FIND ITS PRIME FACTORS) IS AS FOLLOWS:

- TAKE THE GIVEN NUMBER, N, AND DIVIDE IT BY ALL PRIMES STARTING WITH 2 AND UP TO THE PRIME P_N SUCH THAT $P_N \leq \sqrt{N}$, $P_{N+1} > \sqrt{N}$
- IF THE RESTS OF ANY OF THOSE DIVISIONS EQUALS ZERO, THEN P_K IS A PRIME FACTOR OF N. THEN, THE NEW $N = N/P_K$, AND THE LOOP CONTINUES.
- IF NONE OF THE RESTS IS ZERO, THEN N IS ITSELF A PRIME

≡ THAT MEANS THAT WE MUST PERFORM DIVISIONS BY 2, 3, 5, 7, 11, 13, 17, --- AND TEST: (a) IF THE DIVISOR $> \sqrt{N}$; (b) IF THE REST = 0

(TO BE CONTINUED ON THE OTHER SIDE OF THIS PAGE)

Operating Limits and Warnings

- N MUST BE A POSITIVE INTEGER, AND IT WOULD BE ALSO WISE THAT $N < 10^9$. OTHERWISE, ERRONEOUS RESULTS MAY APPEAR.
- THE LAST STEP 136 R/S CANNOT BE CHANGED TO 136 RTN

– THE MAIN TROUBLE OF THE PROGRAM IS TO GENERATE A SEQUENCE OF DIVISORS THAT INCLUDES ONLY PRIME NUMBERS. PROGRAM # 52050D SOLVES THIS BY STORING A LE OF PRIMES IN MAGNETIC CARDS THAT MUST BE LOADED BY THE USER AT PROPER TIMES. THIS SOLVES A PROBLEM AND CREATES 3:

(a) MANY DATA CARDS ARE TO BE USED TO TEST A LARGE NUMBER N
(b) USER CAN NOT BE ABSENT, BECAUSE HE MUST LOAD DATA CARDS.
(c) RUNNING TIME IS GREATLY INCREASED BY THE LOADING PROCESS.

THE PRESENT PROGRAM DOESN'T NEED DATA CARDS OR OTHER TRICKS LIKE THAT, BUT SIMPLY GENERATES A SEQUENCE THAT STARTS WITH 2, 3, 5, 7, --- AND INCLUDES ALL NUMBERS UP TO INT $[\sqrt{N}]$ EXCEPT THOSE THAT ARE MULTIPLES OF 2, 3, 5, OR 7. THIS IS A EXTREMELY EFFICIENT METHOD, BECAUSE THE FIRST NON-PRIME DIVISOR TESTED IS 121, AND AFTERWARDS ONLY MULTIPLES OF 11, 13, ETC WILL PASS THE SELECTION PROCESS. THAT MEANS ALL NUMBERS $N \leq 14641$ OR SO WILL BE TESTED WITH ALL POSSIBLE EFFICIENCY, AND THE REST, TO A VERY NEAR OPTIMUM ONE.

AS AN ILLUSTRATION, THE DIVISORS TESTED ARE

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 121, 127, 131 --- ETC.

(ALL ARE PRIME EXCEPT $121 = 11^2$)

– THIS PROGRAM MAY BE IMPROVED BY SKIPPING ALSO THE MULTIPLES OF 11, 13, ETC, BUT THIS WOULD COST FAR MORE THAN 224 STEPS IF A LINEAR PROGRAM IS TO BE CREATED, AND THE SAVING IN TIME DOES NOT EXCEED A MERE 5% (ASSUMING MORE THAN 224 STEPS AVAILABLE, OF COURSE, WHICH IS NOT THE CASE)

– THE EXECUTION TIMES FOR THE PRESENT PROGRAM ARE (IF N IS EITHER A PRIME OR THE SQUARE OF A PRIME):

N	RUNNING TIME	N	RUNNING TIME
$N < 10$ (I.E = 11)	3 SECONDS	$N < 10^5$ (I.E = 10^5+3)	57 SEC.
$N < 100$ (I.E = 10^2+1)	4 "	$N < 10^6$ (I.E = 10^6+3)	2MIN. 46 SEC.
$N < 1000$ (I.E = 10^3+9)	9 "	$N < 10^7$ (I.E = 10^7+19)	8 MIN. 55 SEC.
$N < 10^4$ (I.E = 10^4+7)	20 "	$N < 10^8$ (I.E = 10^8+7)	28 MIN. 6 SEC.

– IF YOU OWN OR HAVE ANOTHER PROGRAM OF THIS KIND, YOU'LL REALIZE THAT THESE TIMES ARE VERY DIFFICULT TO BEAT. IF YOU ARE INTERESTED IN A SHORTER ROUTINE WITH A MAXIMUM SPEED, HERE IS INCLUDED A ROUTINE 52 STEPS LONG, WHICH IS A SIMPLIFICATION OF THE PRESENT PROGRAM THAT SKIPS ALL MULTIPLES OF 2, 3, OR 5 AS DIVISORS:

```
LBL A      LBL 1      . 6        LBL E
STO A      . 4        GSBE       STO + 9    LAST X
0          GSBE       . 2        RCL A      STO A
STO 9      . 2        GSBE       RCL 9      RCL 9
2          GSBE       . 4        ÷          PAUSE
GSBE       . 4        . 6        LAST X     0
1          GSBE       GSBE       X>Y        GTO E
GSBE       . 2        GTO 1      GTO 0      LBL 0
2          GSBE                  X≥Y        RCL A
GSBE       . 4                   FRAC       CHS
. 2        GSBE                  X≠0        R/S
GSBE                             RTN
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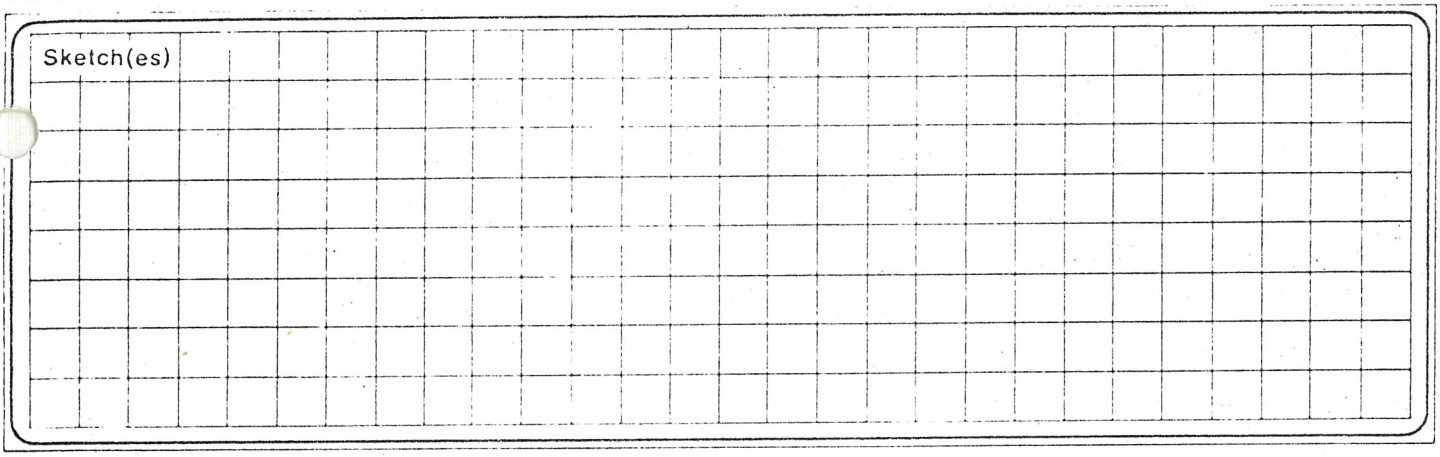
– THIS ROUTINE IS NEARLY 3 TIMES SHORTER, BUT AS IT DOES NOT SKIPS THE MULTIPLES OF 7, IT SOME WHAT SLOWER.

RUNNING TIMES FOR THIS ROUTINE ARE

$N < 10$, (11)	3 SEC.	$N < 10^5$, (10^5+3)	1MIN 1
$N < 10^2$, (10^2+1)	4 SEC.	$N < 10^6$, (10^6+3)	3MIN 8
$N < 10^3$, (10^3+9)	9 SEC.	$N < 10^7$, (10^7+19)	9 MIN 55
$N < 10^4$, (10^4+7)	21 SEC.	$N < 10^8$, (10^8+7)	31 MIN 19

Programmable Description II

Sketch(es)



Sample Problem(s) (1) FIND ALL PRIMES OF THE FORM 1, 11, 111, ETC, UP TO 10^{8+2}

- LOAD PROGRAM, BOTH SIDES;
- TO TEST 1 ⇨ 1 [A] → −1, (− SIGN INDICATES LAST FACTOR), PRIME

11 ⇨ 11 [A] → −11, PRIME (3 SEC)

111 ⇨ 111 [A] → 3 → −37 ⇨ 111 = 3 × 37

1111 ⇨ 1111 [A] → 11 → −101 ⇨ 1111 = 11 × 101

111111 ⇨ 111111 [A] → 41 → −271 ⇨ 111111 = 41 × 271

111111 ⇨ 111111 [A] → 3 → 7 → 11 → 13 → −37 ⇨ 111111 = 3 × 7 × 11 × 13 × 37

11111111 ⇨ 11111111 [A] → 239 → −4649, ⇨ 11111111 = 239 × 4649

111111111 ⇨ 111111111 [A] → 11 → 73 → 101 → 137 ⇨ 111111111 = 11 × 73 × 101 × 137

1111111111 ⇨ 1111111111 [A] → 3 → 3 → 37 → −333667 ⇨ 1111111111 = 3^2 × 37 × 333667

111111111111 ⇨ 111111111111 [A] → 11 → 41 → 271 → −9091 ⇨ 111111111111 = 11 × 41 × 271 × 9091

NOTE = BE EXTREMELY CAREFUL WHEN TESTING NUMBERS GREATER THAN 10^8, BECAUSE FACTORS MAY BE ERRONEOUS.

Solution(s) ≡ (2) FIND ALL PRIMES OF THE FORM N! + 1, N ≤ 12, N ≥ 2

N = 2 ⇨ 2 [N!] 1 + [A] → −3, PRIME

N = 3 ⇨ 3 [N!] 1 + [A] → −7, PRIME

N = 4 ⇨ 4 [N!] 1 + [A] → 5 → −5 ⇨ 4! + 1 = 5^2

N = 5 ⇨ 5 [N!] 1 + [A] → 11 → −11 ⇨ 5! + 1 = 11^2

N = 6 ⇨ 6 [N!] 1 + [A] → 7 → −103 ⇨ 6! + 1 = 7 × 103

N = 7 ⇨ 7 [N!] 1 + [A] → 71 → −71 ⇨ 7! + 1 = 71^2

N = 8 ⇨ 8 [N!] 1 + [A] → 61 → −661 ⇨ 8! + 1 = 61 × 661

N = 9 ⇨ 9 [N!] 1 + [A] → 19 → 71 → 269 ⇨ 9! + 1 = 19 × 71 × 269

Reference(s) N = 10 ⇨ 10 [N!] 1 + [A] → 11 → −329891 ⇨ 10! + 1 = 11 × 329891

N = 11 ⇨ 11 [N!] 1 + [A] → (AFTER 17 MIN. 48 SEC.) → −3991681, PRIME

N = 12 ⇨ 12 [N!] 1 + [A] → 13 → 13 → −2834329 ⇨ 12! + 1 = 13^2 × 2834329

MORE EXAMPLES ON THE OTHER SIDE OF THIS PAGE =

≡ (3) TEST THE FOLLOWING NUMBERS TO SEE WHICH ARE PRIME AMONG THEM

55555551 [A] → 773 → −7187 ⇨ 55555551 = 773 × 7187

31415927 [A] → 31 → 103 → 9739 ⇨ 31415927 = 31 × 103 × 9739

19801979 [A] → 719 → 27541 ⇨ 19801979 = 719 × 27541

72727 [A] → −72727 , PRIME

9393931 [A] → 211 → 211 → −211 ⇨ 9393931 = 211^3

1987654321 [A] → 457 → −4349353 ⇨ 1987654321 = 457 × 4349353

1234567 [A] → 127 → −9721 ⇨ 1234567 = 127 × 9721

3837280200 [A] → 2 → 2 → 2 → 3 → 5 → 5 → 13 → 13 → 13^3 → 41 → −71 ⇨

⇨ 3837280200 = $2^3 \times 3 \times 5^2 \times 13^3 \times 41 \times 71$

≡ (4) FIND THE 1ST PRIME NUMBER GREATER THAN 10^7

CLX EEX 7 ENTER 1 + [A] → 11 R/S ($10^7 + 1$ IS A MULTIPLE OF 11)

EEX 7 ENTER 3 + [A] → 13 R/S ($10^7 + 3$ " " 13)

EEX 7 ENTER 7 + [A] → 941 R/S ($10^7 + 7$ " " 941)

EEX 7 ENTER 13 + [A] → 421 R/S ($10^7 + 13$ " " 421)

EEX 7 ENTER 19 + [A] → −10000019 , PRIME

≡ (5) SIMPLIFY (OR REDUCE) THE FRACTION $\frac{1333333}{7777771}$ TO A SIMPLER FORM

1333333 [A] → 23 → 29 → −1999 ⇨ 1333333 = 23 × 29 × 1999

7777771 [A] → 29 → −268199 ⇨ 7777771 = 29 × 268199 ⇨

⇨ $\frac{1333333}{7777771} = \frac{23 \times 29 \times 1999}{29 \times 268199} = \frac{23 \times 1999}{268199} = \frac{45977}{268199}$

≡ (6) IF YOU ARE AN HP-97 OWNER, YOU MAY CHANGE STEP 130 PAUSE, TO 130 PRINTX. THIS ALLOWS YOU TO BE ABSENT WHILE PROGRAM RUNS. BUT IF YOU OWN AN HP-67 INSTEAD, THERE IS A SIMPLE PROCEDURE, THAT ALLOWS YOU TO BE ABSENT, TOO.

FOR INSTANCE, WHERE YOU TO FACTORIZE 136723602, WHEN SOME OTHER BUSINESS IMPEDES YOU TO WATCH THE WHOLE PROCESS. NO MATTER, PRESS:

136723602 [A] AND GO OUT. SOME TIME LATER, YOU LOOK AT THE DISPLAY AND FIND: −773 ⇨ 773 IS THE NUMBER'S GREATEST PRIME FACTOR

PRESS: 136723602 [X≷Y] [CHS] → 773 (ONE OF THE PRIME FACTORS)

[LASTX] → 719 (ANOTHER PRIME FACTOR)

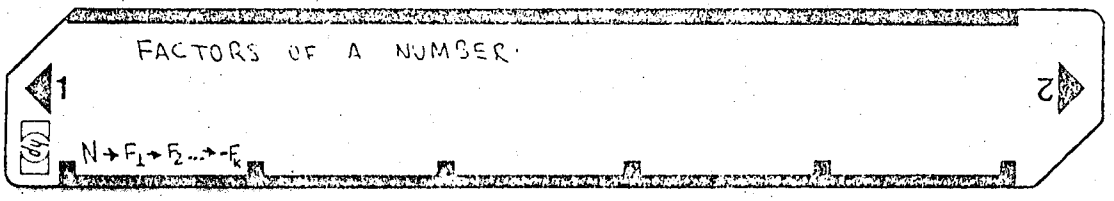
[X] [÷] [A] → 2 → 3 → −41 (2, 3, 41 ARE THE REMAINING FACTORS)

SO, 136723607 = 2 × 3 × 41 × 719 × 773

– THIS IS, YOU MAY RECOVER AT ONCE THE 2 GREATEST PRIME FACTORS, AND WHEN THEY ARE REMOVED BY DIVISION FROM THE ORIGINAL NUMBER, THE REMAINING ON IS QUICKLY FACTORIZED.

ONLY IF THE 2 FACTORS ARE CONSECUTIVE

User Instructions



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	LOAD PROGRAM, BOTH SIDES			
2	INPUT N, AND FIND ITS PRIME FACTORS	N	A	F1
				F2
	F1, F2, ... = PRIME FACTORS			---
	-Fm = LAST PRIME FACTOR			Fk

	(OF COURSE, IF Fm = N, THEN N IS			-Fm
	A PRIME)			
3	FOR ANOTHER N, GO TO 2			
	MAXIMUM RUNNING TIMES ARE:			
	$N \simeq 10$, 3 SECONDS			
	$N \simeq 10^2$, 4 "			
	$N \simeq 10^3$, 9 "			
	$N \simeq 10^4$, 20 "			
	$N \simeq 10^5$, 52 "			
	$N \simeq 10^6$, 2 MIN. 46 SECONDS			
	$N \simeq 10^7$, 8 " 55 "			
	$N \simeq 10^8$, 28 " 6 "			
	RESULTS AREN'T SAFE IS $N \geq 10^9$			
	- IF YOU WANT TO USE THE PROGRAM AS			
	A SUBROUTINE, TAKE NOTE OF THIS:			
	- PROGRAM USES R0, A, E, 01			
	- " " REGISTERS A, 9			
	- " " NO FLAGS			
	- STEP 130 R/S CANNOT BE CHANGED			
	TO 130 PTN			

Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBL A	31 25 11	
	STO A	33 11	STORE N
	0	00	SET DIVISOR TO 0
	STO 9	33 09	
	2	02	TEST 2 AS A FACTOR
	GSB E	31 22 15	
	1	01	TEST 3 AS A FACTOR
	GSB E	31 22 15	
	2	02	TEST 5 AS A FACTOR
010	GSB E	31 22 15	
	2	02	TEST 7 AS A FACTOR
	GSB E	31 22 15	
	4	04	TEST 11 AS A FACTOR
	GSB E	31 22 15	
	*LBL 1	31 25 01	BEGINNING OF LOOP
	2	02	SKIPS 2 NON-PRIME FACTS
	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
020	2	02	SKIPS 2 "
	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
	2	02	SKIPS 2 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
030	4	04	SKIPS 4 "
	GSB E	31 22 15	
	2	02	SKIPS 2 "
	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
040	2	02	SKIPS 2 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
	2	02	SKIPS 2 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
050	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
	8	08	SKIPS 8 "
	GSB E	31 22 15	
	4	04	SKIPS 4 "

STEP	KEY ENTRY	KEY CODE	COMMENTS
	GSB E	31 22 15	
	2	02	SKIPS 2 N.P. FACTOR
	GSB E	31 22 15	
060	4	04	SKIPS 4 "
	GSB E	31 22 15	
	2	02	SKIPS 2 "
	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
	8	08	SKIPS 8 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
070	4	04	SKIPS 4 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
	2	02	SKIPS 2 "
	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
080	2	02	SKIPS 2 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
	2	02	SKIPS 2 "
	GSB E	31 22 15	
090	4	04	SKIPS 4 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
	2	02	SKIPS 2 "
	GSB E	31 22 15	
	6	06	SKIPS 6 "
	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
100	2	02	SKIPS 2 "
	GSB E	31 22 15	
	4	04	SKIPS 4 "
	GSB E	31 22 15	
	2	02	SKIPS 2 "
	GSB E	31 22 15	
	1	01	SKIPS 10 "
	0	00	
	GSB E	31 22 15	
	2	02	SKIPS 2 "
110	GSB E	31 22 15	
	1	01	SKIPS 10 "
	0	00	

REGISTERS

0	1	2	3	4	5	6	7	8	Divisor.
50	S1	S2	S3	S4	S5	S6	S7	S8	S9

I, used	B	C	D	E	I

Program Listing II

STEP	KEY ENTRY	KEY CODE	COMMENTS
	GSB E	31 22 15	
	GTO 1	22 01	END OF LOOP
	* LBL E	31 25 15	TEST SUBROUTINE
	STO + 9	33 61 09	SKIPS KNON-PRIME FACTORS
	RCL A	34 77	RECALL N
	RCL 9	34 09	RECALL DIVISOR
	÷	81	
100	LAST X	35 82	
	X≥Y	32 81	
	GTO 0	22 00	ALL DIVISORS TESTED
	X<Y	25 52	
	FRAC	32 83	IS Rq A FACTOR?
	X≠0	31 61	
	RTN	35 22	NO, RETURN TO LOOP
	LAST X	35 82	YES
	STO A	33 77	NEW N = N/Rq
	RCL 9	34 09	DISPLAY FACTOR
130	PAUSE	35 72	
	0	00	TEST Rq ONCE AGAIN
	GTO E	22 15	
	* LBL 0	31 25 00	LAST FACTOR
	RCL 1	34 77	RECALLS N (OR LAST FACTOR)
	CHS	42	CHANGE SIGN = LAST FACTOR
	R/S	84	STOP
140			
150			
160			
170			
180			
190			
200			
210			
220			

HP-97 OWNERS:

STEP 130 PAUSE

MUST BE CHANGED TO:

130 PRINT X

AND ANOTHER

PRINT X

STATEMENT

MUST BE INSERTED

BETWEEN

135 CHS

136 R/S

LABELS					FLAGS	SET STATUS		
A N fi	B	C	D	E used	0	FLAGS	TRIG	DISP
a	b	c	d	e	1	ON OFF 0 ☐ ■ 1 ☐ ■ 2 ☐ ■ 3 ☐ ■	DEG ■ GRAD ☐ RAD ☐	FIX ■ SCI ☐ ENG ☐ n 0
0 used	1 used	2	3	4	2			
5	6	7	8	9	3			