[VA] SRC #009 - Pi Day 2021 Special Threaded Mode | Linear Mode

03-14-2021, 08:00 PM **Post: #1**

Valentin Albillo
Senior Member

Posts: 705
Joined: Feb 2015
Warning Level: 0%

[VA] SRC #009 - Pi Day 2021 Special

Welcome to my ***SRC #009 - π Day 2021 Special***, a small affair to commemorate that most universal and ubiquitous constant π, ***The Mother of All Constants***.

Why, π is such an über-ubiquitous constant that it appears *everywhere* in *everything* from pure math to applied sciences to stochastic processes and beyond, embedded in the very fabric of the *Universe*. You'll find π as the result of an uncountable *infinity* of non-trivial mathematical expressions, including finite and infinite series and integrals and as a root of equations (*not* polynomial, mind you, π 's a *transcendent* constant), you'll find it in the innards of fractal sets such as the *Mandelbrot set*, you'll find it by throwing needles on a grid, you'll find it by rounding numbers, you'll find it when dealing with quantum mechanics or financial instruments, ...

Thus, though it would seem difficult to find *new*, interesting appearances of π, actually that's not the case at all and I can tap that uncountable infinite set of appearances I mentioned to get ***new*** ones for this *SRC*, so get your *HP* calculator (physical or virtual as you see fit) and use its built-in manual and/or programming capabilities to deal with what follows, you don't need anything else ... :)

- **Note**: this *SRC* **isn't** intended as a challenge or anything of the sort. If you're like me, you'll probably welcome the chance to check for yourself a surprising evaluation or to calculate an intriguing root or getting to know various math-related trivia. This is what the following is all about.
  Also, please **do NOT include *CODE* panels** in your replies to this thread, as it makes it difficult for me to generate the online PDF document from it. I expect you'll kindly comply with this requirement but otherwise you'll risk your carefully crafted code appearing truncated or not at all in the final PDF and thus being irretrievably lost from the online document and making your posting it moot.

New assorted appearances of π and other trivia:

- **a.** Find a real root ***x*** in *[3, 4]* of the following equation:

$$\int_0^x \left(\frac{\sin t}{t}\, e^{t/\tan t}\right)^x .dt = \frac{x\, x^x}{x!}$$

where for aesthetic considerations ***x!*** is considered to also apply to *real* ***x***, not just integers, and the units are in radians.

- **b.** Find a real root ***x*** in *[3, 4]* of the following equation:

$$x = \frac{1}{\Omega} \int_0^x log\left(1 + \frac{\sin t}{t}\, e^{t/\tan t}\right) .dt$$

where ***Ω = LambertW(1)***, the real root of ***y e^y = 1***, *log* is the natural logarithm and the units are in radians.

- **c.** Another most famous transcendental constant also appearing everywhere is ***e*** = *2.718+*. We know that π and ***e*** are related by $e^{i\pi} + 1 = 0$, but we may ask ourselves: is there any other simpler way to get π from ***e*** which does not involve *complex* numbers ? Yes, there is, simply evaluate:

$$4 * (\text{Arctan } e - \text{Arctan } \frac{e-1}{e+1})$$

- **d.** Conversely, the volume enclosed by the *n-dimensional sphere* of radius ***R*** is given by:

$$V_n = \frac{\pi^{\frac{n}{2}} R^n}{\Gamma\left(\frac{n}{2}+1\right)}$$

Go on and evaluate the π*-th root* of the summation for *even* dimensions from *0* to ∞ of the volumes enclosed by the respective n-dimensional unit spheres *(R = 1)*.

- **e.** π also features in a song by Kate Bush (included in her 2005's album "*Aerial*") about a man who's utterly obsessed with the calculation of π (that could describe some of us here at the *MoHPC*). She sings more than a hundred digits of π and comments the following about the experience:

  - *"I really like the challenge of singing numbers, as opposed to words because numbers are so unemotional as a lyric to sing and it was really fascinating singing that. Trying to sort of, put an emotional element into singing about ... a 7 ..., you know, and you really care about that 9.*

    I find numbers fascinating, the idea that nearly everything can be broken down into numbers, it is a fascinating thing; and I think also that we are completely surrounded by numbers now, in a way that we weren't, you know, even 20, 30 years ago, we're all walking around with mobile phones and numbers on our foreheads almost; and it's like, you know, computers...

    I suppose, um, I find it fascinating that there are people who actually spend their lives trying to formulate π*; so the idea of this number, that, in a way is possibly something that will go on to infinity and yet people are trying to pin it down and put their mark on and make it theirs in a way I guess also I think, you know, you get a bit a lot of connection with mathematism and music because of patterns and shapes..."* *(Ken Bruce show, BBC Radio 2, 31 Oct 2005)*

- **f.** Finally, a little serving of trivia. About two years ago a researcher at *Google* set out a new world record by computing some 31 trillion digits of π, namely *31415926535897* digits, to be exact, and was surprised to discover that the very first digits in the output were *31415926535897*... What a coincidence !!

  Also, here you are, a bilingual joke I concocted for this *SRC* that probably only those of you who understand both English and Spanish will get:

  Q: Fear of number ***13*** is called "***Triskaidekaphobia***". How would you call *Fear of number* π ?

  A: *"**Trescatorcephobia**"* *(select to see)*

  Sorry for that. Last, for a really good laugh have a look at just a sample of modern papers on π published in what they say are reputable, peer-reviewed journals:

  - Paper **A**, **B** and **C** in the *IOSR Journal of Mathematics*
  - Paper **D** in the *International Journal of Engineering Inventions*

All your comments are welcome and appreciated. My own results and comments in a few days.

- And **remember**: please **do <u>NOT</u> include *CODE* panels** in your replies to this thread, as it makes it difficult for me to generate the online PDF document from it. I expect you'll kindly comply with this requirement but otherwise you'll risk your carefully crafted code appearing truncated or not at all in the final PDF and thus being irretrievably lost from the online document and making your posting it moot.

  Researching, testing and formatting these *SRC* takes considerable time and effort. Hence, if you *do* enjoy them and would like to see more posted in the future, consider *participating* or at least *commenting* on them so that I get *feedback* of your appreciation. Saying *"Hey, I never post a thing but I do read and enjoy them very much !"* doesn't quite cut it with me, as then I have no way to tell apart sheer laziness from blatant disinterest. Your move.

V.

Find All My HP-related Materials here: **Valentin Albillo's HP Collection**

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03-15-2021, 09:19 PM **Post: #2**

J-F Garnier
Senior Member

Posts: 495
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

Equations (a) and (b) would be good candidates for benchmarking the combined solve and integrate features on various calculators but I can't really solve them between [3,4] because the expressions to integrate are not defined outside [0, π] however I was able to check that π is indeed a solution of (a), up to 30 decimals or so with Free42. Nice expressions, I have no idea of how they were built.

For point (c), well ... it's a nice identity

J-F

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03-15-2021, 09:59 PM **Post: #3**

J-F Garnier
Senior Member

Posts: 495
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

More on item (c):

> **Valentin Albillo Wrote:** (03-14-2021 08:00 PM)
>
> We know that π and ***e*** are related by $\boldsymbol{e^{i\pi} + 1 = 0}$, but we may ask ourselves: is there **any other simpler way to get** π **from *e*** which does not involve *complex* numbers ?

I don't think we can say that the equation $e^{i\pi} + 1 = 0$ can be used to **get** π **from e**. If you try to get π from this expression, you will just end with π = acos(-1).
I don't know -and don't think there is - any relation that can be used to get π from e.

J-F

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03-15-2021, 10:33 PM (This post was last modified: 03-15-2021 10:34 PM by Gerson W. Barbosa.) **Post: #4**

Gerson W. Barbosa
Senior Member

Posts: 1,376
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

> **J-F Garnier Wrote:** (03-15-2021 09:19 PM)
>
> For point (c), well ... it's a nice identity

A nice multi-purpose identity, I would say. Replace *e* with *c*, where *c* is the speed of light (in m/s, mi/s, whatever) and see what you get. It “relates” π to any constant greater than -1. For constants less than -1 divide the result by -3.

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03-16-2021, 12:06 AM (This post was last modified: 03-17-2021 03:10 AM by Albert Chan.) **Post: #5**

Albert Chan
Senior Member

Posts: 1,311
Joined: Jul 2018

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Gerson W. Barbosa Wrote:** (03-15-2021 10:33 PM)
>
> For constants less than -1 divide the result by -3.

The reason for this is because atan(x) only return principle angle, between ±pi/2

Let the constant is x, instead of e:

atan((x-1)/(x+1)) = atan((x - tan(pi/4)) / (1 + tan(pi/4)*x) = atan(tan(atan(x) - pi/4))

If atan(x) - pi/4 ≥ -pi/2 (equivalent to ~~x ≥ -1~~ x > -1): (*)

4*(atan(x) - atan((x-1)/(x+1)) = 4*(atan(x) - (atan(x) - pi/4)) = pi

If atan(x) - pi/4 < -pi/2 (equivalent to x < -1):

4*(atan(x) - atan((x-1)/(x+1)) = 4*(atan(x) - (atan(x) - pi/4 + pi)) = -3*pi

(*) Correction: At x=-1, atan((x-1)/(x+1)) = atan(±∞) is undefined.

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03-16-2021, 12:10 AM **Post: #6**

PeterP
Member

Posts: 126
Joined: Jul 2015

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Valentin Albillo Wrote:** (03-14-2021 08:00 PM)
>
> Last, for a really good laugh have a look at just a sample of modern papers on π published in what they say are reputable, peer-reviewed journals:
>
> - Paper **A**, **B** and **C** in the *IOSR Journal of Mathematics*
> - Paper **D** in the *International Journal of Engineering Inventions*
>
> [/list]**All your comments are welcome and appreciated**. My own results and comments in a few days.

This is astounding I have to say and now I want to go out and publish papers very Sunday as well. Has academic publishing really come to this? I was recently teaching a class here at university on Space Entrepreneurship and during that class one team build a crowd sourced publishing company for academic papers. While their idea had little to do with Space I felt it had a lot of merit given the exorbitant fees academic publishing houses extract from readers without giving anything back to the creators (or the funders of the creators) of that content.

Now, where was my simple proof of Fermat's Last Theorem again? Gotta get that published lest someone steals my brilliant idea...

Cheers,

PeterP

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03-16-2021, 12:12 AM **Post: #7**

PeterP
Member

Posts: 126
Joined: Jul 2015

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Albert Chan Wrote:** (03-16-2021 12:06 AM)
>
> > **Gerson W. Barbosa Wrote:** (03-15-2021 10:33 PM)
> >
> > For constants less than -1 divide the result by -3.
>
> The reason for this is because atan(x) only return principle angle, between ±pi/2
> Let the constant is x, instead of e:
>
> atan((x-1)/(x+1)) = atan((x - tan(pi/4)) / (1 + tan(pi/4)*x) = atan(tan(atan(x) - pi/4))
>
> If atan(x) - pi/4 ≥ -pi/2 (equivalent to x ≥ -1):

4*(atan(x) - atan((x-1)/(x+1)) = 4*(atan(x) - (atan(x) - pi/4)) = pi

If atan(x) - pi/4 < -pi/2 (equivalent to x < -1):

4*(atan(x) - atan((x-1)/(x+1)) = 4*(atan(x) - (atan(x) - pi/4 + pi)) = -3*pi

Thank you Albert for that explanation to the mere mortals like myself of something which is obviously trivial to Gerson (and you). Very neat.

Cheers,

PeterP

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03-16-2021, 02:08 AM **Post: #8**

Albert Chan
Senior Member

Posts: 1,311
Joined: Jul 2018

RE: [VA] SRC #009 - Pi Day 2021 Special

> **J-F Garnier Wrote:** (03-15-2021 09:19 PM)
> Equations (a) and (b) would be good candidates for benchmarking the combined solve and integrate features on various calculators but I can't really solve them between [3,4] because the expressions to integrate are not defined outside [0,π] ...

Just search for [3, pi] instead ...

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03-16-2021, 03:38 PM **Post: #9**

robve
Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Quote:**
> Last, for a really good laugh have a look at just a sample of modern papers on ? published in **what they say** are reputable, peer-reviewed journals:

Predatory journals like these aim to make money by publishing pretty much anything as long as you pay. Predatory and hijacked journals are popping up like bad mushrooms:

https://predatoryjournals.com/journals/

Also well-known reputable conferences are hijacked. I remember attending an IEEE conference to give a technical talk. Some of the expected attendees did not show up because they travelled to the hijacked conference in Seattle. Read the confusion here:

https://academia.stackexchange.com/quest...ces-icws-2

It takes some due diligence to read and cite journal and conference proceeding papers, i.e. look for reputable publishers and professional societies such as AMS and IEEE. Reputable journals are indexed. No data is reported on "IOSR Journal of Mathematics". It is doubtful that these papers are peer reviewed by academics.

- Rob

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-16-2021, 05:59 PM (This post was last modified: 03-16-2021 08:29 PM by robve.) **Post: #10**

robve
Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Quote:**
> **a.** Find a real root *x* in [3, 4] of the following equation:

$$\int_0^x \left(\frac{\sin t}{t} e^{t/\tan t}\right)^x dt = \frac{x\,x^x}{x!}$$

The LHS is an improper integral with the integrand defined on (0,π). The improper integral can be evaluated for x in [3,pi], for example with the excellent HP PRIME or by using a vintage HP or SHARP BASIC calculator using Romberg with midpoint quadrature (for open intervals, see e.g. NR2 Ch4.4). For example, Romberg open interval integration gives 15.93599534 for x=3.141592654 on my SHARP PC-1350. *Edit*: I should add that this value is the same as x^x/Gamma(x)=15.93599533 for x=3.141592654 on my SHARP PC-1350 with `X=PI: GOSUB "GAMMA": PRINT X^X/Y`.

Rewrite the equation to

$$\int_0^x \left(\frac{\sin t}{t} e^{t/\tan t}\right)^x dt - \frac{x^x}{\Gamma x} = 0$$

After some hunting on the interval [3,π] we find the root x=π.

That makes this a remarkable equation, which I am not yet sure where it came from.

SHARP BASIC:

```
' ROMBERG QUADRATURE FOR IMPROPER INTEGRALS WITH OPEN INTERVALS
' Functions to integrate are defined with label "F1", "F2",... should return Y given X
' VARIABLES
' A,B range
' F$,F function label (or number) to integrate
' Y result
' E relative error: integral = Y with precision E (attempts E = 1E-10)
' H step size
' N max number of Romberg steps (=10)
' I iteration step
' U current row
' O previous row
' J,S,X scratch
' A(27..26+2*N) scratch auto-array

100 "QROMO" INPUT "f=F";F
110 INPUT "a=";A
120 INPUT "b=";B
' init and first midpoint step
130 E=1E-9,N=7,F$="F"+STR$ F,H=B-A,X=A+H/2: GOSUB F$: O=27,U=O+N,A(O)=H*Y,I=1
' next midpoint step
140 H=H/3,S=0
150 FOR J=1 TO 3^I STEP 3: X=A+(J-.5)*H: GOSUB F$: S=S+Y,X=A+(J+1.5)*H: GOSUB F$: S=S+Y: NEXT J
' integrate
160 A(U)=H*S+A(O)/3,S=1
170 FOR J=1 TO I: S=9*S,A(U+J)=(S*A(U+J-1)-A(O+J-1))/(S-1): NEXT J
' loop until convergence
180 IF I>5 LET Y=A(U+I): IF ABS(Y-A(O+I-1))<=E*Y+E PRINT Y: END
190 J=O,O=U,U=J,I=I+1: IF I<N GOTO 140
' no convergence, output result with error estimate
200 E=ABS((Y-A(U+N-2))/(Y+E)): PRINT Y,E: END

300 "F1" Y=(SIN X/X*EXP(X/TAN X))^B: RETURN

400 "GAMMA" IF X<=0 LET Y=9E99: RETURN
410 Y=1+76.18009173/(X+1)-86.50532033/(X+2)+24.01409824/(X+3)
420 Y=Y-1.231739572/(X+4)+1.208650974E-3/(X+5)-5.395239385E-6/(X+6)
430 Y=EXP(LN(Y*2.506628275/X)+(X+.5)*LN(X+5.5)-X-5.5): RETURN
```

b. Find a real root *x* in [3, 4] of the following equation:

Again we have an improper integral. We could follow the same strategy as **a.** to numerically solve it.

However, due to Poisson we already know that

$$W(x) = \frac{1}{\pi} \int_0^{\pi} \log\left(1 + x\frac{\sin t}{t} e^{t \cot t}\right) dt$$

Hence,

$$\Omega = W(1) = \frac{1}{\pi} \int_0^{\pi} \log\left(1 + \frac{\sin t}{t} e^{t \cot t}\right) dt$$

Rewrite the given equation and take x=π. By Poisson we have

$$\frac{1}{\Omega} \int_0^{\pi} \log\left(1 + \frac{\sin t}{t} e^{t \cot t}\right) dt = \pi$$

Hence the root of the given equation is at x=π. QED

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-16-2021, 07:36 PM **Post: #11**

Valentin Albillo
Senior Member

Posts: 705
Joined: Feb 2015
Warning Level: 0%

RE: [VA] SRC #009 - Pi Day 2021 Special

Hi all:

Thanks to **J-F Garnier**, **Gerson W. Barbosa**. **Albert Chan**, **PeterP** and **robve** for your interesting posts, much appreciated. This isn't my final results and comments yet but a few intermediate comments to things you say in your posts, read on ...

> **J-F Garnier Wrote:**
> For point (c), well ... it's a nice **identity** :)

If you say so ... this means you think that ***2 + 2 = 4*** is a *"nice identity"* too ?

> **J-F Garnier Wrote:**
> I don't think we can say that the equation e^(i π) + 1 = 0 can be used to get π from e. **If you try to get π from this expression, you will just end with π = acos(-1)**.

Well, you can isolate π in the equation and you get π = **$\log_e$(-1) / i**, which apart from constants ***-1*** and ***i*** features a *logarithm base* ***e*** as a fundamental part of it, and which your *Emu71+Math ROM* readily evaluates as:

```
>LOG((-1,0))/(0,1)

          (3.14159265359, 0),
```
i.e.: π

and I see no *cosine* in that evaluation.

> **J-F Garnier Wrote:**
> I don't know -and **don't think there is** - any relation that can be used to get π from e.

Paraphrasing ***Hamlet***:

"There are more things in heaven and earth, Jean-François, than are dreamt of in your philosophy"

In other words, you bet there are.

> **Gerson W. Barbosa Wrote:**
> A nice multi-purpose **identity**, I would say.

Thanks for your appreciation of the *expression* (not *identity*, unless like *J-F* you consider *2 + 2 = 4* an identity).

> **PeterP Wrote:**
> This is astounding I have to say and now I want to go out and publish papers very Sunday as well. **Has academic publishing really come to this?**

Some of it, regrettably **yes**. See *robve* comments immediately below and my comments on it.

> **robve Wrote:**
> Predatory journals like these aim to make money by publishing pretty much anything as long as you pay. **Predatory and hijacked journals are popping up like bad mushrooms**:

This blog entry explains it all in great detail and also includes a long list of such journals.

Science Spammers

The list includes many *IOSR* journals like the two featuring the four papers I cited.

> **robve Wrote:**
> No data is reported on "IOSR Journal of Mathematics". **It is doubtful that these papers are peer reviewed by academics**.

It isn't *"doubtful"* at all: **they aren't**, period. It's quite *impossible* that any paper claiming that π is actually a simple algebraic value would pass *any* kind of peer review by *real* academics, it would be immediately thrown to the garbage bin to keep company with papers solving the quadrature of the circle and other such nonsense.

> **robve Wrote:**
> The improper integral can be evaluated [...] using a vintage HP or SHARP BASIC calculator **using Romberg** with midpoint quadrature

I seem to remember that in some other post in another thread you said something along the lines of *"every programmer should write their own integration procedure".*

Well, I could agree in principle with that statement, writing quadrature programs is fun, but I've never bothered to write *Romberg*-based ones, as they are extremely *inefficient* in my not-so-humble opinion. Said procedures were OK for very limited *HP* calcs of old such as the slow, *RAM*-starved **HP-34C**, but for powerful models such as the **HP-71B**, say, there are much, much better, faster alternatives, some of which I've programmed in the far past, with excellent results.

I don't know why the *Math ROM* creators used *Romberg* in the ROM instead of a better, faster method but then again, they made many questionable decisions and glaring omissions as well (*J-F* has remedied some of that in his awesome *Math Pac 2* and he's not done with it yet.)

My final results/comments in a few days. Let me remind all interested people that point **d** hasn't been addressed by anyone yet. As it's my long-standing policy, if it finally goes utterly unaddressed I won't comment on it either.

Best regards.
V.

Find All My HP-related Materials here: **Valentin Albillo's HP Collection**

03-16-2021, 07:36 PM **Post: #12**

Posts: 828

Maximilian Hohmann
Senior Member

Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

Hello!

> **Valentin Albillo Wrote:** (03-14-2021 08:00 PM)
>
> **All your comments are welcome and appreciated**

No solution to your challenges, just comments...

> **Valentin Albillo Wrote:** (03-14-2021 08:00 PM)
>
> **e.** π also features in a song by Kate Bush ...

Yes, wonderful, isn't it. What an artist. Seeing and hearing her perform live is one of the top ten items in my bucket list. Unfortunately, at her rate of playing one concert every decade or so, this is probably not going to happen. Not long ago I stumbled across the Wikipedia entry about her song "Cloudbusting" which has a very interesting scientific (or pseudo- or para- scientific) background.

And regarding Pi and Pi day, my favorite science explainer on Wikipedia (not by coincidence his ph.d thesis is "Designing Effective Multimedia for Physics Education") just today realeased a video for the mathematically illiterates (like myself) about how Isaac Newton revolutionsnised the way Pi is calculated: https://www.youtube.com/watch?v=gMlf1ELvRzc

Regards
Max

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03-16-2021, 07:50 PM **Post: #13**

Albert Chan
Senior Member

Posts: 1,311
Joined: Jul 2018

RE: [VA] SRC #009 - Pi Day 2021 Special

For (a), (b), it is simpler to setup as iterative formula, X=F(X), instead of searching in range [3, pi]

```
>X=3 @ P=1E-10 @ W1=.56714329041
>INTEG(0,X,P,(SIN(IX)/IX*EXP(IX/TAN(IX)))^X) * GAMMA(X)/X^(X-1)
3.1415926536
>INTEG(0,X,P,LOGP1(SIN(IX)/IX*EXP(IX/TAN(IX)))) / W1
3.14159265358
```

Both (a),(b) already converged to X=PI (confirmed by X=PI, and run integral again)

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03-16-2021, 08:17 PM **Post: #14**

Albert Chan
Senior Member

Posts: 1,311
Joined: Jul 2018

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Valentin Albillo Wrote:** (03-14-2021 08:00 PM)
>
> [*]**d.** Conversely, the volume enclosed by the *n-dimensional sphere* of radius ***R*** is given by:
>
> $$V_n = \frac{\pi^{\frac{n}{2}} R^n}{\Gamma\left(\frac{n}{2}+1\right)}$$
>
> Go on and evaluate the π*-th root* of the summation for *even* dimensions from *0* to ∞ of the volumes enclosed by the respective n-dimensional unit spheres *(R = 1)*.

sum = 1 + pi/1! + pi²/2! + pi³/3! + ... = e^pi
sum ^ (1/pi) = e

Comment: formula give 1 for volume of 0-dimensional sphere, which seems weird.

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03-16-2021, 09:28 PM **Post: #15**

robve
Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Valentin Albillo Wrote:** (03-16-2021 07:36 PM)
>
> > **robve Wrote:**
> > The improper integral can be evaluated [...] using a vintage HP or SHARP BASIC calculator **using Romberg** with midpoint quadrature
>
> I seem to remember that in some other post in another thread you said something along the lines of *"every programmer should write their own integration procedure".*
>
> Well, I could agree in principle with that statement, writing quadrature programs is fun, but I've never bothered to write *Romberg*-based ones, as they are extremely *inefficient* in my not-so-humble opinion. Said procedures were OK for very limited *HP* calcs of old such as the slow, *RAM*-starved **HP-34C**, but for powerful models such as the **HP-71B**, say, there are much, much better, faster alternatives, some of which I've programmed in the far past, with excellent results.

Which ones you wrote are better? Adaptive Simpson is one of my favorites, but does not allow open intervals, at least not "out of the box". It easily beats Romberg in terms of speed for most integrands as long as they are well behaved. However, compared to other methods it may require much more memory for the recursive calls (with up to 9 parameters!) or a stack to simulate recursion. Recursive calls are typically 20 levels deep and typically more to get a decent accuracy.

Newton-Cotes formulas such as Romberg variants are quite popular. The methods also allow you to monitor the convergence error closely. The HP 71B appears to use a modified version of Romberg to avoid evaluating the endpoints, like my "QROMO" version.

Monte Carlo methods of integration with quasi random number generators are good for integrals over multiple dimensions.

When implementing algorithms, be aware that high-quality numerical analysis code is hard to find. Rolling out your own method that differs from existing methods is risky. My versions closely follow the methods published in NR and elsewhere in the literature (not in fake journals, LOL).

I never posted *"every programmer should write their own integration procedure".* as the quotation suggests. What post are you referring to? I may have encouraged code because the challenges are also about writing and reusing code as you've stated.

- Rob

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-16-2021, 09:41 PM (This post was last modified: 03-16-2021 09:59 PM by J-F Garnier.) **Post: #16**

J-F Garnier
Senior Member

Posts: 495
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Valentin Albillo Wrote:** (03-16-2021 07:36 PM)
>
> > **J-F Garnier Wrote:**
> > For point (c), well ... it's a nice **identity** :)
>
> If you say so ... this means you think that ***2 + 2 = 4*** is a *"nice identity"* too ?

By identity, I meant that the expression is independent from the 'e' value, and always produces π, quite similar to arctan(1/x) = π/2 - arctan(x) (for x>0).
I'm ready to admit that *identity* not the correct word, if you think so.

> **Quote:**
>
> > **J-F Garnier Wrote:**
> > I don't think we can say that the equation e^(i π) + 1 = 0 can be used to get π from e. **If you try to get π from this expression, you will just end with π = acos(-1)**.

Well, you can isolate π in the equation and you get π = **log$_e$(-1) / i**, which apart from constants ***-1*** and ***i*** features a *logarithm base* ***e*** as a fundamental part of it, and which your *Emu71+Math ROM* readily evaluates as:

```
>LOG((-1,0))/(0,1)

           (3.14159265359, 0),
```
i.e.: π

and I see no *cosine* in that evaluation.

Right but how, practically, do you compute a complex logarithm ?
log(z) = log(abs(z)) + i.arg(z) [1]
for z=(-1,0) , log(abs(z)) = 0 and arg(z)=atan2(0,-1) [ANGLE(-1,0) in HP-71's BASIC]
So, we get π **= log$_e$(-1) / i = atan2(0,-1)**
No 'e' value involved anywhere.
[1] This is the method used to compute log(z) in the HP-71's Math ROM, and all RPN/RPL descendants, and very likely the 15C too.

Quote:

> **J-F Garnier Wrote:**
> I don't know -and **don't think there is** - any relation that can be used to get π from e.

Paraphrasing ***Hamlet***:

"There are more things in heaven and earth, Jean-François, than are dreamt of in your philosophy"

In other words, you bet there are.

Looking forward to see examples, if included in your final comments !

J-F

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03-16-2021, 09:48 PM **Post: #17**

Albert Chan
Senior Member

Posts: 1,311
Joined: Jul 2018

RE: [VA] SRC #009 - Pi Day 2021 Special

> **robve Wrote:** (03-16-2021 05:59 PM)
> Rewrite the equation to
>
> $$\int_0^x \left(\frac{\sin t}{t} e^{t/\tan t}\right)^x dt - \frac{x^x}{\Gamma x} = 0$$
>
> After some hunting on the interval [3,π] we find the root x=π.
>
> That makes this a remarkable equation, which I am not yet sure where it came from.

From identity: $\int_0^\pi \left(\frac{\sin t}{t} e^{t/\tan t}\right)^x dt = \frac{\pi x^x}{x!}$, x ≥ 0

AN INTEGRAL REPRESENTATION FOR THE LAMBERT W FUNCTION

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03-16-2021, 10:09 PM **Post: #18**

robve
Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

Quote:

> **c.** Another most famous transcendental constant also appearing everywhere is e = 2.718... is there any other simpler way to get π from e which does not involve complex numbers? Yes, there is, simply evaluate:

$$\pi = 4(\arctan \mathrm{e} - \arctan \frac{\mathrm{e}-1}{\mathrm{e}+1})$$

My HP-71B gives:

>RADIANS
>4*ATAN(EXP(1))-ATAN((EXP(1)-1)/(EXP(1)+1))
3.1415926536

Looks good for sure!

Why is this equal to π? Use the famous identity and the corresponding series

$$\frac{\pi}{4} = \arctan 1 = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

We also have

$$\arctan u - \arctan v = \arctan \frac{u-v}{1+uv}$$

Therefore

$$\arctan \mathrm{e} - \arctan \frac{\mathrm{e}-1}{\mathrm{e}+1} = \arctan \frac{\mathrm{e}-(\mathrm{e}-1)/(\mathrm{e}+1)}{1+\mathrm{e}(\mathrm{e}-1)/(\mathrm{e}+1)}$$

Multiplying both sides by e+1 then expanding and cancelling terms:

$$\arctan \frac{\mathrm{e}(\mathrm{e}+1)-(\mathrm{e}-1)}{\mathrm{e}+1+\mathrm{e}(\mathrm{e}-1)} = \arctan \frac{\mathrm{e}^2+1}{\mathrm{e}^2+1} = \arctan 1$$

Bingo!

- Rob

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-16-2021, 10:26 PM **Post: #19**

Gerson W. Barbosa
Senior Member

Posts: 1,376
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Maximilian Hohmann Wrote:** (03-16-2021 07:36 PM)
> And regarding Pi and Pi day, my favorite science explainer on Wikipedia (not by coincidence his ph.d thesis is "Designing Effective Multimedia for Physics Education") just today realeased a video for the mathematically illiterates (like myself) about how Isaac Newton revolutionsnised the way Pi is calculated: https://www.youtube.com/watch?v=gMlf1ELvRzc

Thanks for the link! Isaac Newton did this and much more during 1665 and 1666, when students were sent home because of a pandemic. Too good there was no Distance Education back then and his professors left him alone for such a long time.

The wp-34s program in this 10-year old thread is an implementation of Newton's method to compute π:

https://www.hpmuseum.org/cgi-sys/cgiwrap...ead=187663

Regards,

Gerson.

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03-16-2021, 10:43 PM (This post was last modified: 03-16-2021 10:43 PM by Gerson W. Barbosa.) **Post: #20**

Gerson W. Barbosa
Senior Member

Posts: 1,376
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Valentin Albillo Wrote:** (03-14-2021 08:00 PM)
> **e.** π also features in a song by Kate Bush ...

Speaking of Art, it features also in a poem by Literature Nobel Prize winner Wisława Szymborska:

Liczba pi (Number pi)

Regards,

Gerson.

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03-16-2021, 11:29 PM (This post was last modified: 03-17-2021 04:37 AM by Gerson W. Barbosa.) **Post: #21**

Gerson W. Barbosa
Senior Member

Posts: 1,376
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

> **J-F Garnier Wrote:** (03-15-2021 09:59 PM)
> I don't know -and don't think there is - any relation that can be used to get π from e.

Et pourtant, il y en a – and yet there is at least one:

http://oeis.org/wiki/A_remarkable_formula_of_Ramanujan

Really remarkable, isn't it?

P.S.: Yet another one (I had forgotten about the Gaussian Integral)

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

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03-17-2021, 01:49 AM **Post: #22**

Valentin Albillo
Senior Member

Posts: 705
Joined: Feb 2015
Warning Level: 0%

RE: [VA] SRC #009 - Pi Day 2021 Special

> **robve Wrote:** (03-16-2021 09:28 PM)
>
> **I never posted *"every programmer should write their own integration procedure"*.** as the quotation suggests. **What post are you referring to?**

This one. I quote:

*"Also, what is the fun of doing math and calc exercises **if we don't implement numerical integration ourselves?**"*

Regards.
V.

Find All My HP-related Materials here: **Valentin Albillo's HP Collection**

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03-17-2021, 03:31 AM **Post: #23**

robve
Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Albert Chan Wrote:** (03-16-2021 08:17 PM)
>
> > **Valentin Albillo Wrote:** (03-14-2021 08:00 PM)
> >
> > [*]**d.** Conversely, the volume enclosed by the *n-dimensional sphere* of radius ***R*** is given by:
> >
> > $$V_n = \frac{\pi^{\frac{n}{2}} R^n}{\Gamma\left(\frac{n}{2}+1\right)}$$
> >
> > Go on and evaluate the *π-th root* of the summation for *even* dimensions from *0* to ∞ of the volumes enclosed by the respective n-dimensional unit spheres *(R = 1)*.
>
> sum = 1 + pi/1! + pi²/2! + pi³/3! + ... = e^pi
> sum ^ (1/pi) = e
>
> Comment: formula give 1 for volume of 0-dimensional sphere, which seems weird.

Yes, it is kind of weird. But this is connecting two seemingly unrelated formulae, which is nice.

1. Taylor series:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

2. The volume of an *n-ball* with radius *R*:

$$V_n = \frac{\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2}+1)} R^n$$

The latter simplifies to

$$\frac{\pi^k}{k!}$$

for *k=2n* and *R=1* (the conditions stated in the challenge). Therefore, the answer is e as the π root of the sum:

$$\sum_{k=0}^{\infty} \frac{\pi^k}{k!} = \mathrm{e}^{\pi}$$

I recognized the Taylor series after simplifying the sum's terms. Whenever you see a factorial in a denominator in a term of a series sum, there may be a Taylor series lurking beneath.

Nice piece of natural pie...

- Rob

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-17-2021, 03:32 AM (This post was last modified: 03-17-2021 12:26 PM by Albert Chan.) **Post: #24**

Albert Chan
Senior Member

Posts: 1,311
Joined: Jul 2018

RE: [VA] SRC #009 - Pi Day 2021 Special

> **robve Wrote:** (03-16-2021 10:09 PM)
>
> $$\arctan u - \arctan v = \arctan \frac{u-v}{1+uv}$$

This is not quite right, LHS has possible range of ±pi, RHS is limited to ±pi/2
Correct identity is more complicated: see Sum of ArcTangents

We could use this instead: atan(u) ± atan(v) = **atan2**(u±v , 1∓uv) (*)

y = atan(x) - atan((x-1)/(x+1)) // y undefined when x = -1
= atan2(x - (x-1)/(x+1), 1 + x*(x-1)/(x+1))
= atan2((x²+1)/(x+1), (x²+1)/(x+1)) // numerator always positive.

If x > -1, 4*y = 4*atan(1) = pi
If x < -1, 4*y = 4*(atan(1) - pi) = -3*pi

(*) Proof is trivial: (1+u*i) * (1±v*i) = (1∓u*v) + (u±v)*i

Phase angle of two sides matched, and we have above atan2 identity

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03-17-2021, 04:03 AM (This post was last modified: 03-17-2021 03:14 PM by robve.) **Post: #25**

robve
Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Albert Chan Wrote:** (03-17-2021 03:32 AM)
>
> > **robve Wrote:** (03-16-2021 10:09 PM)
> >
> > $$\arctan u - \arctan v = \arctan \frac{u-v}{1+uv}$$
>
> This is not quite right, LHS has possible range of ±pi, RHS is limited to ±pi/2

Right. I did not include the two necessary conditions since these hold, note the (mod π):

$$\arctan u - \arctan v = \arctan \frac{u-v}{1+uv} \pmod{\pi}, \quad uv \neq 1$$

EDIT: the arctan identity comes from

$$\tan\alpha \pm \tan\beta = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \, \tan\beta}$$

Since 0<=e<π and 0<=(e-1)/(e+1)<π we have (e.g. verify numerically)

$$\tan\arctan\mathrm{e}=\mathrm{e};\quad \tan\arctan\frac{\mathrm{e}-1}{\mathrm{e}+1}=\frac{\mathrm{e}-1}{\mathrm{e}+1}$$

Then

$$\alpha=\arctan\mathrm{e};\quad \beta=\arctan\frac{\mathrm{e}-1}{\mathrm{e}+1};\quad \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\,\tan\beta}=\frac{\mathrm{e}-\frac{\mathrm{e}-1}{\mathrm{e}+1}}{1+\mathrm{e}\frac{\mathrm{e}-1}{\mathrm{e}+1}}=1$$

Generally, tan has period π

$$\tan(k\pi+\theta)=\theta$$

for any integer *k*.

- Rob

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-17-2021, 04:42 AM **Post: #26**

robve
Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Valentin Albillo Wrote:** (03-17-2021 01:49 AM)
>
> > **robve Wrote:** (03-16-2021 09:28 PM)
> >
> > **I never posted *"every programmer should write their own integration procedure"*.** as the quotation suggests. **What post are you referring to?**
>
> **This one**. I quote:
>
> *"Also, what is the fun of doing math and calc exercises **if we don't implement numerical integration ourselves?**"*

"Me/ourselves" and "implement integration" therefore "all programmers (should) write integration"?

As in "Socrates is a man, Socrates is mortal, therefore all men are mortal"?

Quotations matter.

Not a programmer, a D in Ph'y after passing some BS, humbly not wanting to be a P in the A, just taking some late snacks on vintage stuff to honor those that came before us.

- Rob

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-17-2021, 09:27 AM **Post: #27**

J-F Garnier

Posts: 495

Senior Member

Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Gerson W. Barbosa Wrote:** (03-16-2021 11:29 PM)
>
> > **J-F Garnier Wrote:** (03-15-2021 09:59 PM)
> >
> > I don't know -and don't think there is - any relation that can be used to get π from e.
>
> *Et pourtant, il y en a* – and yet there is at least one:
>
> http://oeis.org/wiki/A_remarkable_formula_of_Ramanujan
>
> Really remarkable, isn't it?
>
> P.S.: Yet another one (I had forgotten about the Gaussian Integral)
>
> $$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

OK, I see what you (and Valentin probably too) mean and I agree of course.
By relation, I was (wrongly) limiting myself to finite expressions, like the arctan expression in Valentin's post. There are obviously many infinite sums and integrals involving e and producing pi in a way or another.

J-F

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03-17-2021, 11:32 AM **Post: #28**

Ángel Martin

Senior Member

Posts: 1,193
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Valentin Albillo Wrote:** (03-14-2021 08:00 PM)
>
> [*]**e.** π also features in a song by Kate Bush (included in her 2005's album *"Aerial"*) about a man who's utterly obsessed with the calculation of π (that could describe some of us here at the *MoHPC*). She sings more than a hundred digits of π and

Love that song ;-)
https://www.youtube.com/watch?v=W8RE2NyAiJg

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03-17-2021, 02:31 PM **Post: #29**

Massimo Gnerucci

Senior Member

Posts: 2,209
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

I love Kate Bush.

Greetings,
Massimo

-+×÷ ↔ *left is right and right is wrong*

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03-18-2021, 03:56 AM **Post: #30**

robve

Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

VA's posts are fascinating and responses are brilliant. His challenges encourages sleuthing using our advanced and vintage HP calculators and perhaps by writing some code to figure this all out.

To return the favor I hereby post two small and related challenges. These two "counter" challenges "invert" VA's common objective (if I may so): instead of writing code and (CAS) expressions, let's figure out what the given code does, find its formula and finally investigate who discovered it (online searching is permitted!). The first question of each of these two should be easy to answer.

If you do not have a HP-71B (I recently acquired mine :-)), then any Basic-capable machine can be used instead. This code is simple enough to easily convert to HPPL, RPN, Forth.

a. Consider the following HP-71B program:

```
10 P=SQR(2)
20 Q=P/2
30 DISP 2/Q
40 P=SQR(2+P)
50 Q=Q*P/2
60 IF P<2 GOTO 30
```

1. What constant does it compute?
2. What is the algebraic formula computed by this code for this constant?
3. Who discovered the formula?
4. Anything else that is interesting about this formula?

b. Consider the following HP-71B program:

```
10 P=1
20 FOR I=2 TO 1000 STEP 2
30 P=P*I/(I-1)
40 DISP 2*P*P/(I+1)
50 NEXT I
```

1. What constant does it compute?
2. What is the algebraic formula computed by this code for this constant?
3. Who discovered the formula?
4. Anything else that is interesting about this formula?

I will reply with the answers after VA posted his conclusions of the pi day challenge.

- Rob

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-18-2021, 03:44 PM **Post: #31**

Albert Chan
Senior Member

Posts: 1,311
Joined: Jul 2018

RE: [VA] SRC #009 - Pi Day 2021 Special

Hi, robve

Thanks for spending the time to add code challenges.
Both codes are calculating pi by 2/(sin(x)/x) at x=pi/2, in 2 different ways.

(a) pi is via Viète's formula:

sin(x)/x = cos(x/2) cos(x/4) cos(x/8) ...

At x = pi/2:
cos(x/2) = cos(pi/4) = √(2) / 2
cos(x/4) = √((1 + cos(x/2))/2) = √(2 + √(2)) / 2
cos(x/8) = √((1 + cos(x/4))/2) = √(2 + √(2 + √(2))) / 2
...

Generated outputs, 2/Q = 2^k * sin(pi/2^k), k = 2, 3, 4, ...
limit(2^k * sin(pi/2^k), k=∞) = 2^k * (pi/2^k) = pi // sin(ε) ≈ ε

(b) pi is via Wallis products

With roots of sin(x) = 0, ±pi, ±2pi, ±3pi, ..., and limit(sin(x)/x, x=0) = 1:

sin(x)/x = (1-(x/pi)²) * (1-(x/(2pi))²) * (1-(x/(3pi))²) ...

At x=pi/2:
LHS = 2/pi ≈ 0.63662
RHS = (1-1/4) * (1-1/16) * (1-1/36) ... = (1*3)/(2*2) * (3*5)/(4*4) * (5*7)/(6*6) ...

BTW, it is more efficient (and accurate !) to calculate P=sin(x)/x, at x=pi/2:
Bonus: code is shorter, and easier to understand.

```
10 P=1
20 FOR I=2 TO 1000 STEP 2
30 P=P-P/(I*I)
40 DISP 2/P
50 NEXT I
```

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03-20-2021, 10:36 PM (This post was last modified: 03-21-2021 02:33 AM by robve.) **Post: #32**

robve
Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Albert Chan Wrote:** (03-17-2021 03:32 AM)
>
> We could use this instead: atan(u) ± atan(v) = **atan2**(u±v , 1∓uv)   (*)
>
> y = atan(x) - atan((x-1)/(x+1))   // y undefined when x = -1
> = atan2(x - (x-1)/(x+1), 1 + x*(x-1)/(x+1))
> = atan2((x²+1)/(x+1), (x²+1)/(x+1))   // numerator always positive.
>
> If x > -1, 4*y = 4*atan(1) = pi
> If x < -1, 4*y = 4*(atan(1) - pi) = -3*pi
>
> (*) Proof is trivial: (1+u*i) * (1±v*i) = (1∓u*v) + (u±v)*i
>
> Phase angle of two sides matched, and we have above atan2 identity

A bit late to reply. Initially I also thought about matching angles between complex numbers in polar coordinates with atan2. Subtracting the angles gives the resulting angle of the complex quotient expressed in polar coordinates: $\arctan e - \arctan\frac{e-1}{e+1} = \mathrm{atan2}(e,1) - \mathrm{atan2}(e-1,e+1)$ then simplifying the quotient of the corresponding complex coordinates $\frac{\sqrt{e^2+1}}{\sqrt{(e-1)^2+(e+1)^2}} \cdot \frac{1+ei}{e+1+(e-1)i} = \frac{1}{\sqrt{2}} \cdot \frac{1}{1-i}$
where the modulus of the denominator is $\sqrt{2}$ and angle $\mathrm{atan2}(-1,1)$ i.e. representing $\frac{1\angle 0}{\sqrt{2}\angle \arctan -1}$. This gives $\arctan e - \arctan\frac{e-1}{e+1} = 0 - \arctan -1 = \arctan 1 = \frac{\pi}{4}$.

Just another way to prove this equation, which is an identity that holds for other values than e by the way (with constraints).

> **Albert Chan Wrote:** (03-17-2021 03:32 AM)
>
> We could use this instead: atan(u) ± atan(v) = **atan2**(u±v , 1∓uv)

Sure, but this is the same formula I had used in my previous post, albeit yours is in disguise using atan2 instead of atan, i.e. atan(u) ± atan(v) = atan((u±v)/(1∓uv)) = atan2(u±v , 1∓uv) the latter by definition and only if 1∓uv is nonzero, i.e. the necessary constraint I mentioned before.

- Rob

Minor edit: fix typo and $\LaTeX$. Second edit: comment on atan2 versus arctan.

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-21-2021, 07:48 PM (This post was last modified: 03-21-2021 10:38 PM by robve.) **Post: #33**

Posts: 71

robve
Member

Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

> **Albert Chan Wrote:** (03-18-2021 03:44 PM)
> Thanks for spending the time to add code challenges.
> Both codes are calculating pi by 2/(sin(x)/x) at x=pi/2, in 2 different ways.

You're very welcome!

Because this is going to be a long post, I will post the two parts **a** and **b** separately.

a.

```
10 P=SQR(2)
20 Q=P/2
30 DISP 2/Q
40 P=SQR(2+P)
50 Q=Q*P/2
60 IF P<2 GOTO 30
```

As Albert replied correctly, the code computes Viète's formula, published in 1593:

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$$

In recurrence form:

$$\lim_{n\to\infty} \prod_{i=1}^{n} \frac{a_i}{2} = \frac{2}{\pi}; \qquad a_1 = \sqrt{2}, \quad a_n = \sqrt{2+a_{n-1}}$$

In functional form:

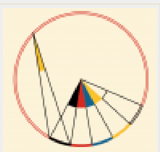
$$\pi \approx \mathrm{viete}(n) = 2/\prod_{i=1}^{n} v(n)/2; \qquad v(n) = \begin{cases} \sqrt{2} & n=1 \\ \sqrt{2+v(n-1)} & n>1 \end{cases}$$

Viète obtained his formula by comparing the areas of regular polygons with 2^n and 2^{n+1} sides inscribed in a circle. The first term in the product, $\frac{\sqrt{2}}{2}$, is the ratio of areas of a square and an octagon, the second term is the ratio of areas of an octagon and a hexadecagon, etc. - Wikipedia

See also "Mathematics: From the Birth of Numbers" by Jan Gullberg.

This directly relates to Archimedes's famous work (ca.225BC) on approximating the area of a circle by polygons inside and outside the circle squeezing the circle: "*At each stage, he needed to approximate sophisticated square roots, yet he never faltered. When he reached the 96-gon, his estimate was* $\frac{6336}{2017\frac{1}{4}} > 3\frac{10}{71}$." - "Journey Through Genius" by William Dunham.

Note that Euclid's Elements does not prove the ratio of the radius of the circle to its circumference 2π. Sometimes confused with Euclid VI.33 that proves an important property of angles and arcs but does not compare them to circles with different radii "*in equal circles* [emphasis mine], *angles, whether at the center or circumference, are in the same ratio to one another as the arcs on which they stand.*" - Oliver Byrne's Euclid 1847 VI.33



[image credit: Oliver Byrne's Euclid 1847]

Viète's formula can also be written as

$$\frac{\pi}{2} = \frac{1}{\sqrt{\frac{1}{2}}} \cdot \frac{1}{\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}} \cdot \frac{1}{\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}}} \cdots$$

In recurrence form:

$$\lim_{n\to\infty} \prod_{i=1}^{n} \frac{1}{a_i} = \frac{\pi}{2}; \qquad a_1 = \sqrt{\frac{1}{2}}, \quad a_n = \sqrt{\frac{1 + a_{n-1}}{2}}$$

In functional form:

$$\pi \approx \mathrm{viete}(n) = 2/\prod_{i=1}^{n} v(n); \qquad v(n) = \begin{cases} \sqrt{1/2} & n = 1 \\ \sqrt{(1 + v(n-1))/2} & n > 1 \end{cases}$$

While not based on Viète's formula, with our calculators we can quickly integrate the unit quarter circle defined by $1 = x^2 + y^2$, i.e. integrate $y = f(x) = \sqrt{1 - x^2}$ to get π, using a square root:

$$\pi = 4\int_0^1 \sqrt{1 - x^2}\, dx$$

Proof (I've simplified this somewhat):

$$4\int_0^1 \sqrt{1 - x^2}\, dx = 4\int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2\theta}\cos\theta\, d\theta = 4\int_0^{\frac{\pi}{2}} \sqrt{\cos^2\theta}\cos\theta\, d\theta = 4\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta = \pi$$

Programs I wrote today to illustrate Viete's formula

My own functional programming language Husky:

```
v(1) := (r,r) where r := sqrt(2)/2;
v(n) := (t,q) where t := p*q
              where q := sqrt(2+2*r)/2
              where (p,r) := v(n-1).
viete(n) := 2/p where (p,r) := v(n).
> viete(20).
```

and a list-based version:

```
prod := foldr(*, 1).
v(1) := [sqrt(2)/2];
v(n) := [sqrt(2+2*x)/2, x | xs] where x.xs := v(n-1).
viete(n) := 2/prod(v(n)).
> viete(20).
```

Haskell:

```
v 1 = (r,r) where r = (sqrt 2)/2
v n = (t,q) where t = p*q
                  q = (sqrt (2+2*r))/2
                  (p,r) = v (n-1)
viete n = 2/r where (r,_) = v n
main = putStrLn (show (viete 20))
```

and a list-based version:

```
prod = foldr (*) 1
viete n = 2/(prod (v n))
v 1 = [(sqrt 2)/2]
v n = (sqrt (2+2*x))/2 : x : xs where x:xs = v (n-1)
main = putStrLn (show (viete 20))
```

Prolog:

```
viete(N, P) :- v(N, R, _), P is 2/R.
v(1, R, R) :- R is sqrt(2)/2, !.
v(N, T, Q) :- M is N-1, v(M, P, R), Q is sqrt(2+2*R)/2, T is P*Q.
?- viete(20, P).
```

C (version with convergence check)

```
#include <stdio.h>
#include <math.h>
int main() {
  double p = sqrt(2), q = p/2;
  while (p < 2)
    q *= (p = sqrt(2+p))/2;
  printf("%.17g\n", 2/q);
}
```

My own MiniC C-like language:

```
int main() {
  float p, q;
  p = sqrt(2.0);
  q = p/2;
  while (p < 2.0)
    q *= (p = sqrt(2+p))/2;
  print 2/q;
}
$ ./minic viete.c
$ java viete
```

Java (version with convergence check):

```
import java.lang.*;
public class Viete
{
  public static void main(String[] arg)
  {
    double p = Math.sqrt(2), q = p/2;
    while (p < 2)
      q *= (p = Math.sqrt(2+p))/2;
    System.out.println(2/q);
  }
}
```

Python (version with convergence check):

```
from math import sqrt
def viete():
   p = sqrt(2)
   q = p/2
   while p < 2:
      p = sqrt(2+p)
      q *= p/2
   print(2/q)
if __name__ == "__main__":
   viete()
```

HPPL (version with convergence check):

```
EXPORT viete()
BEGIN
  LOCAL p = √2, q = p/2;
  REPEAT
    p := √(2+p);
```

```
    q := q*p/2;
  UNTIL p >= 2;
  RETURN 2/q;
END;
```

HP-71B FORTH with MultiMod (forthcoming: where can I find the pdf manual?)

Edit: Thanks to rprosperi's help to locate the missing FORTH manual, here is my HP-71B FORTH program:

```
FVARIABLE P
FVARIABLE Q
: VIETE
  2. SQRT P STO
  2. F/ Q STO
  BEGIN
    2. P RCL F+ SQRT P STO
    2. F/ Q RCL F* Q STO
    2. P RCL X>=Y?
  UNTIL
  2. Q RCL F/ F.
;
VIETE
```

HP-71B FORTH significantly extends ANS FORTH. Couldn't be happier with MultiMod installed on my HP-71B!

- Rob

Minor edit: fixed typo and added list-based functional versions of Husky and Haskell programs and HP-71B FORTH.

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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03-21-2021, 08:00 PM **Post: #34**

rprosperi
Senior Member

Posts: 4,591
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

> **robve Wrote:** (03-21-2021 07:48 PM)
> HP-71B FORTH with MultiMod (forthcoming: where can I find the pdf manual?)

Not sure which manual you're asking about.

MultiMod Manual is here.

The 71B Forth/Assembler Manual is part of the MoHPC Document Set (which I presume you have by now) but is also available here if you don't have that.

--Bob Prosperi

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03-21-2021, 10:31 PM **Post: #35**

robve
Member

Posts: 71
Joined: Sep 2020

RE: [VA] SRC #009 - Pi Day 2021 Special

> **rprosperi Wrote:** (03-21-2021 08:00 PM)
>> **robve Wrote:** (03-21-2021 07:48 PM)
>> HP-71B FORTH with MultiMod (forthcoming: where can I find the pdf manual?)
>
> The 71B Forth/Assembler Manual is part of the MoHPC Document Set (which I presume you have by now) but is also available here if you don't have that.

Great, many thanks! I had done some searches online but didn't find it before. I immediately ordered the MoHPC Document Set and used the link to the pdf to figure out how to use floating point in HP-71B FORTH and its editor. It's very easy. Within minutes after skimming the documentation I had my program entered as `SCREEN`, running and spitting out pi.

The program:

```
FVARIABLE P
FVARIABLE Q
: VIETE
  2. SQRT P STO
  2. F/ Q STO
  BEGIN
    2. P RCL F+ SQRT P STO
    2. F/ Q RCL F* Q STO
    2. P RCL X>=Y?
  UNTIL
  2. Q RCL F/ F.
;
VIETE
```

- Rob

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

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Yesterday, 05:06 PM **Post: #36**

John Keith
Senior Member

Posts: 655
Joined: Dec 2013

RE: [VA] SRC #009 - Pi Day 2021 Special

Thanks, robve, nice to see the algorithm expressed in so many languages. For completeness here is an RPL version:

```
\<< 2 \v/ DUP 2 /
DO SWAP 2 + \v/ SWAP OVER * 2 /
UNTIL OVER 2 \>=
END SWAP DROP 2 SWAP /
\>>
```

It gets the result 3.1415926536 after 19 iterations. Takes about 1.2 seconds on an HP-28S.

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Yesterday, 06:33 PM **Post: #37**

Albert Chan
Senior Member

Posts: 1,311
Joined: Jul 2018

RE: [VA] SRC #009 - Pi Day 2021 Special

> **robve Wrote:** ⇒ (03-21-2021 07:48 PM)
>
> Proof (I've simplified this somewhat):
>
> $$4\int_0^1 \sqrt{1-x^2}\,dx = 4\int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2\theta}\cos\theta\,d\theta = 4\int_0^{\frac{\pi}{2}} \sqrt{\cos^2\theta}\cos\theta\,d\theta = 4\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta = \pi$$

A comment about the last (missing) step.
Instead of using half-angle formula, cos(x/2)^2 = (1+cos(x))/2, then integrate, fold the integral.

$$\int_a^b f(x)\,dx = \int_a^b \frac{f(x)+f(a+b-x)}{2}\,dx$$

$$4\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta = 2\int_0^{\frac{\pi}{2}} (\cos^2\theta\ +\ \sin^2\theta)\,d\theta = 2\int_0^{\frac{\pi}{2}} 1\,d\theta = \pi$$

This is the same trick used in [VA] Short & Sweet Math Challenge #25.

> **Valentin Albillo Wrote:** ⇒ (02-28-2021 03:18 AM)
>
> ***My original solution for "Concoction the Third: Weird Integral"***

$$\int_1^{\varphi} \frac{\Gamma \ln(\varphi^2 - x)}{\Gamma \ln x + \Gamma \ln(\varphi^2 - x)}\, dx$$

What's so weird about this integral?

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Today, 11:20 PM **Post: #38**

Valentin Albillo
Senior Member

Posts: 705
Joined: Feb 2015
Warning Level: 0%

RE: [VA] SRC #009 - Pi Day 2021 Special

Hi, **all**:

Thanks to all of you who posted messages and/or comments to this *SRC #009 - π Day 2021 Special* (namely **J-F Garnier**, **Gerson W. Barbosa**, **Albert Chan**, **PeterP**, **robve**, **Maximilian Hohmann**, **Ángel Martín** and **Massimo Gnerucci**), for your interest and valuable contributions, much appreciated.

Now, these are my original results and comments for the various parts of this *SRC*:

- **Caveat**: *Don't expect anything ground-breaking here, this is a simple SRC, not a full-fledged challenge or even a challenge proper, and some of you already gave the correct results and explained them thoroughly as well, so there's not much to add, this will be brief.*

---

a. The root of this equation in `[3,4]` is indeed $X = \pi \sim 3.14159265359$. The base integral is:

$$I(x) = \int_0^{\pi} \left(\frac{\sin t}{t}\, e^{t/\tan t}\right)^x . dt = \frac{\pi x^x}{x!} \quad \text{for } x \geq 0$$

so that we have:

$I(\pi) = \pi\ \pi^{\pi}/\pi! = 15.9359953238$, $I(e) = \pi\ e^{e}/e! = 11.1735566407$

and this simple *HP-71B* command-line expression will evaluate ***I(X)*** and $\pi\ X^X/X!$ for any given $X \geq 0$:

```
>INPUT X @ INTEGRAL(0,PI,0,(SIN(IVAR)*EXP(IVAR/TAN(IVAR))/IVAR)^X);PI*X^X/GAMMA(X+1)

     ?1        3.14159265359     3.14159265359     {  π  }
     ?2        6.28318530717     6.2831853072      {  2 π  }
     ?EXP(1)   11.1735566407     11.1735566407     {  π e^e / e!  }
     ?PI       15.9359953238     15.9359953238     {  π π^π / π!  }
```

Additional Notes:

- **robve** asked which quadrature procedures have I written that are better than *Romberg*. Well, actually several for various machines, and using them I've been able to compute definite integrals (even difficult ones) with high precision (say *100 digits* and more) very fast. For the *HP-71B*'s implementation I've achieved speeds *faster* than the ones achieved by the *Math ROM*'s *Romberg*-based quadrature, despite the latter having the tremendous advantage of being implemented in *assembly language* (vs. my *BASIC* code) and using *15-digit/50,000-exponent* precision for extended accuracy (vs. the *12-digit/499-exponent* available to my *BASIC* code).

  I'll say no more about it as my implementation is the subject matter of my article ***"VA036 - Boldly Going - Outsmarting INTEGRAL"***, to be published soon.

---

b. The root of this equation in `[3,4]` is also $X = \pi \sim 3.14159265359$. This time the base integral is:

$$W_0(x) = \frac{1}{\pi} \int_0^{\pi} \log\left(1 + x \frac{\sin t}{t} e^{t \cot t}\right) dt.$$

which is a representation of the ubiquitous *Lambert **W*** function as a parametric definite integral, which though not new (there are other various integral representations) seems to me rather awesome nevertheless: a *definite integral* solves a *transcendental equation*, $W_0(X)\ \mathbf{e}^{W_0(X)} = X$

Particularizing its value for `X = 1` and exchanging $\mathbf{W_0(1)}$ and π we get:

$$\pi = \frac{1}{W_0(1)} \int_0^{\pi} \log\left(1 + \frac{\sin t}{t} e^{t \cot t}\right) \mathrm{d}t.$$

from where the equation is then obtained. This *HP-71B* program lets us try different values of `X` and returns the difference between the *LHS* and the *RHS*, to see how it approaches `0` when `X` approaches π:

```
1  DEF FNF(T)=LN(1+SIN(T)/T*EXP(T/TAN(T)))
2  DEF FNI(X)=INTEGRAL(0,X,0,FNF(IVAR))
3  DESTROY ALL @ W=FNROOT(0,1,FVAR*EXP(FVAR)-1)
4  INPUT X @ DISP FNI(X)/W-X @ GOTO 4

[RUN]
      ? .1        .131342478087
      ? .5        .63098466783
      ? 1        1.1030252775
      ? 1.5      1.27459847697
      ? 2        1.08227897742
      ? 2.5       .6404117652
      ? 3         .14159265358
      ? 3.14      .00159265358
      ? 3.1415    .00009265358
      ? PI       -.00000000001
```

For `X` > π, it gives an error: `ERR L1:LOG(neg)`. As expected, for `X` = π it returns ~ `0` but notice that for `X ≥ 3` it returns ~ π - `X`. Using a graphing calculator to plot the above values into a continuous graph will show why.

---

c. Though this seems to be a striking *finite* evaluation which gives π in terms of ***e*** in a much simpler and direct way than *Euler's formula* and without involving *complex* values, the magic is tarnished somewhat by the fact that ***e*** isn't really needed here, as the basic *identity* is:

$$\pi = 4 * (\ \mathit{Arctan}\ X - \mathit{Arctan}\ \frac{X-1}{X+1}\ )$$

which is valid for all $X > -1$, as some of you explained and proved. That said, there are many interesting particular cases which you can use to trick your colleagues into believing you've discovered a brand-new, amazing evaluation for π. For instance:

- Using π *itself (!)* to get π: $\pi = 4 * (\mathit{Arctan}\ \pi - \mathit{Arctan}\ \frac{\pi-1}{\pi+1})$

  `>4*(ATN(PI)-ATN((PI-1)/(PI+1))) -> 3.14159265359`

- Using the ***Golden Ratio*** ϕ to get π: $\pi = 4 * (\mathit{Arctan}\ \phi - \mathit{Arctan}\ \frac{\phi-1}{\phi+1})$

  `>P=(1+SQR(5))/2 @ 4*(ATN(P)-ATN((P-1)/(P+1))) -> 3.14159265360`

- Using the current year ***(2021)*** to get π: $\pi = 4 * (\mathit{Arctan}\ 2021 - \mathit{Arctan}\ \frac{2020}{2022})$

  `>4*(ATN(2021)-ATN(2020/2022)) -> 3.14159265358`

You can also trick them by using your phone number, your birthday or any number personally related to you or the person being tricked !

Additional Notes:

- **J-F Garnier** commented that he doesn't think there's a relation which can be used to get π from ***e*** but added he was thinking about *finite* formulae. Indeed, there are various relatively simple infinite series involving ***e*** and returning π that would do fine.

  He also insisted that deriving π from *Euler's formula* as a *logarithm* base ***e*** (namely $\pi = \log_{\mathbf{e}}(-1) / \boldsymbol{i}$) doesn't mean that ***e*** is involved because he can use *arctan* instead and this is the method used to compute *log(z)* in various *HP* calculators. To this I say that the computing methods used are irrelevant, we're talking here about *theoretical* math, not *practical* implementation details, and theoretically the derivation of π from *Euler's formula* holds and fundamentally involves ***e*** as the log base. I could give a pertinent example related to cubic equations to make it clearer but it would make this too long.

Gerson suggested replacing ***e*** by ***c*** (the speed of light in whatever units) in the formula and correctly gave the range of constants and resulting values, as well as a link to an interesting formula which, well, links ***e*** and π but I don't consider that a *bona-fide* way to derive π from ***e*** or vice versa, and the other example he gave is a *definite integral*, also quite unsatisfactory to me for the purpose: all sorts of integrals have π as a result, thus demonstrating that π is ubiquitous but nothing else. Just change π to **5** and you'll see what I mean: a definite integral is no way to *"derive" 5* from anything.

Last, **Albert Chan** and **robve** also explained several times and in various ways why the identity works.

---

d. The summation for even dimensions from **0** to infinity of the volumes enclosed by the respective *n-dimensional* unit spheres *(R = 1)* is $e^{\pi} \sim 23.1406926328$ (*aka Gelfond's constant*) and thus its π-th root is ***e***, or conversely we could say that π is its natural logarithm. The π-th root of the summation is readily obtained with the *HP-71B* by executing this from the command line:

```
>V=0 @ FOR D=0 TO 50 STEP 2 @ V=V+PI^(D/2)/FACT(D/2) @ NEXT D @ V^(1/PI)

        2.71828182846
```

which agrees with ***e*** ~ *2.71828182846*. For the 12-digit *HP-71B* we stop at dimension **50** because $\pi^{25}/25! \sim 1.73.10^{-13}$, so adding further terms won't affect the result.

Also, as I read for the first time in some Martin Gardner's book many decades ago, the volume enclosed by the *n-dimensional* unit sphere tends to **0** with growing *n*, and reaches a *maximum* for a (fractional) dimension between **5** (Vol = $8\ \pi^2/15 \sim 5.264$) and **6** (Vol = $\pi^3/6 \sim 5.168$). See if you can find this *unique* dimension and the corresponding *maximum volume*.

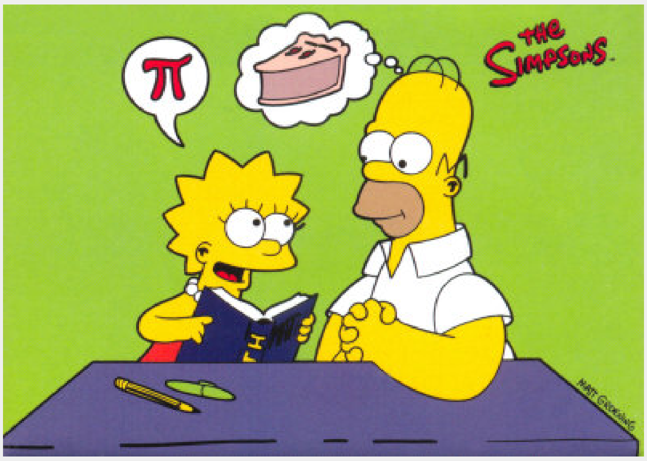
Additional Notes:

- **Albert Chan** commented that the formula gives **1** for the volume of a *0-dimensional* sphere (whatever its *"radius"*), which seems weird to him. Well, that's *by definition*. Come to that, the formula also gives **2** for the volume of the *1-dimensional* unit *"sphere"*, which is but a *line segment* that obviously has no *3D "volume"*, and it also gives π for the volume of the *2-dimensional* unit *"sphere"*, which is but a *2D circle* with no *3D "volume"* either. We tend to think of *volume* in terms of dimensions **3** or greater but mathematically that's not necessarily so.

---

e. The song "π" by Kate Bush is indeed awesome, as is most of her music (*"Cloudbusting"*, mentioned in this thread, certainly is, as is the video for it, almost a whole tragic movie told in a few minutes), and part of it appears in *The Simpsons*' 26th-season finale, *"Mathlete's Feat"*, featuring about one minute of the song or so.

π also appears in at least two other episodes which I watched at the time: in one of them, Prof. Frink unexpectedly says aloud that π is exactly equal to **3** as a way to get some much needed attention (he sure gets it !), and in another Homer and Marge are visiting some school for *"Snotty Girls and Mama's Boys"* (i.e., gifted children) and two of them are singing a hand-clapping song with lyrics they've concocted to help them remember a few digits of π. There may be many more ... (episodes featuring π, that is).



Additional Notes:

- **Ángel Martín** briefly visited the thread to express his love for the song (and **Massimo Gnerucci** did likewise for the artist herself), as did **Maximilian Hohmann**, who also mentioned *"Cloudbusting"* and its pseudo-scientific background, which I knew about from reading Martin Gardner's and James Randi's books in the distant past.

---

f. First, those `31,415,926,535,897` decimal places were reported by Emma Haruka Iwao on 2019's π *Day* after 121 days of computation. Then, on January 2020 Tim Mullican computed ***50 trillion digits*** in some 300 days, which if I'm not wrong it's the current world record as of 2021.

The resulting string of digits passes all *normality* tests, where we consider a real number *R* to be *normal* in some base *B* if any string of *N* base-*B* digits appears in the base-*B* representation of *R* with frequency B^{-N}, e.g: in base *10* every digit *0-9* appears *1/10*-th of the time, every 2-digit sequence *00-99* appears *1/100*-th of the time, etc.

It can be proved that *almost all* real numbers are normal in any and all bases *B*, but rigurously proving that a *"naturally-occurring"* real *R* (i.e., not artificially defined), say π, is normal for even just *one* base *B* (say base *2* or *10)* is *excruciatingly difficult* and not a single result is known so far, though the expectations are that all irrational numbers actually are, e.g. if you compute a trillion bits of $\sqrt{2}$, you'll find about the same number of *0's* and *1's* and about the same number of *00's*, *01's, 10's* and *11's*, etc.

Not all is lost, however. If we can't (yet) prove than π is normal in base 2 or 10, say, we can try to *estimate* the credibility of the decision *"π is **not** normal"*, which somewhat resembles *probabilistic primality testing*, where we can't rigurously prove that a certain number is prime in reasonable time but we may certify it as a *probable* prime if we can quickly prove that the probability of it being composite is *extremely small*. In the case of π's normality, this has been done by analyzing the first *16 trillion* bits of π, and the result is that the decision *"π is not normal"* has credibility $4.3497.10^{-3064}$, which in my dictionary is akin to *"impossible"*.

Second, the bilingual joke about *"Fear of number π"* being called *"Trescatorcephobia"* is a pun on *3.14* being usually said as *"tres catorce"* in Spanish.

Finally, re the *"peer reviewed"* papers demonstrating that π's value is some *algebraic* number, I'm astonished that anyone would give them credit (let alone publish them) except for non-academic reasons. Next category: papers succinctly proving *Riemann's Hypothesis* or *Fermat's Last Theorem* in a few pages, almost casually.

Additional Notes:

- **Peter P** commented *"where was my simple proof of Fermat's Last Theorem again? Gotta get that published lest someone steals my brilliant idea..."*. **LOL !** 🙂

  FLT is *almost* false, in the sense that there are infinitely many almost-*counterexamples* where the arbitrarily large *LHS* and *RHS* differ by just **1** (two other almost-counterexamples appeared in *The Simpsons* as well though not so close, namely $1782^{12} + 1841^{12} = 1922^{12}$ and $3987^{12} + 4365^{12} = 4472^{12}$). Also, a nice *"flipped"* variant of *FLT* can actually be proved using a truly *"brilliant"* idea.

  robve gave very interesting comments and links re so-called *predatory journals*, as well as a related personal experience while attending an *IEEE* conference.

That's all for now. Again, **thanks** for your interest and participation in this *SRC*, glad you enjoyed it !

Best regards.
V.

Find All My HP-related Materials here: **Valentin Albillo's HP Collection**

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