Program Description I

Program Title: ELLIPTIC LOWPASS FILTER DESIGN

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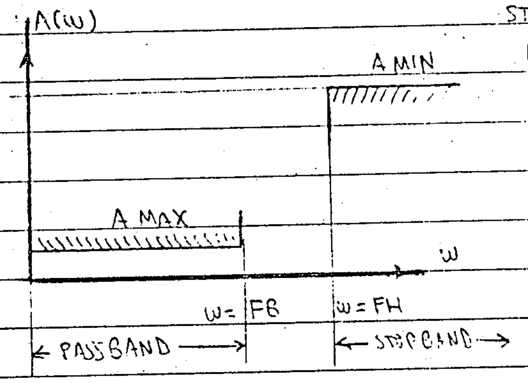
Program Description, Equations, Variables: THIS PROGRAM IS INTENDED AS A HELP WHEN DESIGNING ELLIPTIC LOWPASS FILTERS (OR HIGHPASS, BANDPASS, BANDSTOP ONCE THEY HAVE BEEN TRANSFORMED INTO A NORMALIZED LOWPASS). THE PROGRAM CAN BE USED TO FIND THE MINIMUM DEGREE OF AN ELLIPTIC LOWPASS FILTER THAT MEETS THE STANDARD REQUIREMENTS AMIN, AMAX, FB, FH, WHERE:



AMIN = MINIMUM STOPBAND LOSS

AMAX = MAXIMUM PASSBAND LOSS

FB = UPPER PASSBAND EDGE

FH = UPPER STOPBAND EDGE

IF THE DEGREE N IS $N \leq 30$, THE PROGRAM ALSO FINDS THE ZEROS & POLES OF ATTENUATION AND, USING CARD 2, CAN FIND THE LOSS AT ANY DESIRED FREQUENCY, EITHER MANUALLY OR AUTOMATICALLY. IN THE CASE YOU PREFER AN AUTOMATIC PLOT, THERE IS A CHOICE BETWEEN LINEAR OR LOGARITHMIC SWEEP.

EQUATIONS USED ARE:

– CONSIDER THE ELLIPTIC INTEGRAL OF THE 1ST KIND $\equiv U(\varphi, k) = \int_0^{\varphi} \frac{dx}{\sqrt{1-k^2\sin^2 x}}$

– THEN, THE COMPLETE ELLIPTIC INTEGRAL IS EXPRESED AS:

$$K(k) = U(\pi/2, k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}$$

~~Operating Limits and Warnings~~ AND THE COMPLEMENTARY COMPLETE ELLIPTIC INTEGRAL:

$$K'(k) = K(k') \quad , \text{WHERE } k' = \sqrt{1-k^2}$$

– THE ELLIPTIC SINE: $SN(u, k) = \sin\varphi$

– THEN, WE COMPUTE $XL = FH/FB$, $\varepsilon = 10^{0.1\,AMAX} - 1$

$$L = \sqrt{\frac{10^{0.1\,AMIN} - 1}{10^{0.1\,AMAX} - 1}}$$

(TO BE CONTINUED ON THE OTHER SIDE OF THIS PAGE)

- THE MINIMUM DEGREE IS FOUND BY USING:

$$m = \text{INT}\left[\frac{\mathbb{K}\left(\frac{1}{X_L}\right)\cdot\mathbb{K}'\left(\frac{1}{L}\right)}{\mathbb{K}'\left(\frac{1}{X_L}\right)\cdot\mathbb{K}\left(\frac{1}{L}\right)}\right] + 1$$

- ONCE THE NECESSARY DEGREE IS FOUND, THE ZEROS & POLES OF ATTENUATION MAY BE COMPUTED AND STORED (IF $m \leq 30$)

IN NORMALIZED FORM ≡

FOR $i = 1, 2, \ldots \text{INT}\left[\frac{m}{2}\right]$

NORMALIZED ZEROS OF ATTENUATION

- N IS EVEN ≡ $X'Z_i = \text{sn}\left[\frac{(2i-1)\mathbb{K}\left(\frac{1}{X_L}\right)}{m}, \frac{1}{X_L}\right]$
- N IS ODD ≡ $X'Z_i = \text{sn}\left[\frac{2i\,\mathbb{K}\left(\frac{1}{X_L}\right)}{m}, \frac{1}{X_L}\right]$

NORMALIZED POLES OF ATTENUATION

- N ODD (OR EVEN) ≡ $X'P_i = \frac{X_L}{X'Z_i}$

BOTH, ZEROS & POLES ARE DENORMALIZED BY

$$XZ_i = X'Z_i \times FB$$
$$XP_i = X'P_i \times FB$$

- THE LOSS AT ANY FREQUENCY ω, IS COMPUTED AS FOLLOWS:

$$A(\omega) = 10 \log\left[1 + \varepsilon R_m^2\right]$$

WHERE $R_m(x, L) = C\,x^{1-NI} \cdot \prod_{i=1}^{\text{INT}(m/2)} \left(\frac{x^2 - XZ_i^2}{x^2 - XP_i^2}\right)^2$

AND $C = \frac{1}{R_m(FB)}$, $(\omega = x \text{ IN } R_m(x, L))$

- THE ROUTINES FOR COMPUTATION OF:
  - COMPLETE ELLIPTIC INTEGRAL OF THE FIRST KIND, $\mathbb{K}(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}$
  - COMPLEMENTARY " " " " " " , $\mathbb{K}'(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-(1-k^2)\sin^2 x}}$
  - ELLIPTIC SINE ≡ $\text{SN}(u, k) = \sin\varphi$

ARE ALSO AVAILABLE FROM KEYBOARD WITHOUT DISTURBING THE STORED ZEROS & POLES

- THE ALGORITHMS THAT ARE USED ARE AS FOLLOWS ≡

- $\boxed{\mathbb{K}(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}}$ ⇒ DATA ≡ k, $\varphi_0 = \pi/2$, $P_0 = 1$, $X_0 = \sqrt{1-k^2}$

LOOP

$$X_{i+1} = \frac{2}{1+\sin(\cos^{-1}X_i)} - 1, \quad P_{i+1} = P_i\,(1 + X_{i+1})$$

$$\varphi_{i+1} = \frac{\varphi_i + \sin^{-1}\left[\sin(\cos^{-1}X_i)\cdot\sin\varphi_i\right]}{2}$$

IF $X_{i+1} = 0$, EXIT

THEN, $\mathbb{K}(k) = P_{i+1} \cdot \text{LN}\left[\text{TAN}\left(\text{TAN}^{-1}1 + \frac{\varphi_{i+1}}{2}\right)\right]$

- $\boxed{\text{sn}(u, k)}$ ⇒ FIRSTLY, $\text{sn}(u/8, k)$ IS COMPUTED BY MEANS OF A TAYLOR SERIES EXPANSI[ON]

$$\text{sn}(u/8, k) \doteq \frac{u}{8} - (1+k^2)\frac{(u/8)^3}{3!} + (1+14k^2+k^4)\cdot\frac{(u/8)^5}{5!} - (1+135k^2+135k^4+k^6)\frac{(u/8)^7}{7!}$$

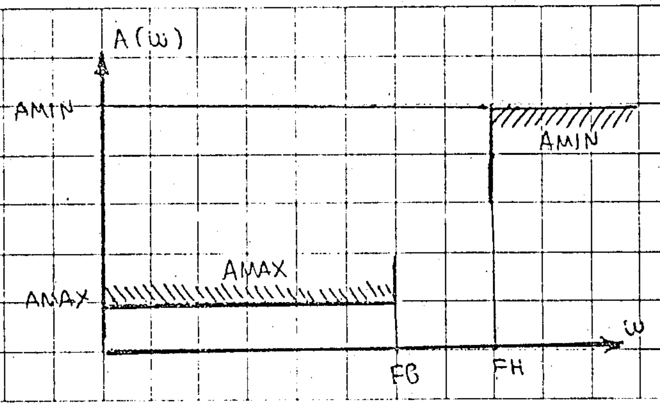
AND THEN, THE DUPLICATION FORMULA $\text{sn}\,2u = \frac{2\,\text{sn}\,u\sqrt{1-\text{sn}^2 u}\sqrt{1-k^2\text{sn}^2 u}}{1-k^2\text{sn}^4 u}$

IS APPLIED 3 TIMES TO YIELD THE DESIRED VALUE $\text{sn}(u, k)$

- THIS PROCEDURE IS VERY FAST (11 SECONDS FOR ANY ARGUMENT) AND QUITE ACCURATE. (

Program Description II

Sketch(es)



Sample Problem(s) NOW, LET'S SEE ALL FUNCTIONS A LITTLE CLOSER:

CARD 1 (BOTH CARDS WORKS ON RADIANS MODE)

CARD 1 DOES NOT DEPEND ON CARD 2 TO PERFORM ITS FUNCTIONS. IN FACT, CARD 2 IS ONLY FOR EVALUATING THE LOSS ONCE ZEROS & POLES HAVE BEEN COMPUTED BY CARD 1

LBL A INPUT: AMIN [ENTER] AMAX [ENTER] FH [ENTER] FB [A] → m (DEGREE)

CHARACTERISTICS
- IT TAKES ABOUT 87 SECONDS (ON THE AVERAGE)
- AMIN, AMAX, --- ARE USED, BUT ONLY FB, AMIN ARE STORED
- IT DESTROYS PREVIOUS ZEROS & POLES IN R8, R9

LBL B PRESS [B] (DEGREE m MUST HAVE BEEN PREVIOUSLY COMPUTED, OF COURSE)

→ (1) → XZ1 → XP1 (DSP9)
→ (2) → XZ2 → XP2 (DSP9) WHERE m = INT(m/2)
--- --- --- --- ---

Solution(s)

→ (m) → XZm → XPm → 0.00

CHARACTERISTICS
- TO BE USED ONCE m HAS BEEN COMPUTED
- ONLY IF m ≤ 30 (m ODD or EVEN)
- IT TAKES ABOUT ½×13 SECONDS PER DEGREE
  (I.E = IF DEGREE IS 2×3, THEN CALCULATION OF ZEROS & POLES WILL LAST ABOUT 40 SECONDS)
- ZEROS ARE STORED IN R1 THROUGH R15
  (POLES NEED NOT BE STORED, THEY ARE FUNCTION OF ZEROS)

Reference(s) LBL C ALLOWS REVIEWING BOTH ZEROS & POLES AT ANY TIME

PRESS [C] → (1) → XZ1 → XP1 (DSP9)
→ (2) → XZ2 → XP2 (DSP9)
--- --- ---
→ (X) → XZx → XPx → 0.00 , (x = INT(m/2))

(CONTINUES ON THE OTHER SIDE OF THIS PAGE)

$\boxed{\text{LBL D}}$ COMPUTES $\mathbb{K}(K) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-K^2\sin^2 x}}$

— SIMPLY, INPUT K $\boxed{D} \rightarrow \mathbb{K}(K)$

$\boxed{\text{LBL d}}$ COMPUTES $\mathbb{K}'(K) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-(1-K^2)\sin^2 x}}$; INPUT K $\boxed{FD} \rightarrow \mathbb{K}'(K)$

CHARACTERISTICS

- IT TAKES FROM 16 TO 32 SECONDS. ABOUT 22 ON THE AVERAGE
- K MUST NOT BE $K < 10^{-4}$ (FOR $K < 10^{-4}$, $\mathbb{K}(K) = \pi/2$)
  OR $K = 1$ (IF $K = 1$, $\mathbb{K}(K) = \infty$)
- USING LBLD OR LBLd DOES NOT ALTER STORED ZEROS
- ACCURACY IS 8 DECIMALS OR BETTER

$\boxed{\text{LBL e}}$ STORES K FOR $sn(u,K)$ COMPUTATIONS : K $\boxed{FE} \rightarrow K$

$\boxed{\text{LBL E}}$ COMPUTES $sn(u,K)$. K IS ASSUMED TO BE IN RE ; u $\boxed{E} \rightarrow sn(u,K)$

CHARACTERISTICS

- IT TAKES 11 SECONDS
- $0 \le K \le$ SOME UNDEFINED VALUE
- IT DOES NOT ALTER STORED ZEROS
- ACCURACY IS 8 DECIMALS OR BETTER FOR NON-OUT OF RANGE VALUES OF K

$\boxed{\text{Card 2}}$ IS AN OPTIONAL COMPLEMENT TO CARD 1. ONCE CARD 1 HAS DETERMINED DEGREE, AND ZEROS & POLES, THE LOSS FOR ANY FREQUENCY CAN BE COMPUTED. THERE ARE SEVERAL OPTIONS:

$\boxed{\text{LBL A}}$ COMPUTES THE LOSS (IN DECIBELS) FOR A GIVEN ω (NOTE: THE FIRST ω TAKES MORE TIME TO BE COMPUTED THAN ALL SUBSEQUENTS)
ω $\boxed{A} \rightarrow A(\omega)$ (DSP 4)

$\boxed{\text{LBL B}}$ SETS LINEAR SWEEP : $\boxed{B} \rightarrow 0.0000$ (LINEAR ≡ $\omega_1 = \omega_0 + \Delta$, $\omega_2 = \omega_1 + \Delta$, etc

$\boxed{\text{LBL C}}$ SETS LOGARITHMIC SWEEP: $\boxed{C} \rightarrow 1.0000$ (LOGARITH ≡ $\omega_1 = \omega_0 \cdot \Delta$, $\omega_2 = \omega_1 \cdot \Delta$, etc

$\boxed{\text{LBL D}}$ STARTS SWEEP : ω_0 $\boxed{ENTER}$ Δ $\boxed{D}$ $\rightarrow (\omega_0) \rightarrow A(\omega_0)$
$\rightarrow (\omega_1) \rightarrow A(\omega_1)$
$\rightarrow (\omega_2) \rightarrow A(\omega_2)$, etc.

(LINEAR SWEEP IS SELECTED WHEN CARD 2 IS LOADED)

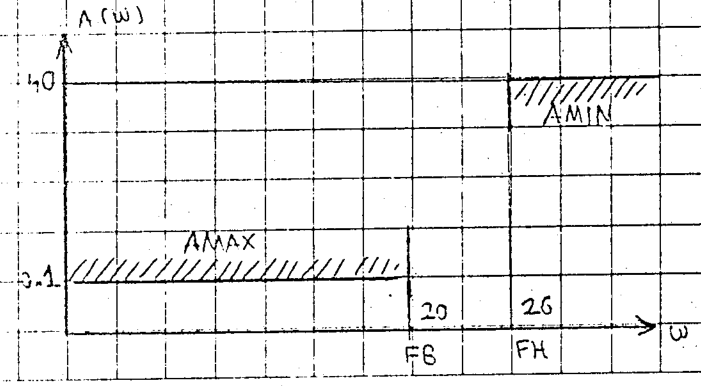
SWEEP CAN BE STOPPED BY PRESSING $\boxed{R/S}$ AFTER $A(\omega_i)$ IS PRINTED.
(THIS IS, AS SOON AS THE PRINTER WORKS OR THE DISPLAY FLASHES, PRESS $\boxed{R/S}$)
OTHERWISE, AN UNDESIRED $\boxed{R/S}$ MAY HAVE BEEN PERFORMED.

$\boxed{\text{LBL E}}$ IS A REVIEWING ROUTINE. IT REVIEWS ZEROS & POLES (SEE LBLC ON CARD 1)
IT'S INCLUDED TO AVOID RELOADING OF CARD 1 FOR REVIEWING PURPOSES.

Program Description II

Sketch(es)

AMAX = 0.1
AMIN = 40
FB = 20
FH = 26



Sample Problem(s) FIND ALL NECESSARY INFORMATION ABOUT AN ELLIPTIC LOWPASS FILTER THAT MEETS THE FOLLOWING REQUIREMENTS (SEE SKETCH)

AMAX = 0.1 dB , AMIN = 40 dB , FB = 20 , FH = 26

a) DEGREE: – LOAD CARD 1, BOTH SIDES
– INPUT FILTER REQUIREMENTS : 40 [ENTER] 0.1 [ENTER] 26 [ENTER] 20 [A] → 6.00

SO, THE MINIMUM DEGREE IS 6 , WE NEED A 6TH-ORDER E.LOWPASS FILTER

b) LOSS AS A FUNCTION OF FREQUENCY
– WE WANT TO MAKE A DRAWING OF THE LOSS AS A FUNCTION OF FREQUENCY. TO OBTAIN THIS GRAPHIC, WE MUST FIND THE MOST RELEVANT ABCISSAE:

(1) ZEROS & POLES : [B] → (1) → 6.245011340 → 82.60501346

Solution(s)

→ (2) → 15.62227766 → 33.28579438
→ (3) → 19.56561718 → 26.57723471 → 0.00

$\underbrace{\qquad\qquad}_{\text{ZEROS}}$ $\underbrace{\qquad\qquad}_{\text{POLES}}$

(2) FREQUENCIES OF MAXIMUM PASSBAND LOSS
– THESE ARE $fe_i = FB \times sn\left(\frac{2iK}{n}\right)$; $i = 0, 1, 2, 3$

– PROCEED AS FOLLOWS: [DSP 4] 0 [E] → $fe_0 = 0.0000$

IE: $fe_1 = 20\, sn\left[\frac{2K}{6}, \frac{20}{26}\right]$ ⇒ [RCL 0] 2 [×] 6 [÷] [E] 20 [×] → $fe_1 = 11.6623$
[RCL 0] 4 [×] 6 [÷] [E] 20 [×] → $fe_2 = 18.1792$

Reference(s) (1) APPROXIMATION METHODS FOR ELECTRONIC FILTER DESIGN – AUTHOR: RICHARD W. DANIELS – MAC GRAW-HILL BOOK COMPANY – PAGES 51 TO 82

(2) MODERN FILTER THEORY AND DESIGN
AUTHOR: EDITED BY GABOR C. TEMES & SANJIT K. MITRA – JOHN WILEY & SONS – PAGES 31 TO 53

$\boxed{ELLC}$ $\boxed{E}$ 20 $\boxed{X}$ → $f_{e3} = 20.0000$

(3) FREQUENCIES OF MINIMUM STOPBAND LOSS

— THESE ARE GIVEN BY $f_{Ei} = \dfrac{FB \times FH}{f_{ei}}$

$$f_{E1} = \frac{20 \times 26}{11.6623} = 44.5881 \quad , \quad f_{E2} = \frac{520}{18.1742} = 28.6041$$

$$f_{E3} = \frac{20 \times 26}{20} = 26.0000 \quad , \quad f_{E0} = \frac{20 \times 26}{0} \to \infty$$

— WE ARE READY FOR THE LOSS COMPUTATION . LOAD CARD 2 (ITS ONLY SIDE)

— LET'S CALCULATE SOME POINTS MANUALLY

THE ZEROS = 0.2950 $\boxed{A}$ → 0.0000
15.6223 $\boxed{A}$ → 0.0000
19.5056 $\boxed{A}$ → 0.0000

THE F_i OF MAX. PASSBAND LOSS = 0 $\boxed{A}$ → 0.1000 dB
11.6623 $\boxed{A}$ → 0.1000 dB
18.1742 $\boxed{A}$ → 0.1000 dB
20 $\boxed{A}$ → 0.1000 dB

THE NEAR-POLES = 82.6050 $\boxed{A}$ → 161.3803 dB
33.2857 $\boxed{A}$ → 139.7303 dB
26.5772 $\boxed{A}$ → 133.7020 dB

THE F_i OF MIN. STOPBAND LOSS =
44.5881 $\boxed{A}$ → 46.8540
28.6041 $\boxed{A}$ → 46.8540
26 $\boxed{A}$ → 46.8540
$\infty \Rightarrow$ 100 000 $\boxed{A}$ → 46.8540

— AND, FINALLY, A GENERAL SWEEP . SELECT LINEAR SWEEP:

$\boxed{B}$ → 0.0000 , 0 $\boxed{ENTER}$ 2 $\boxed{D}$ → (0.0000) → 0.1000 → (2.0000) → 0.0783
→ (4.0000) → 0.0311 → (6.0000) → 0.0006 --- etc

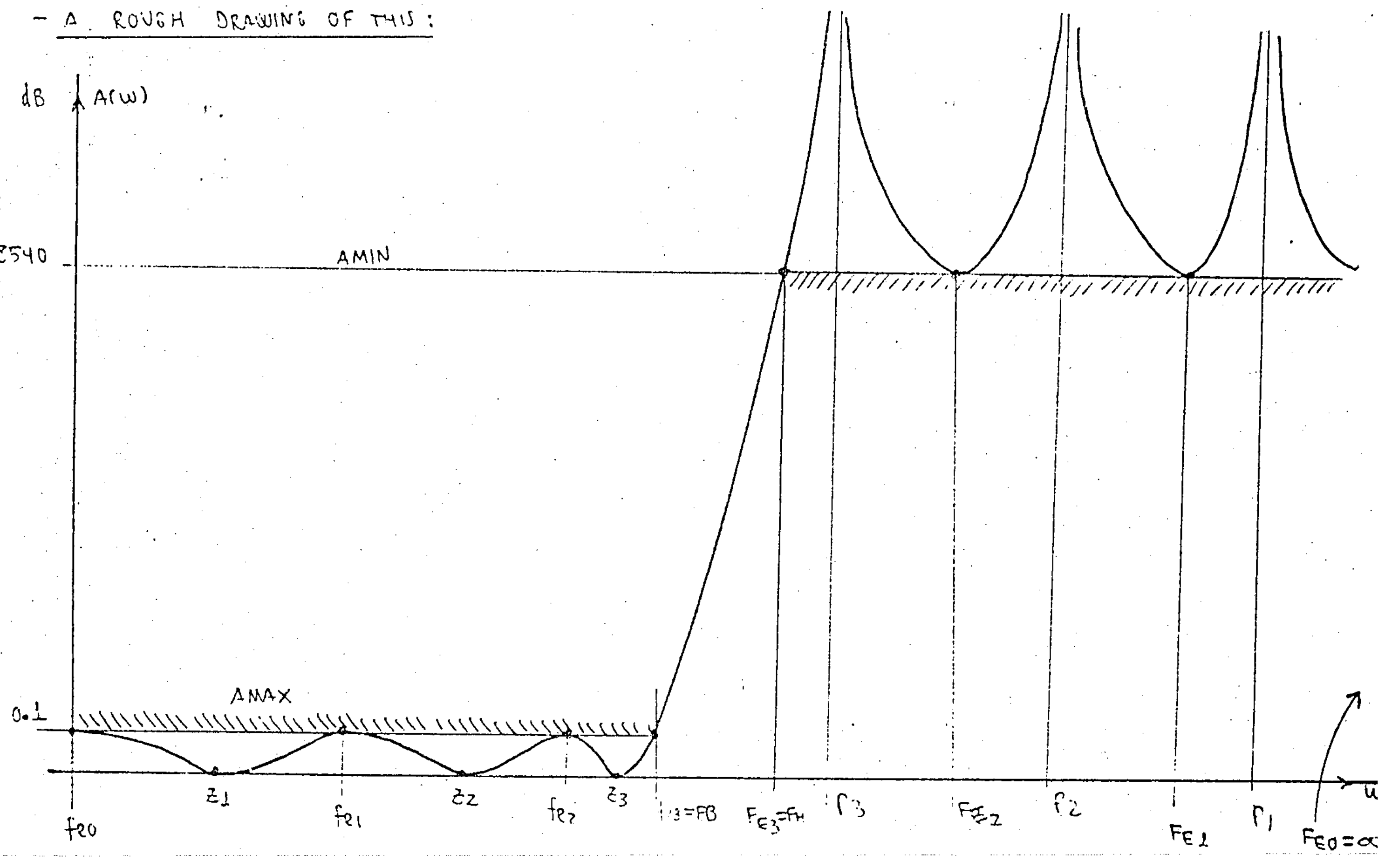
WE THUS FORM THE TABLE :

ω	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36
A(ω)	0.1000	0.0783	0.0311	0.0006	0.0200	0.0748	0.0987	0.0434	0.0036	0.0980	0.1000	9.1868	24.8645	46.8540	47.3712	48.6069	56.9113	63.0148	52.7390

PASS BAND (LOSS ≤ 0.1000)

STOPBAND (LOSS ≥ 46.8540)

— A ROUGH DRAWING OF THIS :



User Instructions

ELLIPTIC LOWPASS FILTER DESIGN - CARD 1				
			K → K'(K)	STO K
DEGREE AMIN↑AMAX↑FH↑FB	→ ZEROS & POLES	REVIEW	K → K(K)	u → Sn(u, K)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	LOAD CARD 1, BOTH SIDES				
2	INPUT AMIN, AMAX, FH, FB, AND COMPUTE DEGREE m	AMIN dB	ENTER		
		AMAX dB	ENTER		
		FH Hz	ENTER		
		FB Hz	A		DEGREE m
3	OPTIONAL: FIND ZEROS & POLES		B		→(1)→Z1→P1
					→(2)→Z2→P2
					etc
4	OPTIONAL: REVIEW ZEROS & POLES		C		→(i)→Zi→Pi
					etc
5	OPTIONAL: COMPUTE K(K)	k	D		K(K)
6	OPTIONAL: COMPUTE K'(K)	k	f	D	K'(K)
7	OPTIONAL: COMPUTE Sn (u, K)	k	f	E	k
		u	E		Sn (u, K)
8	OPTIONAL: LOAD CARD 2, SIDE 1				
9	TO COMPUTE THE LOSS AT SOME FREQUENCY ω	ω Hz	A		A(ω) dB
10	TO PERFORM A SWEEP OF FREQUENCIES:				
	a) SELECT LINEAR OR		B		0.0000
	b) SELECT LOGARITHMIC MODE		C		1.0000
	START SWEEP:	ω0 Hz	ENTER		
		Δ	D		→(ωi)→A(ωi)
	TO STOP SWEEP, PRESS R/S AFTER A(ωi)				etc
11	TO REVIEW ZEROS and POLES		E		→(i)→Zi→Pi
					etc.
12	FOR ANOTHER CASE, GO TO APPROPRIATE STEP.				

CARD 2

ELLIPTIC LOWPASS FILTER DESIGN - CARD 2 -				
		SWEEP		
ω → A(ω)	LINEAR	LOGARITHMIC	ω0↑Δ → A(ωi)	REVIEW

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	* LBL A	31 25 11	DEGREE COMPUTATION
	STO D	33 14	STORES FB
	X≷Y	35 52	
	÷	81	
	STO E	33 15	STORES $K = 1/X_L$
	R↓	35 53	
	GSB 8	31 22 08	COMPUTES ε
	STO A	33 11	STORES ε
	X≷Y	35 52	
010	GSB 8	31 22 08	} COMPUTES $\frac{1}{L}$
	÷	81	
	√	31 54	
	GSB 9	31 22 09	} COMP. & STORES $\frac{K(\frac{1}{L})}{K'(\frac{1}{L})}$
	STO C	33 13	
	RCL E	34 15	} COMP. $K(1/X_L)/K'(1/X_L)$
	GSB 9	31 22 09	
	RCL C	34 13	} $m = \left[\frac{K(\frac{1}{X_L}) \cdot K'(\frac{1}{L})}{K'(\frac{1}{X_L}) \cdot K(\frac{1}{L})}\right] + 1$
	÷	81	
	1	01	
020	+	61	
	INT	31 83	
	STO C	33 13	
	RTN	35 22	
	* LBL 8	31 25 08	AUXILIAR SUBROUTINE
	.	83	
	1	01	COMPUTES EITHER $10^{0.1\,AMAX} - 1$
	×	71	
	10^x	32 53	
	1	01	OR $10^{0.1\,AMIN} - 1$
030	−	51	
	RTN	35 22	
	* LBL 9	31 25 09	AUXILIAR SUBROUTINE
	STO 9	33 09	
	GSB D	31 22 14	$K(\frac{1}{X_L})$ OR $K(\frac{1}{L})$.
	STO 8	33 08	
	STO 0	33 00	
	RCL 9	34 09	
	GSB d	32 22 14	$K'(\frac{1}{X_L})$ OR $K'(\frac{1}{L})$
	STO ÷ 8	33 81 08	
040	RCL 8	34 08	$K(1/L)/K'(\frac{1}{L})$ OR ---
	RTN	35 22	
	* LBL D	31 25 14	$K(K)$ COMPUTATION
	P≷S	31 42	
	SIN^{-1}	32 62	} $\sqrt{1-x^2}$
	COS	31 63	
	P≷S	31 42	
	* LBL d	32 25 14	$K'(K)$ COMPUTATION
	P≷S	31 42	
	STO 9	33 09	STORE X_0
050	1	01	
	STO 8	33 08	STORE P_0
	SIN^{-1}	32 62	} STORE $\varphi_0 = \pi/2$
	STO 7	33 07	
	RCL 9	34 09	RCL X_0
	* LBL 3	31 25 03	LOOP
	COS^{-1}	32 63	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	SIN	31 62	$\sqrt{1-x^2}$
	ST I	35 33	} $1 + \sin(\cos^{-1} x_i)$
	ISZ	31 34	
060	RCL 7	34 07	} NEW $\varphi_i \to \varphi_{i+1}$
	SIN	31 62	
	×	71	
	SIN^{-1}	32 62	
	STO + 7	33 61 07	
	2	02	
	STO ÷ 7	33 81 07	
	RCL I	35 34	
	÷	81	
	STO × 8	33 71 08	$P_i \to P_{i+1}$
070	1	01	
	−	51	
	STO 9	33 09	NEW $X_i \to X_{i+1}$
	X≠0	31 61	$X_{i+1} = 0$?
	GTO 3	22 03	NO, ITERATE
	RCL 7	34 07	YES;
	2	02	
	÷	81	
	1	01	
	TAN^{-1}	32 64	} $K(K) = P \times LN[TAN[----]]$
080	+	61	
	TAN	31 64	
	LN	31 52	
	RCL 8	34 08	
	×	71	
	P≷S	31 42	
	RTN	35 22	
	* LBL B	31 25 12	ZEROS & POLES
	CF 0	35 61 00	
	RCL C	34 13	} STORES INT($m/2$) AND SETS F0 IF $m = \dot{2}$
090	2	02	
	÷	81	
	ENTER↑	41	
	INT	31 83	
	STO B	33 12	
	ST I	35 33	
	X=Y	32 51	
	SF 0	35 51 00	
	* LBL 6	31 25 06	LOOP
	RCL I	35 34	
100	2	02	
	×	71	
	0	00	
	F0?	35 71 00	EVEN?
	e^x	32 52	YES = SUBSTRACT 1
	−	51	} $u_i = \frac{2i - \{^0_1\}}{m} \cdot K(\frac{1}{X_L})$
	RCL C	34 13	
	÷	81	
	RCL 0	34 00	
	×	71	
110	GSB E	31 22 15	$\sin(u_i, 1/X_L)$
	RCL D	34 14	} $\times z_i$
	×	71	

REGISTERS

0	1	2	3	4	5	6	7	8	9
$K(\frac{1}{L})$	z_1	z_2	z_3	z_4	z_5	z_6	z_7	used, z_8	used, z_9

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
z_{10}	z_{11}	z_{12}	z_{13}	z_{14}	z_{15}	used	used	used	used

A	B	C	D	E	I
ε, used	AMIN, INT($m/2$), used	m	FB	$K = \frac{1}{X_L}$	used.

STEP	KEY ENTRY	KEY CODE	COMMENTS
	STO(i)	33 24	STORE XZi
	DSZ	31 33	
	GTO 6	22 06	LOOP
	* LBL C	31 25 13	DISPLAY & REVIEW ROUTINE
	0	00	RESET INDEX TO 0
	ST I	35 33	
	* LBL 0	31 25 00	LOOP
120	ISZ	31 34	
	RC I	35 34	
	DSP 0	23 00	} DISPLAY i
	PAUSE	35 72	
	DSP 9	23 09	
	RCL D	34 14	
	x^2	32 54	
	RCL(i)	34 24	
	−x−	31 84	DISPLAY XZi
	RCL E	34 15	} COMPUTE Xpi
130	×	71	
	÷	81	
	−x−	31 84	DISPLAY XPi
	RCL B	34 12	} ALL DISPLAYED?
	RC I	35 34	
	x≠y	32 61	
	GTO 0	22 00	NO, REPEAT
	CLX	44	YES, RESET
	DSP 2	23 02	
	RTN	35 22	
140	* LBL e	32 25 15	STORE K
	STO E	33 15	
	RTN	35 22	
	* LBL E	31 25 15	Sm(u,x) COMPUTATION
	P⇄S	31 42	
	SF 3	35 51 03	
	SF 2	35 51 02	
	8	08	u → u/8
	÷	81	
	RCL E	34 15	
150	x^2	32 54	
	STO 9	33 09	
	STO 7	33 07	
	STO 6	33 06	
	1	01	Sm(u,K) is
	+	61	
	STO 8	33 08	approximated
	STO×6	33 71 06	by a
	x^2	32 54	
	RCL 7	34 07	Taylor'
160	1	01	
	2	02	series
	×	71	
	+	61	expansion
	STO 7	33 07	
	RCL 8	34 08	
	×	71	
	RCL 6	34 06	
	5	05	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	N!	35 81	$Sm(u,x) = \frac{u}{8} - (1+k^2)\frac{(u/8)^3}{3!} + (1+14k^2+k^4)\frac{(u/8)^5}{5!} + \cdots$
170	STO 6	33 06	
	×	71	
	+	61	
	×	71	
	7	07	
	N!	35 81	
	÷	81	
	×	71	
	RCL 7	34 07	
	RCL 6	34 06	
180	÷	81	
	−	51	
	×	71	
	×	71	
	RCL 8	34 08	
	6	06	
	÷	81	
	+	61	
	×	71	
	×	71	
190	1	01	
	−	51	
	×	71	
	CHS	42	
	* LBL 4	31 25 04	DUPLICATION FORMULA
	STO 6	33 06	
	STO×6	33 71 06	
	2	02	
	×	71	
	1	01	
200	RCL 6	34 06	
	GSB 7	31 22 07	
	GSB 7	31 22 07	
	−	51	
	÷	81	
	F2?	35 71 02	DUPLICATE AGAIN
	GTO 4	22 04	
	F3?	35 71 03	DUPLICATE ONCE MORE
	GTO 4	22 04	
	P⇄S	31 42	
210	RTN	35 22	
	* LBL 7	31 25 07	AUXILIAR SUBROUTINE
	−	51	
	√x	31 54	
	×	71	
	1	01	
	RCL 9	34 09	
	RCL 6	34 06	
	STO×6	33 71 06	
	×	71	
220	RTN	35 22	
			HP97 OWNERS: USE THOSE 4 FREE STEPS TO INSERT "SPACE" AND CHANGE "PAUSE" TO "PRINT X"

LABELS

A	B	C	D	E
A DEGREE	B → ZEROS, POL.	C REVIEW	D K → Σ(K)	E u → Sm(u,K)
a	b	c	d K → Σ'(K)	e STO K
0 used REVIEW	1	2	3 used Σ(K)	4 used Sm(u,K)
5	6 used Z, P	7 used Sm(u,K)	8 used DEGREE	9 used DEGREE

FLAGS

0	ODD/EVEN
1	
2	used Sm(u,K)
3	used Sm(u,K)

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☐
GRAD	☐
RAD	☒

DISP	
FIX	☒
SCI	☐
ENG	☐
n	2

DON'T FORGET

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	31 25 11	ω → A(ω)
-	DSP4	23 04	SET DISPLAY
-	F1?	35 71 01	FIRST TIME?
-	GTO 0	22 00	YES,
-	*LBL3	31 25 03	RETURN LABEL
-	GSB1	31 22 01	GO TO LOOP AND RETURN
-	RCL0	34 00	$\}\ \varepsilon . C^2$
120 -	×	71	
-	1	01	
010	+	61	A(ω) =
-	LOG	31 53	
-	1	01	$10 \log[1+\varepsilon C^2 R_m(\omega)^2]$
-	0	00	
-	×	71	
-	RTN	35 22	
-	*LBL0	31 25 00	COMPUTE CONSTANT C
-	CF1	35 61 01	
130 -	RCLA	34 11	$\varepsilon \to R_0$
-	STO 0	33 00	
020	R↓	35 53	$\omega \to R_A$
-	STO A	33 11	
-	RCLD	34 14	FB
-	GSB1	31 22 01	Rm(FB) = 1/C
-	STO÷0	33 81 00	$\varepsilon . C \to R_0$
-	RCLA	34 11	RECALL ω
-	GTO 3	22 03	
-	*LBL1	31 25 01	COMPUTATION OF Rm(ω)
140 -	x²	32 54	
-	STO B	33 12	
030	GSB4	31 22 04	RESET I AND ODD/EVEN
-	ST I	35 33	
-	RCLB	34 12	
-	F2?	35 71 02	EVEN ?
-	1	01	
-	*LBL2	31 25 02	
-	RCLB	34 12	LOOP
-	RCL(i)	34 24	
150 -	x²	32 54	
-	−	51	
040	×	71	
-	RCLB	34 12	
-	RCLD	34 14	
-	x²	32 54	
-	RCL(i)	34 24	EVALUATES $\prod_{i=1}^{INT(\frac{m}{2})} \frac{X^2 - Xz_i^2}{X^2 - Xp_i^2}$
-	÷	81	
-	RCLE	34 15	
-	÷	81	
160 -	x²	32 54	
-	−	51	
050	÷	81	
	DSZ	31 33	
	GTO 2	22 02	
	x²	32 54	
	RTN	35 22	
	*LBL4	31 25 04	AUXILIAR SUBROUTINE
	CF2	35 61 02	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCLC	34 13	COMPUTES INT($\frac{m}{2}$)
170	2	02	
-	÷	81	
060	ENTER↑	41	
	INT	31 83	
-	x=y	32 51	EVEN?
-	SF2	35 51 02	YES, SF2
-	RTN	35 22	
-	*LBLB	31 25 12	LINEAR SWEEP
-	CF0	35 61 00	
-	0	00	
180 -	RTN	35 22	
-	*LBLC	31 25 13	LOGARITHMIC SWEEP
070	SF0	35 51 00	
-	1	01	
-	RTN	35 22	
-	*LBLD	31 25 14	SWEEP
-	P⇄S	31 42	
-	STO 9	33 09	STORE Δ
-	x⇄y	35 52	
-	*LBL5	31 25 05	LOOP
190 -	PAUSE	35 72	PAUSES ωi
-	STO 8	33 08	STORES ωi
080	P⇄S	31 42	
-	GSB A	31 22 11	COMPUTES A(ωi)
-	−x−	31 84	
-	0	00	
-	P⇄S	31 42	
-	RCL8	34 08	ωi
-	RCL9	34 09	Δ
-	F0?	35 71 00	LOGARITHMIC?
200 -	×	71	YES, $\omega_{i+1} = \omega_i \cdot \Delta$
-	+	61	NO, $\omega_{i+1} = \omega_i + \Delta$
090	GTO 5	22 05	CONTINUE SWEEP
-	*LBLE	31 25 15	REVIEW ROUTINE
-	0	00	
-	ST I	35 33	
-	*LBL6	31 25 06	
-	ISZ	31 34	
-	RCI	35 34	
-	DSP0	23 00	DISPLAYS i
210 -	PAUSE	35 72	
-	DSP9	23 09	
100	RCLD	34 14	
-	x²	32 54	
-	RCL(i)	34 24	
-	−x−	31 84	DISPLAYS Xz_i
-	RCLE	34 15	
-	×	71	
-	÷	81	
-	−x−	31 84	DISPLAYS Xp_i
220 -	GSB4	31 22 04	ALL DISPLAYED?
-	RCI	35 34	
110	x≠y	32 61	
-	GTO 6	22 06	
112	CLX	44	END OF REVIEW.

LABELS

A	B	C	D	E
ω → A(ω)	LINEAR	LOGARITH.	ω↑Δ→A(ωi)	REVIEW
a	b	c	d	e
0 used	1 used	2 used	3 used	4 used
5 used	6 used	7	8	9

FLAGS

0	
1	FIRST TIME?
2	ODD/EVEN
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	■	DEG	☐	FIX	■
1	■	☐	GRAD	☐	SCI	☐
2	☐	■	RAD	■	ENG	☐ 4
3						