MoHPC The Museum of HP Calculators

[VA] SRC #008 - 2021 is here ! - Printable Version

[VA] SRC #008 - 2021 is here ! - Valentin Albillo - **01-02-2021 01:57 AM**

... and what better way to welcome **2021** than a little *2021-themed SRC* ?

Welcome to my *SRC #008 - 2021 is here !*, a small nice one to commemorate 2021's arrival. See if you can deliver **using your HP calculator**, physical or emulated.

- Please **NO Lua**, **NO Python**, **NO Excel**, **NO Mathematica**, etc. This is the *Museum of **HP Calculators***, so if you can't or won't use an *HP* calculator please post your results somewhere else, there are plenty of threads suitable for you to show off your non-*HP* achievements. **Thank you**.

That said, try your *HP*-hand with these three:

1) Let's partition **2021** into a set of positive integer numbers that add up to **2021**. Find the set of such numbers whose product is *maximum*, and output that maximum in all its full glory.

For instance, we could have ***2021** = 1 + 1 + ... + 1* (*2021 1's*) and their product would be *1 x 1 x ... x 1* = ***1***, which doesn't quite cut it. We could also have ***2021** = 137 + 682 + 1202* and their product would be *137 x 682 x 1202* = ***112307668***, which is much better but still far from the maximum as well.

2) Numerically evaluate as accurately as possible this nice definite integral using your favorite *HP* calc (*per standard notation, the | ... | vertical bars mean "absolute value")*:

$$\int_0^{\pi} \frac{\log\left(\dfrac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\dfrac{1}{2}}\sin\theta\,|^{2.021}\right)}{2.021\ \cos\theta\ \sin\theta}\,d\theta$$

and for extra points, see if you can *symbolically* recognize the resulting value. You'll need a sufficiently accurate value to do it, though ... :)

3) It is trivially easy to express the fraction **4/2021** as the sum of 4 numbers of the form **1/N** where N is a positive integer, namely:

4/2021 = 1/2021 + 1/2021 + 1/2021 + 1/2021

See if you can do it with just **3** such numbers.

My own solutions and comments in a few days.

- *P.S.: If you manage to solve them, do not spoil the fun for others: include your solutions in a CODE block leaving a number of blank lines at the beginning so that they can't be seen without willingly scrolling.*

V.

P.S.: Edited to include a third case, see post #21 below.

RE: [VA] SRC #008 - 2021 is here ! - Gene - **01-02-2021 02:49 AM**

Happy New Year Valentin!

My idea for #1 came after a few minutes thought. I will wait a bit to post it, but it came without an HP being used - no cheating either. I will verify that my thoughts work with an HP, but ...

Gene

RE: [VA] SRC #008 - 2021 is here ! - RMollov - **01-02-2021 06:42 AM**

#1 is interesting for me and I think I know the answer even though I can't prove it. Furthermore I'm really curious how much a calculator helps in this case; there must be some theoretical mathematical basis under and not calculations.

Thanks

RE: [VA] SRC #008 - 2021 is here ! - Nihotte(lma) - **01-02-2021 05:09 PM**

> *(01-02-2021 01:57 AM)Valentin Albillo Wrote:*
> ...
>
> My own solutions and comments in a few days.
>
> - *P.S.: If you manage to solve them, do not spoil the fun for others: include your solutions in a CODE block leaving a number of blank lines at the beginning so that they can't be seen without willingly scrolling.*
>
> **V.**

Thanks for the challenge Valentin Albillo !

Code:

```
1) at 07.09 PM 2021/01/02
2) at 10.31 PM 2021/01/02
.
.
.
.
.
.
.
.
.
.
At first, I wanted to try some different calculation configurations
I asked help to my hp35s
```

.
1)
```
 **  LBL  D
      RCL  L
      STO  I
      CLx
      STO  C
      STO  D
007 RCL  N
      STO+  C
      LOG
      STO+  D
      DSE  I
      GTO  D007
      RCL  R
      STO+  C
      LOG
      STO+  D
      RCL  C
      RCL  D
      RTN
```

With finally :
673 STO L // L is the number of loops
3   STO N // N is the incremental constant
2   STO R // R is the residual number to reach 2021
In the spirit, XEQ D001 evaluates L * N + R (LOL)
It's quite complicated but I have been making several tests around "RCL I" instead of just "RCL N"

My approach started from the way I laid the foundations of the problem :

1a) Σ for i=1..n of the xi gives 2021 : x1 + x2 + ... + xn = 2021
1b) Π for i=1..n of the xi is maximized : x1 * x2 * ... * xn is as greater as possible
It means also that LOG(Π for i=1..n of xi) is as greater as possible
Or Σ for i=1..n of LOG(xi) is maximized

I have tested several configurations but it seems to me that 673x3 + 2 gives the best result
With a result of 2021 and 2,5329955 x 10^321 [it is ALOG(LOG(3)x673 + LOG(2))]

2)
For this time, I'm already waiting after my hp35s result !!
I'm sorry it's INTEGRATING !!!
See you later !

So, I'm coming back

With my first generation of program, the hp35s didn't gives any satisfying result...
I've made a second approach :
- where SIN(π) gives -2,06761537357 x 10^-13 in RAD MODE on the hp35s, SIN(180) gives 0 in DEG MODE !
--> I prefered to make my calculations in DEG MODE

- because 2.021 x COS(180°) x SIN(180°) gives 0
--> I prefered to insert a test in my function on the hp35s, of course to avoid the error of division by 0

There is the program I used in the spirit of the HP15C :

```
   ** LBL  I
      STO  A
      RCL  A
      SIN
      STO  I
      LASTx
      COS
      STO  J
      RCL  I
      ABS
      RCL  E
      y^x
      2
      ÷
      RCL  J
      2
      1/x
      RCL  I
      x
      RCL  E
      x√y
      +
      ABS
      RCL  E
      y^x
      +
      LOG
      RCL  E
      RCL  J
      x
      RCL  I
      x
      x=y?
      GTO  I037
      ÷
      RTN
 037 1
      RTN
```

With this usage :
- 2.021 [blue] STO E
- [yellow] FN= I
- MODE 1 DEG
- 0 ENTER 180
- [yellow] ∫ A
==> INTEGRATING and ∫= 180 in DEG MODE
Since then, in RAD MODE, I think the expected result would be π...

Now, what I can regret, while wishing my answers are correct with with theory and not just by practice, is not being able to prove what I am saying in mathematical terms.
I am impatiently awaiting the answers of the specialists!

RE: [VA] SRC #008 - 2021 is here ! - J-F Garnier - **01-02-2021 10:24 PM**

Hi Valentin,

Thanks for this nice New Year present !

I was inspired by your 2nd problem:

> *(01-02-2021 01:57 AM)Valentin Albillo Wrote:* **2)** Numerically evaluate as accurately as possible this nice definite integral using your favorite *HP* calc (*per standard notation, the | ... | vertical bars mean "absolute value")*:
>
> $$\int_0^{\pi} \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\frac{1}{2}}\,\sin\theta\,|^{2.021}\right)}{2.021\ \cos\theta\ \sin\theta}\,d\theta$$

I had no special problem to evaluate it on the HP-71 (actually Emu71:-) with the Math ROM, after a simple operation.

> *Quote:*and for extra points, see if you can *symbolically* recognize the resulting value. You'll need a sufficiently accurate value to do it, though ...

Using your great "Identifying Constants" program, I found the symbolic value, a nice surprise, that lead me to another conclusion about the expression to integrate :-)

Thanks again,

J-F

RE: [VA] SRC #008 - 2021 is here ! - robve - **01-03-2021 07:33 PM**

Thanks for posting. A good motivation to write some code on HP Prime. You did not mention that RPN is required...

#1 I couldn't find a way to run HP Prime programs with bigint (only up to 64bit), so the full decimal result is computed and displayed with HP Prime CAS in the code below.

Here is another puzzle: **Compute the smallest set(s) of *square numbers* that sum up to 2021. In other words, what are the sets of fewest square numbers that sum up to 2021 such that each square number is used at most once?**

For example, 1+9+36 sums up to 46 and 1=1^2, 9=3^2, 36=6^2 are square numbers. The size of this set is three, which is minimal since no two square numbers sum up to 46. Another solution is 1+4+16+25=46, but that solution has four square numbers and thus is not a minimal set.

There may be multiple minimal sets of square numbers for 2021. A set is minimal if no other set exists that is smaller.

Code:

```
Since we want to write some code, let's verify that powers of 3 give the max,
by brute force trying all products from 2 to sqrt(2021).
We use log here, which means we can simply sum up instead of using products,
e.g. to prevent overflow on some calculators (but HP Prime handles large floats.)

EXPORT T2021()
BEGIN
  LOCAL N := 2021, M := 0, J, K, R, X;
  FOR I FROM 2 TO SQRT(N) DO
    K := FLOOR(N/I);
    R := N-K*I;
    IF R=0 THEN
      R := 1;
    END;
    X := K*LN(I)+LN(R);
    IF X>M THEN
      M := X;
      J := I;
    END;
  END;
  MSGBOX(J);
  MSGBOX(e^M);
END;

After playing with this, it is interesting that 3 is best.
The reason is that 3 is close to the number e, which you can easily see is the best number to pick, because we have that
N/3*LN(3)<N/e
and
N/2*LN(2)<N/e
for any N>0 e.g. N=2021. Graphing N/x*LN(x) shows x=e is optimal.

The full number is displayed in HP Prime CAS:

2*3^673

25329955218868262923281496551277939397110796485699809049268130708906002579675557 8
808413238305232345813667540825378197488286425521231426450591180750065933882457334
555696326219823274718662880838327321358161962623367953348770606025349498189611266
452088571663048389902914200391654464495707679152072175924067160473978181030784 6
```

RE: [VA] SRC #008 - 2021 is here ! - Dwight Sturrock - **01-03-2021 10:11 PM**

RE #1

With a brute force approach, I searched for the largest product of:

(2021/n)^n

Code:

```
Quickly got to 9.999e99 on my 15C, so switched to Free 42.
Determined that the answer appeared to be in the 700-800 range
for "n" so wrote and ran this program:

HP42S

LBL VA2021
2021
RCL 00
/
RCL 00
Y^X
PSE
1
STO +00
GTO VA2021
RTN

resulting in:
7.781840369084335804554270907184552e322
when n= ~743

2021 / 743 = ~2.72, very close to e
```

RE: [VA] SRC #008 - 2021 is here ! - J-F Garnier - **01-04-2021 10:12 AM**

> *(01-02-2021 10:24 PM)J-F Garnier Wrote:* I was inspired by [the] 2nd problem:
> ...
> I had no special problem to evaluate it on the HP-71 (actually Emu71:-) with the Math ROM, after a simple operation.

Here is my solution, not for the HP-71 but the 32S (not even the 32SII) that is, as I mentioned several times, my favourite machine for simple calculations.
Well, I cheated a bit since I used the observation I made in my previous message :-) so the execution time is reduced to less than two minutes including keystrokes. Not bad for this machine....

Code:

```
Program for the 32S/32SII:
LBL V
RCL X
COS
STO C
LASTx
SIN
STO S
2
SQRT
/
+
x²
RCL S
x²
2
/
+
LN
RCL C
/
RCL S
/
2
/
RTN


Keystokes:
RAD
FN= V
FIX 10
0
PI 2 /
§FN dX
>>  §=0.925275413
STO A
PI 2 /
PI
§FN dX
>> §=1.542125688
RCL+ A
>> 2.4674011003   ***
Exactly the expected constant (to 10 places).

J-F
```

RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-04-2021 04:00 PM**

Code:

```
For #2, it seems constant 2.021 can be changed, and still gives same integral result.
Why ?


Let k = 2.021, c = 2^(-1/k)

I = ∫ [(ln(|sin(x)|^k/2 + |cos(x)+c*sin(x)|^k) / (k*sin(x)*cos(x))] dx  , x = 0 .. pi

Let y=pi/2-x, dy=-dx:  sin → cos, cos → sin:

I = ∫ [(ln(|cos(y)|^k/2 + |sin(y)+c*cos(y)|^k) / (k*sin(y)*cos(y))] dy  , y = -pi/2 .. pi/2
  = ∫ f(y) dy            , y = -pi/2 .. pi/2
  = ∫ [f(y) + f(-y)] dy  , y = 0 .. pi/2

          |cos(y)|^k/2 + |sin(y)+c*cos(y)|^k        sec(y)^2
```

```
I = ∫ ln(-----------------------------------) × (------------) dy  , y = 0 .. pi/2
          |cos(y)|^k/2 + |sin(y)-c*cos(y)|^k       k * tan(y)

          1/2 + |t+c|^k       1 + t^2
  = ∫ ln(---------------) × (---------) dy  , where t = tan(y)      , y = 0 .. pi/2
          1/2 + |t-c|^k        k * t

For HP-71B INTEGRAL command, it preferred smooth curve.
So, we split the integral into 2 parts, when |t-c| = 0, or x = atan(c)

10 INPUT "K? ";K @ C=2^(-1/K) @ P=.0000000001
20 DEF FNT(T)=LN((.5+ABS(T+C)^K)/(.5+ABS(T-C)^K))*(1/T+T)
30 T=TIME @ S=ATAN(C)
40 S1=INTEGRAL(0,S,P,FNT(TAN(IVAR)))/K
50 S2=INTEGRAL(S,PI/2,P,FNT(TAN(IVAR)))/K
60 DISP S1;"+";S2;"=";S1+S2,TIME-T

>RUN
k? 2.021
 .936151026289 + 1.53125007399 = 2.46740110028                  .44
>RUN
k? 1
 1.0306547334 + 1.43674636688 = 2.46740110028                   .55
>RUN
k? 2
 .937458075515 + 1.52994302478 = 2.4674011003                   .44
>RUN
k? 3
 .891862933413 + 1.57553816686 = 2.46740110027                  .66
>PI*PI/4
 2.46740110027
```

RE: [VA] SRC #008 - 2021 is here ! - StephenG1CMZ - **01-04-2021 04:59 PM**

On the HP Prime, I get a result for number 2 if I use the constant 2.021 (2.021 approximately 2).
But if that "." is meant to be a thousands separator rather than decimal, representing the year 2021 rather than the year 2 :-), I get undef (in home mode).
(And to get the answers others are giving, remember to change log into Ln)

RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-04-2021 05:16 PM**

Code:

```
Post #10, we transform I by using substitution: y = pi/2-x, dy = -dx

We can continue substitutions: t = tan(y), dt = (1+t^2) dy
This shift integration limit from y = 0 .. pi/2, to t = 0 .. infinity

One more substitution, t = (1-u)/u, dt  = (-1/u^2) du
This shift back integration limit back to finite: u = 0 .. 1

This remove all use of trigonometric functions, or constant PI :)

10 INPUT "K? ";K @ C=2^(-1/K) @ P=.0000000001
20 DEF FNU(U)=LN((.5+ABS((1-U)/U+C)^K)/(.5+ABS((1-U)/U-C)^K))/(U*(1-U))
30 T=TIME @ S=1/(C+1)
40 S1=INTEGRAL(0,S,P,FNU(IVAR))/K
50 S2=INTEGRAL(S,1,P,FNU(IVAR))/K
60 DISP S1;"+";S2;"=";S1+S2,TIME-T

>RUN
k? 2.021
 1.53125007397 + .936151026314 = 2.46740110028                  .66
>RUN
k? 1
 1.43674636688 + 1.03065473339 = 2.46740110027                  .66
>RUN
k? 2
 1.52994302476 + .937458075515 = 2.46740110028                  .65
>RUN
k? 3
 1.57553816687 + .891862933407 = 2.46740110028                  .66
```

reference: http://fmnt.info/blog/20180818_infinite-integrals.html

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RE: [VA] SRC #008 - 2021 is here ! - Gene - **01-04-2021 06:56 PM**

For problem 1, I don't understand the Y^X function showing up in the answers... ?

A + B + C = 2021 (or however many #'s you propose in your solution)

Then A x B x C = Big number.

505 + 505 + 505 + 505 + 1 works, but the product would be 505 x 505 x 505 x 505 x 1 or 6.5038 x 10^10.

Biggest number I can see is a portion of the factorial.

For example 63! = (roughly) 1.98 x 10^87. The sum of 63, 62, ... down to 2 is 2015. So with a little adjustment, one could get 2021 as a sum and something in the 10^80-something as a product.

Think I've missed something :-) but don't see where. Wouldn't be the first time!

RE: [VA] SRC #008 - 2021 is here ! - ijabbott - **01-04-2021 07:24 PM**

Here's my quick and dirty RPL solution for #1:

Code:

```
« DUP
  IF 4 >
```

```
  THEN
    3 IDIV2
    CASE
      DUP 0 == THEN DROP 1 END
      DUP 1 == THEN DROP 1 - 4 END
    END
    3 ROT ^ *
  END
»
'MAXPROD' STO
2021 MAXPROD
{resulting string of 322 digits here!}
->NUM
2.53299552189E321
```

RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-04-2021 08:08 PM**

> *(01-04-2021 06:56 PM)Gene Wrote:* Biggest number I can see is a portion of the factorial.

No, we would like as many equal numbers as possible.
Say, we partition number N into n parts.

AM-GM inequality:

\(\Large{x_1\,+\,x_2\,+\,x_3\,+\,\cdots\,+\,x_n \over n} \normalsize
≥ \sqrt[n]{x_1\;x_2\;x_3\;\cdots\;x_n}\)

LHS = N/n = constant
RHS = LHS, i.e. maximized products, when \(x_1 = x_2 = x_3 = \cdots = x_n\)

RE: [VA] SRC #008 - 2021 is here ! - Gene - **01-04-2021 08:24 PM**

Ah, now I see it more clearly.

For example... 2 + 2 + 2... + 2 + 21 (suppose there are one thousand 2's there).

Sum is 2021.

Product is 2^1000 x 21 which is huge... and still not the biggest.

ty

RE: [VA] SRC #008 - 2021 is here ! - ijabbott - **01-04-2021 09:27 PM**

Some clues:

Code:

```
* Splitting a 1 out of the number makes things worse unless (unless the number is 1).
  E.g. 2 -> 1+1 -> 1*1 = 1 which is less than 2.
* Splitting 4 -> 2+2 -> 2*2 = 4, so splitting out a 4 is the same as splitting out 2 2s.
* Splitting 5 -> 2+3 -> 2*3 = 6 which is larger than 5.
  Therefore split the number only into 2s, 3s and 4s (or just into 2s and 3s since 4s can be split into 2 2s).
* Splitting 6 -> 2+2+2 -> 2*2*2 = 8, but 6 -> 3+3 -> 3*3 = 9.
  9 is larger than 8, so splitting out 3s is preferable to splitting out 2s (or 4s),
  except when that would leave a 1 remaining.
```

Conclusion:

Code:

```
* If N > 3 and N ≡ 0 (mod 3) then only split out 3s.
* If N > 3 and N ≡ 1 (mod 3) then split out a single 4 (or 2 2s) and only split out 3s for any remaining part.
* If N > 3 and N ≡ 2 (mod 3) then split out a single 2 and only split out 3s for the remaining part.
* If N < 4 then leave it as is.
```

For example:

Code:

```
 7 -> 4+3 -> 4*3 = 12 (or 7 -> 2+2+3 -> 2*2*3 = 12)
 8 -> 2+3+3 -> 2*3*3 = 18
 9 -> 3+3+3 -> 3*3*3 = 27
10 -> 4+3+3 -> 4*3*3 = 36
```

RE: [VA] SRC #008 - 2021 is here ! - telemachos - **01-04-2021 09:36 PM**

As to No. 1, my candidate for greatest product exceeds my candidate for second-greatest by 12.5%. I'll explain later. I obtained these results without coding, but I knew the route to a solution many years ago.

Valentin, while we wait for your solutions, would you be willing to tell us how you came to choose the number 5, in gold, as your avatar/icon? I ask because the image was a puzzle until yesterday, when I saw that number on a postcard.

RE: [VA] SRC #008 - 2021 is here ! - Valentin Albillo - **01-05-2021 12:52 AM**

.
Hi, **telemachos**:

> *(01-04-2021 09:36 PM)telemachos Wrote:* Valentin, while we wait for your solutions, **would you be willing to tell us how you came to choose the number 5, in gold, as your avatar/icon?**
>
> I ask because the image was a puzzle until yesterday, when I saw that number on a postcard.

Well, being *very* fond of both the number **5** and fine art, I've liked ***Charles Demuth***'s 1928 painting *"**I Saw the Figure 5 in Gold**"* since I saw it for the first time many decades ago, and I thought it would make a very nice, classy, artistic avatar for me in the *MoHPC*'s forums, which it does.

Full info on the painting **here**

Thanks for your interest in both my *SRC #008* and my avatar ! :)

Best regards.
V.

RE: [VA] SRC #008 - 2021 is here ! - robve - **01-05-2021 04:39 AM**

My solution for #1 is in my earlier post with the non-CAS program listing, followed by the full number printed with HP Prime CAS. But if you really can't accept the number printed with HP Prime CAS, then below is a simple non-CAS program to produce the full decimal result as a list of digits:
Code:

```
EXPORT POWER3()
BEGIN
  L0:=MAKELIST(0,I,1,322);
  L0(SIZE(L0)):=2;
  FOR N FROM 1 TO 673 DO
    L1:=L0;
    FOR T FROM 1 TO 2 DO
      C:=0;
      FOR I FROM SIZE(L0) DOWNTO 1 DO
        D:=L0(I)+L1(I)+C;
        IF D>9 THEN
          D:=D-10;
          C:=1;
        ELSE
          C:=0;
        END;
        L0(I):=D;
      END;
    END;
  END;
  PRINT(L0);
END;
```

RE: [VA] SRC #008 - 2021 is here ! - Valentin Albillo - **01-05-2021 07:32 PM**

Hi all:

Thanks for your interest in this *SRC #008*, many more replies than I expected, many correct solutions and last but not least, *A-Chan* finally saw the light and actually *refrained* from posting solutions using everything but *HP* calcs in my threads, and has posted **HP-71B** code ! *Shocking !* :)

To express my gratitude, and as we say in Spain *"No hay dos sin tres"*, I've edited my original post to include a ***third*** **2021**-related question, namely this one:

- *It is trivially easy to express the fraction **4/2021** as the sum of 4 numbers of the form **1/N** where N is a positive integer, namely:*

  4/2021 = 1/2021 + 1/2021 + 1/2021 + 1/2021

  *See if you can do it with just **3** such numbers.*

As I've added this third *"teaser"* I'll delay posting my own solutions to all three for a few days, to allow you to ponder it and post your own solution.

Thanks again and best regards.
V.

RE: [VA] SRC #008 - 2021 is here ! - Maximilian Hohmann - **01-05-2021 08:07 PM**

Hello!

> *(01-05-2021 07:32 PM)Valentin Albillo Wrote:* ... and last but not least, *A-Chan* finally saw the light and actually *refrained* from posting solutions using everything but *HP* calcs...

This may be due to the fact that most older HP calcs (which are the ones I - as well as many others - prefer) can't handle the 3-digit exponents of puzzle #1 :-)

Regards
Max

RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-05-2021 08:42 PM**

(01-05-2021 07:32 PM)Valentin Albillo Wrote: _

- *It is trivially easy to express the fraction **4/2021** as the sum of 4 numbers of the form **1/N** where N is a positive integer, namely:*

  4/2021 = 1/2021 + 1/2021 + 1/2021 + 1/2021

  *See if you can do it with just **3** such numbers.*

Code:

```
2021 = 43 * 47

1/(43*k1) + 1/(47*k2) + 1/(2021*k3) = 4/2021

47/k1 + 43/k2 + 1/k3 = 4

if k = k1 = k2, we have 90/k + 1/k3 = 4

k3 = 1/(4-90/k) = k/(4*k-90) = integer

k=24 → k3=4    → 4/2021 = 1/1032 + 1/1128 + 1/8084
k=30 → k3=1    → 4/2021 = 1/1290 + 1/1410 + 1/2021

Of course, there are many more solutions, if k1 ≠ k2
```

RE: [VA] SRC #008 - 2021 is here ! - Gerson W. Barbosa - **01-05-2021 11:57 PM**

I chose #2 only and I chose this one not because it's hard but because it's easy. :-)

Incidentally I got the same result when solving this equation for *x* on my HP-50g:

\[e^{3x+x\cdot e^{-x}}= 2021+\frac{2^{3+ \sqrt{2}}}{30+3^{3}\cdot 2^{\sqrt{2}}}\]

Not nearly as beautiful as Valentín's integral, but at least easier to key in.

RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-06-2021 02:20 PM**

(01-04-2021 05:16 PM)Albert Chan Wrote: _Post #10, we transform I by using substitution: y = pi/2-x, dy = -dx

We can continue substitutions: t = tan(y), dt = (1+t^2) dy
This shift integration limit from y = 0 .. pi/2, to t = 0 .. infinity

One more substitution, t = (1-u)/u, dt = (-1/u^2) du
This shift back integration limit back to finite: u = 0 .. 1

Another way is made use of symmetry.
Code:

```
I = ∫ f(y) dy, y = -pi/2 .. pi/2
  = ∫ [f(y) + f(-y)] dy, y = 0 .. pi/2         // Fold
  = ∫ g(y) dy, y = 0 .. pi/2
  = ∫ [g(y) + g(pi/2-y)] dy, y = 0 .. pi/4     // Fold again

Now, let t = tan(y), dt = (1+t^2) dy
This shift integration limit from y = 0 .. pi/4, to t = 0 .. 1

Integrand looks more complicated, but it actually converge faster !

Note: t = tan(y), tan(pi/2-y) = cot(y) = 1/t

10 INPUT "K? ";K @ C=2^(-1/K) @ P=.0000000001
20 DEF FNF(T,F)=LN((.5+(T+C)^K)*(.5+(F+C)^K)/((.5+ABS(T-C)^K)*(.5+(F-C)^K)))/T
30 T=TIME
40 S1=INTEGRAL(0,C,P,FNF(IVAR,1/IVAR))/K
50 S2=INTEGRAL(C,1,P,FNF(IVAR,1/IVAR))/K
60 DISP S1;"+";S2;"=";S1+S2,TIME-T

>RUN
k? 2.021
 1.88084223982 + .586558860435 = 2.46740110026                   .44
>RUN
k? 1
 1.47906894032 + .98833215996 = 2.46740110028                    .44
>RUN
k? 2
 1.87491615102 + .59248494925 = 2.46740110027                    .44
>RUN
k? 3
 2.08306009111 + .38434100916 = 2.46740110027                    .44
```

RE: [VA] SRC #008 - 2021 is here ! - Gene - **01-06-2021 03:54 PM**

Couple of solutions:

4/2021 = 1/506 + 1/348128 + 1/16362016
4/2021 = 1/506 + 1/348458 + 1/15664771

Using Egyptian fractions. HP-41C program in the Test Stat rom. Program originally in the PPCCJ. Form is a series of 1/N fractions where each N must be different.

RE: [VA] SRC #008 - 2021 is here ! - Nihotte(lma) - **01-06-2021 08:06 PM**

(01-02-2021 05:09 PM)Nihotte(lma) Wrote: _
Code:

```
1) at 07.09 PM 2021/01/02
```

```
2) at 10.31 PM 2021/01/02
.
.
.
.
.
.
.
.
.
.
Since then, in RAD MODE, I think the expected result would be π...
```

I'm coming back on the second point. I didn't want to stay on a half-failure...

Code:

```
2) at 08.06 PM 2021/01/06
.
.
.
.
.
.
.
.
.
.
.
I choose to enter Valentin Albillo's equation with the EQUATION editor of the HP48
Without integral sign, I obtain this result :
'LN(ABS(SIN(x))^2.021+ABS(COS(X)+XROOT(2.021,1/2)*SIN(X))^2.021)/(2.021*COS(X)*SIN(X))'
By the PLOT function in DEG mode and the H-View of -180 --> 180 with AUTOSCALE, I can see the full graph !
It shows that the graph of the function is defined in 3 parts and meets 2 limits : around -90° and +90°
(I have chosen to work in DEG mode rather than RAD for accuracy reasons on my calculators)

As I have seen it in other posts in this thread (thanks to J-F Garnier and Albert Chan, for example), I decided to calculate the integral in 2 parts : 0-->90 and 90--
>180 and natural logarithm
So, I'm coming back on the HP15C : the rest of the events take place on the HP15C in which I trust !

Here is the program for the HP15C  :

   ** f LBL C
      COS
      STO 2
      g LASTx
      SIN
      STO 1
      g ABS
      RCL 0
      y^x
      2
      ÷
      RCL 2
      2
      1/x
      RCL 0
      1/x
      y^x
      RCL 1
      x
      +
      g ABS
      RCL 0
      y^x
      +
      g LN
      RCL 0
      RCL 2
      x
      RCL 1
      x
      g TEST 5 (x=y?)
      GTO 8
      ÷
      g RTN
    * f LBL 8
      1
      g RTN


With this usage in 2 parts  :

- 2.021 STO 0
- g DEG
- 0 ENTER 90
- f ∫xy C
==> 52.95704983

- 2.021 STO 0
- g DEG
- 180 ENTER 90 (because 90 ENTER 180 falls in ERROR 8 : SOLVE ?)
- f ∫xy C
==> -88.41461966
And so 88.41461966 in the normal way by 90 ENTER 180

With the conversion of the results in ->RAD it gives :
52.95704983 --> f RAD : 0.924274882
88.41461966 --> f RAD : 1.543126220

And for the whole integral it gives :
52.95704983 + 88.41461966 --> 141.3716695 --> f RAD : 2.467401102
```

RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-06-2021 11:49 PM**

Another way to calculate I, simpler and faster 🙂

Code:

```
Let f(t) = ln((0.5+|t+c|^k)/(0.5+|t-c|^k)), where c = 2^(-1/k)

k*I = ∫(f(t)/t, t = 0 .. inf)
    = (∫f(t)/t, t = 0 .. c) + (∫f(t)/t, t = c .. inf)
    = (∫f(t)/t, t = 0 .. c) + (∫f(1/t)/t, t = 0 .. 1/c)

10 INPUT "k? ";K @ C=2^(-1/K) @ P=.0000000001
20 DEF FNF(T)=LN((.5+(T+C)^K)/(.5+ABS(T-C)^K))
30 T=TIME
40 S1=INTEGRAL(0,C,P,FNF(IVAR)/IVAR)/K
50 S2=INTEGRAL(0,1/C,P,FNF(1/IVAR)/IVAR)/K
60 DISP S1;"+";S2;"=";S1+S2,TIME-T

>RUN
k? 2.021
 .936151026284 + 1.53125007398 = 2.46740110026                 .38
>RUN
k? 1
 1.0306547334 + 1.43674636689 = 2.46740110029                  .33
>RUN
k? 2
 .937458075505 + 1.52994302476 = 2.46740110026                 .38
>RUN
k? 3
 .89186293341 + 1.57553816687 = 2.46740110028                  .39
```

RE: [VA] SRC #008 - 2021 is here ! - Valentin Albillo - **01-07-2021 10:37 PM**

Hi all:

First of all, thanks to all of you for the warm welcome to my **SRC #008** and the many contributions by **Nihotte(lma)**, **Vincent Weber**, **J-F Garnier**, **robve**, **Dwight Sturrock**, **Albert Chan**, **StephenG1CMZ**, **Albert Chan** *(again)*, **Gene**, **ijabbott**, **telemachos**, **Maximilian Hohmann**, **Gerson W. Barbosa** and last but not least, **Albert Chan**, thanks a lot to all of you for your interest and your solutions and quite varied approaches :)

These are **my original solutions**:

1) Let's partition **2021** into a set of positive integer numbers that add up to 2021. Find the set of such numbers whose product is *maximum*, and output that maximum in all its full glory.

It's easy to see that only using just the terms **2** and **3** will produce the maximum product, because:

- if a term = 5 then we can replace it by *2+3*, which contributes the same to the sum, 5, but increases the product as *2*3 = 6 > 5*.
  The same is true for terms >5 (e.g.: *6 = 3+3* but *3*3 = 9 > 6*, *7 = 3+2+2* but *3*2*2 = 12 >7*, etc.)
- if a term = 4 then it can be replaced by *2+2*, which contributes the same for both the sum and the product.
- if a term = 1 then *1+2* can be replaced by *3*, as *1*2 = 2 < 3*, and *1+3* can be replaced by *2+2*, as *1*3 = 3 < 2*2 = 4*.

Thus only terms **2** and **3** will need to be considered, and furthermore as three 2's can be replaced by two 3's, which have the same sum *(6)* but *2*2*2 = 8 < 3*3 = 9*, then at most two 2's can be part of the solution. As **2021** = *3 * 673 + 2*, then the maximum product will be the product of *673 3's* and *one 2*, i.e: **Max. P = $2*3^{673}$**.

In general, for integer $N = 3 * q + r$, where *q* is the *quotient* and *r* is the *reminder*, the maximum products are:

- if r = 0: Max. P = 3^q *(no 2's, all are 3's)*
- if r = 1: Max. P = 2^2*3^{q-1} *(two 2's, the rest are 3's)*
- if r = 2: Max. P = $2*3^q$ *(one 2, the rest are 3's)*

For *real N* all summands must be equal because of the *Arithmetic-Geometric Means* inequality, and they must be as close to ***e*** *= 2.718+* as possible, which is the reason behind the need for a majority of *3's* in the integer *N* case, as *3* is the integer closer to ***e***.

Also, as I am so adamant in *HP* calcs being used to solve the questions I propose, you may wonder where do *HP* calcs intervene in my solution ? Well, to fulfill the *"output that maximum in all its full glory"* part of the task, that's where, and to that effect I've concocted this *5-line (220-byte)* program for the ***HP-71B***, which exactly computes the 322 digits of the max. product **P = $2*3^{673}$** in half a second using the excellent J-F Garnier's ***Emu71*** emulator:

```
1   DESTROY ALL @ OPTION BASE 0 @ DIM A(36) @ K=10^9 @ A(0)=2*3^13 @ P=0
2   FOR J=1 TO 110 @ MAT A=(729)*A @ FOR I=0 TO P @ A(I+1)=A(I+1)+A(I) DIV K
3   A(I)=MOD(A(I),K) @ NEXT I @ P=P+SGN(A(P+1)) @ NEXT J @ J=0
4   DISP STR$(A(P)); @ FOR I=P-1 TO 0 STEP -1 @ J=J+1 @ IF J=5 THEN DISP @ J=0
5   A$=STR$(A(I)) @ DISP RPT$("0",9-LEN(A$));A$; @ NEXT I @ DISP

      Line 1      initializes
      Lines 2-3   compute the 322-digit product
      Lines 4-5   output the result

    >RUN

        2532995521886826292328149655127793939711079
      648569980904926813070890600257967555788084132
      383052323458136675408253781974882864255212314
      264505911807500659338824573345556963262198232
      747186628808383273213581619626233679533487706
      060253494981896112664520885716630483899029142
      003916544644957076791520721759240671604739781
      810307846
```

2) Numerically evaluate as accurately as possible this nice definite integral using your favorite *HP* calc (*per standard notation, the | ... | vertical bars mean "absolute value")*:

$$\int_0^{\pi} \frac{\log\left(\dfrac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\dfrac{1}{2}}\,\sin\theta\,|^{2.021}\right)}{2.021\ \cos\theta\ \sin\theta}\,d\theta$$

and for extra points, see if you can *symbolically* recognize the resulting value. You'll need a sufficiently accurate value to do it, though ... :)

The funny thing about this integral is that *its value remains the* ***same*** if you replace *2.021* by ***any*** positive real value !!

This means that the integral's value is the same for *2.021* or *0.2021*, *20.21*, *2021*, *Pi*, *e*, the *Golden Ratio*, your age, your phone number, etc. So it doesn't actually matter whether the *"."* in *2.021* is taken as the *decimal point* or as the *thousands separator*, as one of you wondered :)

Let's compute it using ***Free42 Decimal*** for extra accuracy. We'll need this short *32-step (60-byte)* program which defines a generalized version of the function to integrate (you can specify *2.021* or any other constant):

```
LBL "SRC82"      Y^X          1/X             ABS
MVAR "X"          2           X<>Y            RCL "X"
MVAR "Z"          /           STO ST T        STOx ST T
RCL "Z"          X<>Y          x              Y^X
SIN               2           RCL "Z"          +
ENTER            RCL "X"      COS             LN
ABS              1/X          STOx ST T       RCL/ ST Y
RCL "X"          Y^X           +              END
```

and now, using the built-in *Integrate* application (the *"S"* are *Integral* symbols):

```
 [Sf(x)]   -> Select Sf(x) Program
 [SRC82]   -> Set Vars; Select Svar
              [X][Z]

 2.021 [X] -> X=2.021
       [Z] -> [LLIM][ULIM][ACC]    [S]

  0 [LLIM] -> LLIM=0
[PI] [ULIM] -> ULIM=3.14159265359
 1E-6 [ACC] -> ACC=0.000001
       [S] -> Integrating -> 2.46739926409
 [+][SHOW] -> 2.467401100272339423194...
```

All that remains now is to *symbolically recognize* this value, for example using my ***IDENTIFY*** program (listing, description and examples here), and the result is **Pi^2/4**, which evaluates to ***2.467401100272339****65470...* so we've got 15 correct digits. You may vary the *2.021* value above and re-compute the integral, to check that indeed you get the same result.

3) Decompose **4/2021** into a sum of three fractions of the form ***1/K*** where *K* is a positive integer.

As we saw in **(1)** above, **2021** is a number of the form *N = 3*m + 2* and for such numbers we have the algebraic identity:

4/(3*m + 2) = 1/(3*m + 2) + 1/(m + 1) + 1 /((m + 1) * (3*m + 2))

so we'll use this trivial program to ask for *N* (which must be of the form *3*m + 2*) and output the resulting denominators:

```
DESTROY ALL @ INPUT N
IF MOD(N,3)#2 THEN DISP "Not of form 3*m+2" @ END
M=N DIV 3 @ DISP N;M+1;N*(M+1)

   >RUN
   ? 2021
               2021       674       1362154
```

so: **4/2021 = 1/2021 + 1/674 + 1/1362154**, and of course many other solutions are possible as well.

As another example, using the program for **2027**, which is a *prime* number and thus can't be factorized, we get:

```
   >RUN
   ? 2027
               2027       676       1370252
```

so *4/2027 = 1/2027 + 1/676 + 1/1370252*, and so on and so forth.

Note that as the first summand fraction is *1/N*, the remaining two add up to *3/N*, thus we also have a decomposition of *3/N* into *two 1/K* fractions.

Thanks again for your interest and contributions.

Best regards.
V.

Edited to include a slightly optimized version of the BASIC code (30 bytes shorter)

RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-07-2021 11:40 PM**

$$\int_0^{\pi} \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\frac{1}{2}}\sin\theta|^{2.021}\right)}{2.021\ \cos\theta\ \sin\theta} d\theta$$

> *(01-07-2021 10:37 PM)Valentin Albillo Wrote:*
>
> The funny thing about this integral is that *its value remains the* ***same*** if you replace *2.021* by ***any*** positive real value !!

Can you explain why ?

With the help of HP Prime emulator, here is the proof for k = 2:

I = ∫(ln(sin(x)^2/2 + (cos(x)+sin(x)/sqrt(2))^2) / sin(2x), x = 0. .. pi)
= ∫(ln(1+sin(2x)/sqrt(2)) / sin(2x), x = 0. .. pi)

I = ∫(ln(1+sin(2y)/sqrt(2)) / sin(2y), y = -pi/2. .. pi/2) -- y = pi/2-x, dy=-dx
= ∫(2*atanh(sin(2y)/sqrt(2)) / sin(2y), y = 0. .. pi/2) -- ln((1+x)/(1-x)) = 2*atanh(x)

I = ∫(atanh(sin(z)/sqrt(2)) / sin(z), z = 0. .. pi) -- z = 2*y, dz=2*dy
= ∫(2*atanh(sin(z)/sqrt(2)) / sin(z), z = 0. .. pi/2) -- sin(z) symmetry around pi/2

I = ∫(2*atanh(sqrt(2)*t/(1+t^2))/t, t = 0. .. 1) -- t = tan(y/2), dt=(1+t^2)/2*dx

CAS> series(atanh(√2*t/(1+t^2))/(√2),t,0,18,polynom)

t -1/3*t^3-1/5*t^5+1/7*t^7+1/9*t^9 -1/11*t^11-1/13*t^13+1/15*t^15+1/17*t^17

2*atanh(√(2)*t/(1+t*t))/t
= √(8) * (1 - t^2/3 - t^4/5 + t^6/7 + t^8/9 - t^10/11 - t^12/13 + t^14/15 + t^16/17 - ...)

Integrate term by term, t from 0 to 1:

$I = \sqrt{8} * (1 - 1/3^2 - 1/5^2 + 1/7^2 + 1/9^2 - 1/11^2 - 1/13^2 + 1/15^2 + 1/17^2 - ...)$

Cas> s1 := simplify(sum(1/(2k-1)^2, k=1..inf))   // pi^2/8
Cas> s2 := simplify(sum(1/(8k-1)^2 + 1/(8k+1)^2, k=1..inf)) // (√(2)*pi^2+2*pi^2-32)/32
Cas> simplify(sqrt(8) * (2 - s1 + 2*s2))   // **I = pi^2/4**

I can proof s1 by hand, but not s2 (any help appreciated)

RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-08-2021 02:28 AM**

> *(01-06-2021 11:49 PM)Albert Chan Wrote:* _Let f(t) = ln((0.5+|t+c|^k)/(0.5+|t-c|^k)), where c = 2^(-1/k)
>
> k*I = ∫(f(t)/t, t = 0 .. inf)
> = (∫f(t)/t, t = 0 .. c) + (∫f(t)/t, t = c .. inf)
> = (∫f(t)/t, t = 0 .. c) + (∫f(1/t)/t, t = 0 .. 1/c)

Code:

```
I should have gone further, and scale both limits, from 0 to 1:

k*I = (∫f(c*t)/t, t = 0 .. 1) + (∫f(c/t)/t, t = 0 .. 1) = (∫g(t), t = 0 .. 1)

What is nice is c = 2^(-1/k) is not needed in the code !
Another hint that value of k does not matter ...

Combined parts into 1 integral, the code also run faster.

10 INPUT "k? ";K @ P=.0000000001
20 DEF FNG(T,F)=LN((1+(1+T)^K)*(1+(F+1)^K)/((1+(1-T)^K)*(1+(F-1)^K)))/T
30 T=TIME
40 DISP INTEGRAL(0,1,P,FNG(IVAR,1/IVAR))/K;TIME-T

>RUN
k? 2.021
 2.46740110026  .28
>RUN
k? 1
 2.46740110027  .3
>RUN
k? 2
 2.46740110026  .27
>RUN
k? 3
 2.46740110026  .27
```

For k=1, g(t) is simple enough that Wolfram Alpha can proof it (after a few seconds)

I = ∫(ln((2+t)/(2-t)*(2t+1))/t, t = 0 .. 1) = pi^2/4

RE: [VA] SRC #008 - 2021 is here ! - J-F Garnier - **01-08-2021 10:04 AM**

> *(01-07-2021 10:37 PM)Valentin Albillo Wrote:* _... I've concocted this unoptimized *5-line (250-byte)* program for the ***HP-71B***, which exactly computes the 322 digits of the max. product **P = 2*3**^{673} in half a second using the **excellent J-F Garnier's *Emu71*** emulator

Thanks :-)
Emu71/DOS is more than 20 years old but still going strong !

> *(01-07-2021 11:40 PM)Albert Chan Wrote:* _
>
> $$\int_0^{\pi} \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\frac{1}{2}}\sin\theta|^{2.021}\right)}{2.021\ \cos\theta\ \sin\theta} d\theta$$
>
>> *(01-07-2021 10:37 PM)Valentin Albillo Wrote:* _
>>
>> The funny thing about this integral is that *its value remains the **same*** if you replace *2.021* by ***any*** positive real value !!
>
> Can you explain why ? ???

Yes, please Valentin; I was expecting to get an explanation or at least some indications.

I used this effect to actually compute the integral on the HP-32S with the value k=2 to replace the exponentiation operations with simple squaring, for a much faster computation.

Also I computed the integral in two parts, from 0 to pi/2 and pi/2 to pi, since the pi/2 value causes a 0/0 limit of the function. The HP algorithm is robust when this kind of limit is at the edges, but not always when it is within the integration range especially when using high target accuracy (I used accuracy= 1E-10).

J-F

RE: [VA] SRC #008 - 2021 is here ! - EdS2 - **01-08-2021 02:32 PM**

I second these calls for an explanation of the inner workings of this ingenious and unexpected integral - but I have another request too: how, Valentin, did you discover or construct this integral?

RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-12-2021 04:27 AM**

Re-posting this from Spence function thread, per Valentin Albillo's request.

> *(01-11-2021 07:16 PM)Albert Chan Wrote:* _

$$\int_0^{\pi} \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\frac{1}{2}}\sin\theta|^{2.021}\right)}{2.021\ \cos\theta\ \sin\theta} d\theta$$

> *(01-07-2021 10:37 PM)Valentin Albillo Wrote:*
>
> The funny thing about this integral is that *its value remains the **same*** if you replace *2.021* by ***any*** positive real value !!

I think Spence's function (Dilogarithms) is the key to proof I = pi^2/4, for any k

After transformation, we have I = ∫(g(t) dt, t = 0 .. 1)

\(\exp(g(t)·k\,t) = \Large
{(1+(1+t)^k)·(1+({1\over t}+1)^k) \over (1+(1-t)^k)·(1+({1\over t}-1)^k)} =
{(1+(1+t)^k)·(t^k+(1+t)^k) \over (1+(1-t)^k)·(t^k+(1-t)^k)}\)

For now, assume k is positive integer → numerator is polynomial of t, with degree 2k.

Let W = exp((2n+1)/k*pi*i), n = 0 .. k-1. In other words, W = roots of x^k = -1

roots of numerator , t = W-1, 1/(W-1)
roots of denominator, t = 1-W, 1/(W+1)

Consider just 1 of W (we have k of them), factor it out, scaled away k, and integrate:

\(\displaystyle \int _0^1 \ln\left({(1-{t \over w-1})·(1-t(w-1)) \over
(1-{t \over 1-w})·(1-t(w+1)) }\right) {dt \over t} \normalsize
= -Li_2({1\over w-1}) + Li_2({1\over 1-w}) - Li_2(w-1) + Li_2(w+1) \)

I = average of all the k pieces, dropped imaginary parts (since I is real)

For example, for k = 7, these are the integral pieces.

```
>>> from mpmath import *
>>> Li2 = lambda x: polylog(2, x)
>>> k = 7
>>> W = [exp((2*n+1)/k*pi*1j) for n in range(k)]
>>> for w in W: print -Li2(1/(w-1)) + Li2(1/(1-w)) - Li2(w-1) + Li2(w+1)
...
(2.46740110027234 + 5.0951007958138j)
(2.46740110027234 + 2.21291211873152j)
(2.46740110027234 + 0.940187304494618j)
(2.46740110027234 + 1.22410719457408e-16j)
(2.46740110027234 - 0.940187304494618j)
(2.46740110027234 - 2.21291211873152j)
(2.46740110027234 - 5.0951007958138j)
```

All the real parts have the same size = pi^2/4 🙂

What happened if k is not integer ? Lets try k = 2.021
Amazingly, for any non-zero real k, real part of the integral piece still = pi^2/4

```
>>> w = exp(pi*1j/2.021)
>>> print -Li2(1/(w-1)) + Li2(1/(1-w)) - Li2(w-1) + Li2(w+1)
(2.46740110027234 + 1.85770404643526j)
```

Note: this only show how integral evaluated to the same value, for different k's
It is still not a proof, but much closer than what I had before ...

RE: [VA] SRC #008 - 2021 is here ! - Valentin Albillo - **01-14-2021 02:42 AM**

Hi, all:

Thanks for your interest in my ***SRC #008***, some comments to your posts and some answers to your questions:

> *Nihotte(lma) Wrote:*I have tested several configurations but it seems to me that 673x3 + 2 gives the best result With a result of 2021 and **2,5329955 x 10^321** ...
> I'm coming back on the second point. I didn't want to stay on a half-failure... and for the whole integral it gives : [...] **2.467401102**

You gave the first correct solution to *Case 1* but forgot to also give the exact 322-digit result as requested. Not bad ! And your value for the integral using the **HP-15C** is correct as well, congratulations.

> *J-F Garnier Wrote:*Using your great "Identifying Constants" program, I found the symbolic value, a nice surprise, that lead me to another conclusion about the expression to integrate :-)

Hehe, you got it, both the symbolic value ($Pi^2/4$) and the fact that the integral's value doesn't depend on the constant being *2.021* and thus you could use 2 instead to speed up the numerical integration using the *HP32S* no less. And thanks a lot for your kind comments and the link to my *Article*.

> *robve Wrote:*Thanks for posting. A good motivation to write some code on HP Prime. You did not mention that RPN is required...

Indeed, the *Prime* is an HP calculator alright. And your fine solution is the very first one to give the full 322-digit correct value for *Case 1*.

> *Albert Chan Wrote:(many, many things spread over many, many posts)*

Thanks for your abundant, relentless and interesting approaches both numerical and theoretical, Albert, and most especially for using HP calculators to obtain them, as I requested. At least you did for your very first posts ...

> *StephenG1CMZ Wrote:*But if that "." is meant to be a thousands separator rather than decimal, representing the year 2021 rather than the year 2 , I get undef ...

As you may have seen, the dot may be taken as the *decimal* separator or the *thousands* separator and it willl make no difference to the result 🙂

> *ijabbott Wrote:*Some clues: Splitting a 1 [...]

Your detailed explanation mimics my original solution and is quite didactic for people who would like to understand the logic behind the result, well done.

*Maximilian Hohmann Wrote:*This may be due to the fact that most older HP calcs (which are the ones I - as well as many others - prefer) can't handle the 3-digit exponents of puzzle #1 :-)

Well, that shouldn't be a problem, many of my former *Challenges* and *Short & Sweet Math Challenges* did require multiprecision values. Remember for instance my *"Sweet & Short Math Challenges #15: April 1st Spring Special"*, where I posted a 9-line program for the **HP-71B** which computed and printed in full the exact solution to the *"Take 5"* sub-challenge, namely 2^{65536}-3, which is a **19,729**-digit integer. There were **RPL** programs doing likewise, too.

*Gerson W. Barbosa Wrote:*I chose #2 only and I chose this one not because it's hard but because it's easy. :-) [...] Not nearly as beautiful as Valentín's integral, but at least easier to key in.

Hehe, Gerson, you're playing tricks here, your (very nice, by the way) expression evaluates to **2.46740110027**18... while the correct value is $Pi^2/4$ = **2.46740110027**23... (Very) close but no cigar !

*Gene Wrote:*Couple of solutions: [...] Using Egyptian fractions. HP-41C program in the Test Stat rom.

Very correct, if different from my original solution. And a very useful reference, thanks.

*Albert Chan Wrote:*Can you explain why ?

*J-F Garnier Wrote:*Yes, please Valentin; I was expecting to get an explanation or at least **some indications**.

*EdS2 Wrote:*I second these calls for an explanation of the inner workings of this ingenious and unexpected integral

Thanks for your interest and well, my goal with these *SRC*'s and *Challenges* is to show some interesting HP-calc programming techniques (mostly for the classic models), as well as some interesting, little-known and even *unexpected* math topics, hopefully for the enjoyment of (at least some) *MoHPC* forum's readers. Also, if I can provide technical details or proofs which aren't too lengthy and/or complicated, I'll usually do it.

For instance, this is not the first time I post a definite integral allegedly dependent on some parameter but actually being independent of it, such as the two integrals featured in my *Short & Sweet Math Challenge #18: April 1st, 2007 Spring Special*, namely:

$$I1 = \int_0^{Inf} 1/((1+x^2)*(1+x^{4.012007}))\ .dx$$

and

$$I2 = \int_0^{Pi/2} 1/(1+Tan(x)^{4.012007})\ .dx$$

which actually are one and the same upon a simple change of variables and their value is Pi/4. The proof of their being independent of the parameter (the *4.012007* value) is short enough that I did post my own proof of the fact in *Message #54* in that thread.

However, for the present integral the proof is not that easy or short, actually it's quite long and would require a lot of time and cumbersome math *"typesetting"* to post it here, which regrettably is beyond the scope of these *SRC*'s. I'll give however a link to *"some indications"* for a very similar integral, which you can find here.

The proof for the similar integral I posted (I don't have a link to the proof, sorry) essentially goes along the same lines. Also, if you think that an integral with a parameter whose value is independent of it is something quite unexpected, wait till you see the next integral I'll post in a future *SRC*, it's *10x* as much !

Best regards to all and thanks for participating.
V.

RE: [VA] SRC #008 - 2021 is here ! - Vincent Weber - **01-14-2021 02:59 PM**

Hi Valentin,

And not one word about my attempt to give a general, mathematical (albeit certainly flawed in many ways) solution for #1 ? Ok, ok, I didn't try to compute it for utmost precision with an HP calculator... but I gave it some thoughts. Just teasing

Cheers,

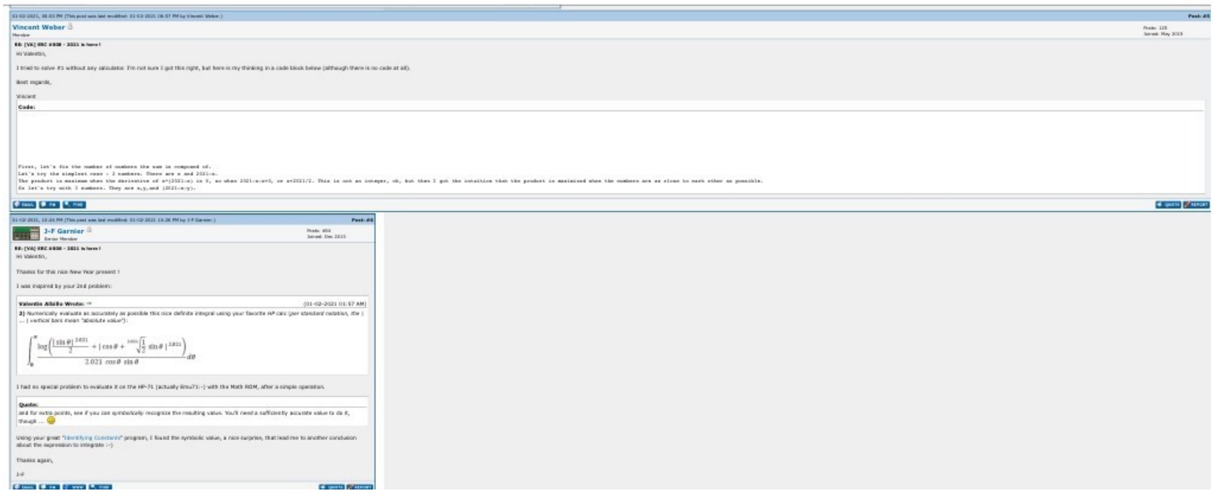
Vincent

RE: [VA] SRC #008 - 2021 is here ! - Valentin Albillo - **01-14-2021 07:51 PM**

.
Hi, Vincent:

(01-14-2021 02:59 PM)Vincent Weber Wrote: And not one word about my attempt to give a general, mathematical (albeit certainly flawed in many ways) solution for #1 ?

Matter of fact I couldn't read your message. I don't know what browser or app you used to format and post it, but this is how it does appear in my browser and yes, that infinitely long message is the one you posted ...



... . As you may see, it dwarfs all other messages in the thread, in fact it extends much beyond the limits of the browser's window and forces me to zoom a lot to be able to read all other messages which are scaled down to unreadable small text, while yours can't be scrolled horizontally any further but is cropped instead so I can't read it properly either, it's truncated.

I considered asking you to remove it and re-post it again properly formatted but I didn't want to ruffle any feathers and preferred to say nothing about the matter though this will probably affect the future creation of the thread's PDF as well, I'll have to edit it somehow.

I *do* appreciate your continued interest in my productions, Vincent, but next time you'd like to post something in one of my threads, please make sure to format it properly, everyone else does.

Thanks and regards.
V.

RE: [VA] SRC #008 - 2021 is here ! - Vincent Weber - **01-14-2021 11:18 PM**

Hi Valentin,
My bad ! I posted this using my phone, not realizing how bad the formatting would be.
I have deleted the original post.
Now that the challenge is over, please find below my original text without the code tags, for your kind consideration.

Many thanks and best regards,

Vincent

First, let's fix the number of numbers the sum is composed of.
Let's try the simplest case - 2 numbers. There are x and 2021-x.
The product is maximum when the derivative of x*(2021-x) is 0, so when 2021-x-x=0, or x=2021/2. This is not an integer, ok, but then I got the intuition that the product is maximized when the numbers are as close to each other as possible.
So let's try with 3 numbers. They are x,y,and (2021-x-y).
The product is maximized when both x and y partial derivatives are zero, so when 2021-2x-y=0 and 2021-x-2y=0. This simple system leads to x=y=2021/3. Again, non-integer, but confirms the intution that the numbers need to be equal !
So I was bald engouh to take this intuition for granted for any number of numbers
So now, what this number of numbers should be ?
Well if x is this number, then 2021/x is each equal number contributing to the sum, and the product is (2021/x)^x, which is also equal to e^(x.ln(2021/x)).
Composition derivation leads to a derivative of
(ln(2021/x)+x.-2021/x^2*x/2021)*...= (Ln(2021/x)-1)*... ("..." being the original product function which is never going to be zero).

Which leads to an optimal number of numbers of x=2021/e = 743.48... non integer of course.So each number should ideally be equal to e=2.718... the closest integer that comes to mind is obviously 3. You can fit 673 times 3 in 2021. 3×673=2019, remains 2. As for the product, 3^673*2=2.53E321, which is, well, quite a lot maybe not as much as you expected though , Valentin ?
I could write a stupid brute force algorithm for the fast free42, trying every single possibility, but I am too lazy for that

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RE: [VA] SRC #008 - 2021 is here ! - Albert Chan - **01-14-2021 11:49 PM**

> *(01-14-2021 11:18 PM)Vincent Weber Wrote:* Well if x is this number, then 2021/x is each equal number contributing to the sum, and the product is (2021/x)^x ...

Instead of letting x = number of products, it may be better letting x = base.

P = x ^ (2021/x)
ln(P) = (2021/x) * ln(x)
ln(P)/2021 = ln(x)/x

We like to maximize P, so find the local extremum.

0 = (x*(1/x) - ln(x)*1) / x^2 → 1 = ln(x) → x = e

> *Quote:*e=2.718... the closest integer that comes to mind is obviously 3

The safer way is not going for closest integer, but actually check value of ln(x)/x

3^2 > 2^3
2 ln(3) > 3 ln(2)
ln(3)/3 > ln(2)/2 = ln(4)/4

This showed ln(x)/x maximized when x=e, and we should pack as many 3's as possible.
(with the exception of N mod 3 = 1, since 3+1 > 3×1)

2021 = 3*673 + 2 → P = 3^673 * 2

RE: [VA] SRC #008 - 2021 is here ! - Valentin Albillo - **01-15-2021 01:50 AM**

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Hi, Vincent:

> *(01-14-2021 11:18 PM)Vincent Weber Wrote:* My bad ! I posted this using my phone, not realizing how bad the formatting would be. I have deleted the original post.

That sort of explains it. Thanks for deleting the post, that will make generating a properly formatted PDF much easier.

> *Quote:*Now that the challenge is over, please find below my original text without the code tags, for your kind consideration.

But of course, my pleasure. Let's see:

> *Quote:*First, let's fix the number of numbers the sum is composed of. Let's try the simplest case - 2 numbers. There are x and 2021-x. The product is maximum when the derivative of x*(2021-x) is 0, so when 2021-x-x=0, or x=2021/2. This is not an integer, ok, but then I got the intuition that the product is maximized when the numbers are as close to each other as possible.

Totally correct, and a logical intuition.

> *Quote:*So let's try with 3 numbers. They are x,y,and (2021-x-y).
> The product is maximized when both x and y partial derivatives are zero, so when 2021-2x-y=0 and 2021-x-2y=0. This simple system leads to x=y=2021/3. Again, non-integer, but confirms the intution that the numbers need to be equal !

Again, quite correct and indeed reinforces said intuition.

> *Quote:*So I was bald engouh to take this intuition for granted for any number of numbers

Very plausible, the previous cases are motivation enough to hypothesize that if the intuition was correct for the cases *N=2* and *N=3* it's reasonable to assume that it might also be true for *general N* and see where that leads us. So far so good.

> *Quote:*So now, what this number of numbers should be ? Well if x is this number, then 2021/x is each equal number contributing to the sum, and the product is (2021/x)^x, which is also equal to e^(x.ln(2021/x)). Composition derivation leads to a derivative of
>
> (ln(2021/x)+x.-2021/x^2*x/2021)*...= (Ln(2021/x)-1)*... ("..." being the original product function which is never going to be zero).
>
> Which leads to an optimal number of numbers of x=2021/e = 743.48... non integer of course.

Correct reasoning, *nihil obstat*.

> *Quote:*So each number should ideally be equal to e=2.718... the closest integer that comes to mind is obviously **3.** You can fit 673 times 3 in 2021. 3×673=2019, remains **2**. As for the product, **3^673*2=2.53E321**, which is, well, quite a lot maybe not as much as you expected though , Valentin ?

The reasoning is crystal-clear and the conclusion, *correct*. And I *did* expect exactly as much from you, Vincent

Congratulations are in order and I'm glad I finally got to see your original message, it's been an interesting read for me and I'm sure that many other readers of this thread will find it very interesting (and enlightening!) as well.

Thanks again for your fine contribution, it sure helps a lot to achieve my stated main goal for these *SRC's*. Hope you enjoyed it all and I fully expect to see you participate in the next one !

Best regards.
V.

RE: [VA] SRC #008 - 2021 is here ! - Vincent Weber - **01-15-2021 07:24 AM**

Many thanks, Valentin, for your kind words ! Much appreciated.

Many thanks also to Albert, for rightfully pointing that rounding to the closest integer is not good enough, that indeed needs to be tested.

Best regards,

Vincent

RE: [VA] SRC #008 - 2021 is here ! - Nihotte(lma) - **01-15-2021 08:56 PM**

> *(01-02-2021 05:09 PM)Nihotte(lma) Wrote:*
> Code:
>
> ```
> 1) at 07.09 PM 2021/01/02
> 2) at 10.31 PM 2021/01/02
>
> 2) at 08.06 PM 2021/01/06
> ```

I push this post to come back on several details that I have taken the time to explore a bit since.

First point :
It seems to me that ultimately the best way to get an error 8 by wanting to calculate an integral is still to run an f SOLVE on a range of a function where there is no 0. This time, I therefore suppose that the anomaly was about 12 inches (30 centimeters) above the calculator. I think that it is indeed preferable to launch an integral calculation by f INTEGR because I never reproduced the error thus!

Second point :
I finally managed to calculate the integral brilliantly using the HP35s. It is indeed simple as long as you use the calculator correctly. The technique consists of not simply reproducing a program in the spirit of the HP15C.
I mean: the HP15C don't need an intermediate storage register to calculate a function on which we can then evaluate an integral. On the other hand, this technique is perhaps less easy with the help of the HP35s which asks to know the derivative variable.
In this case, avoid reproducing my way of proceeding and avoid using STO A followed by one or more RCL A. Only RCL A is required, in fact!
This is all the more correct, since it is easy to enter the equation via EQU and to launch the calculation of the integral from the equation present on the screen. You just need to provide the name of the derivation variable (i.e. A) when the HP35s asks for it and everything is fine!

On the other hand, it seems that the computation of integrals remains more efficient by taking into account the limit of the functions evaluated. It is therefore more judicious, according to my observations, to evaluate 0 -> 90, then 180 -> 90 (or, as Albert Chan pointed out to me 0 -> -90) rather than 0 -> 180 directly because the calculation runs up against the limits around 90.

Last point :
Eventually the HP35s gave me a clean result using RAD directly with π based values instead of DEG and starting values in degrees.

Thanks again to Valentin Albillo who offered us this challenge which turned out to be rich in lessons and questions for all those who took part!