HP Forums / **HP Calculators (and very old HP Computers)** / **General Forum** ▼ / **[VA] SRC #008 - 2021 is here !**

🔧 **NEW REPLY**

[VA] SRC #008 - 2021 is here !	**Threaded Mode	Linear Mode**

01-02-2021, 02:57 AM (This post was last modified: 01-05-2021 08:31 PM by Valentin Albillo.)   **Post: #1**

Valentin Albillo 👤
Senior Member

Posts: 709
Joined: Feb 2015
Warning Level: 0%

[VA] SRC #008 - 2021 is here !

... and what better way to welcome **2021** than a little *2021-themed SRC* ?

Welcome to my *SRC #008 - 2021 is here !*, a small nice one to commemorate 2021's arrival. See if you can deliver **using your HP calculator**, physical or emulated.

- Please **NO Lua**, **NO Python**, **NO Excel**, **NO Mathematica**, etc. This is the *Museum of **HP Calculators***, so if you can't or won't use an *HP* calculator please post your results somewhere else, there are plenty of threads suitable for you to show off your non-*HP* achievements. **Thank you**.

That said, try your *HP*-hand with these three:

1) Let's partition **2021** into a set of positive integer numbers that add up to **2021**. Find the set of such numbers whose product is *maximum*, and output that maximum in all its full glory.

For instance, we could have **2021** = *1 + 1 + ... + 1 (2021 1's)* and their product would be *1 x 1 x ... x 1 = **1***, which doesn't quite cut it. We could also have **2021** = *137 + 682 + 1202* and their product would be *137 x 682 x 1202 = **112307668***, which is much better but still far from the maximum as well.

2) Numerically evaluate as accurately as possible this nice definite integral using your favorite *HP* calc (*per standard notation, the | ... | vertical bars mean "absolute value"*):

$$\int_0^\pi \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + \left|\cos\theta + \sqrt[2.021]{\frac{1}{2}}\sin\theta\right|^{2.021}\right)}{2.021\,\cos\theta\,\sin\theta}\,d\theta$$

and for extra points, see if you can *symbolically* recognize the resulting value. You'll need a sufficiently accurate value to do it, though ... 😊

3) It is trivially easy to express the fraction **4/2021** as the sum of 4 numbers of the form **1/N** where N is a positive integer, namely:

   4/2021   =   1/2021 + 1/2021 + 1/2021 + 1/2021

See if you can do it with just **3** such numbers.

My own solutions and comments in a few days.

- *P.S.: If you manage to solve them, do not spoil the fun for others: include your solutions in a CODE block leaving a number of blank lines at the beginning so that they can't be seen without willingly scrolling.*

V.

P.S.: Edited to include a third case, see post #21 below.

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01-02-2021, 03:49 AM   **Post: #2**

Gene 👤
Moderator

Posts: 1,117
Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !
Happy New Year Valentin!

My idea for #1 came after a few minutes thought. I will wait a bit to post it, but it came without an HP being used - no cheating either. I will verify that my thoughts work with an HP, but ...

Gene

[EMAIL] [PM] [FIND]
[QUOTE] [REPORT]

01-02-2021, 07:42 AM (This post was last modified: 01-02-2021 07:42 AM by RMollov.)
Post: #3

RMollov 🔒
Member

Posts: 251
Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !

#1 is interesting for me and I think I know the answer even though I can't prove it. Furthermore I'm really curious how much a calculator helps in this case; there must be some theoretical mathematical basis under and not calculations.

Thanks

[EMAIL] [PM] [FIND]
[QUOTE] [REPORT]

01-02-2021, 06:09 PM (This post was last modified: 01-02-2021 09:39 PM by Nihotte(lma).)

Nihotte(lma) 🔒
Member

RE: [VA] SRC #008 - 2021 is here !

> **Valentin Albillo Wrote:** ⇒
>
> ...
>
> My own solutions and comments in a few days.
>
> - *P.S.: If you manage to solve them, do not spoil the fun for others: include your solutions in a CODE block leaving a number of blank lines at the be*
>   *be seen without willingly scrolling.*
>
> **V.**

Thanks for the challenge Valentin Albillo !

Code:

```
1) at 07.09 PM 2021/01/02
2) at 10.31 PM 2021/01/02
.
.
.
.
.
.
.
.
```

[EMAIL] [PM] [FIND]

01-02-2021, 11:24 PM (This post was last modified: 01-02-2021 11:26 PM by J-F Garnier.)
Post: #5

J-F Garnier 🔒
Senior Member

Posts: 506
Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !

Hi Valentin,

Thanks for this nice New Year present !

I was inspired by your 2nd problem:

> **Valentin Albillo Wrote:** ⇒   (01-02-2021 02:57 AM)
>
> **2)** Numerically evaluate as accurately as possible this nice definite integral using your favorite *HP* calc (*per standard notation, the | ... | vertical bars mean "absolute value")*:
>
> $$\int_0^\pi \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\frac{1}{2}}\sin\theta|^{2.021}\right)}{2.021\ \cos\theta\ \sin\theta}\,d\theta$$

I had no special problem to evaluate it on the HP-71 (actually Emu71:-) with the Math ROM, after a simple operation.

> **Quote:**
> and for extra points, see if you can *symbolically* recognize the resulting value. You'll need a sufficiently accurate value to do it, though ... 😊

Using your great "Identifying Constants" program, I found the symbolic value, a nice surprise, that lead me to another conclusion about the expression to integrate :-)

Thanks again,

J-F

01-03-2021, 08:33 PM (This post was last modified: 02-16-2021 03:38 AM by robve.)   **Post: #6**

robve 🔓   Posts: 115
Member   Joined: Sep 2020

RE: [VA] SRC #008 - 2021 is here !

Thanks for posting. A good motivation to write some code on HP Prime. You did not mention that RPN is required...

#1 I couldn't find a way to run HP Prime programs with bigint (only up to 64bit), so the full decimal result is computed and displayed with HP Prime CAS in the code below.

Here is another puzzle: **Compute the smallest set(s) of *square numbers* that sum up to 2021. In other words, what are the sets of fewest square numbers that sum up to 2021 such that each square number is used at most once?**

For example, 1+9+36 sums up to 46 and 1=1^2, 9=3^2, 36=6^2 are square numbers. The size of this set is three, which is minimal since no two square numbers sum up to 46. Another solution is 1+4+16+25=46, but that solution has four square numbers and thus is not a minimal set.

There may be multiple minimal sets of square numbers for 2021. A set is minimal if no other set exists that is smaller.

```
Since we want to write some code, let's verify that powers of 3 give the max,
by brute force trying all products from 2 to sqrt(2021).
We use log here, which means we can simply sum up instead of using products,
e.g. to prevent overflow on some calculators (but HP Prime handles large floats.)

EXPORT T2021()
BEGIN
  LOCAL N := 2021, M := 0, J, K, R, X;
  FOR I FROM 2 TO SQRT(N) DO
    K := FLOOR(N/I);
    R := N-K*I;
    IF R=0 THEN
      R := 1;
    END;
    X := K*LN(I)+LN(R);
    IF X>M THEN
      M := X;
      J := I;
    END;
  END;
  MSGBOX(J);
  MSGBOX(e^M);
END;

After playing with this, it is interesting that 3 is best.
The reason is that 3 is close to the number e, which you can easily see is the best number to pick, because we have that
N/3*LN(3)<N/e
and
N/2*LN(2)<N/e
for any N>0 e.g. N=2021. Graphing N/x*LN(x) shows x=e is optimal.

The full number is displayed in HP Prime CAS:

2*3^673

25329955218868262923281496551277939397110796485699809049268130708906002579675557880841323830523234581366754082537819748828642552123142645059118075006593388245733455556963262198232747186628808383273213581619626233679533487706060253494981896112664520885716630483899029142003916544644957076791520721759240671604739781810307846
```

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

01-03-2021, 11:11 PM   **Post: #7**

Dwight Sturrock 🔓   Posts: 134
Member   Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !

RE #1

With a brute force approach, I searched for the largest product of:

(2021/n)^n

Code:

```
Quickly got to 9.999e99 on my 15C, so switched to Free 42.
Determined that the answer appeared to be in the 700-800 range
```

01-04-2021, 11:12 AM (This post was last modified: 01-04-2021 11:14 AM by J-F Garnier.)   **Post: #8**

J-F Garnier 🔒
Senior Member

RE: [VA] SRC #008 - 2021 is here !

> **J-F Garnier Wrote:** ➡   (01-02-2021 11:24 PM)
>
> I was inspired by [the] 2nd problem:
> ...
> I had no special problem to evaluate it on the HP-71 (actually Emu71:-) with the Math ROM, after a simple operation.

Here is my solution, not for the HP-71 but the 32S (not even the 32SII) that is, as I mentioned several times, my favourite machine for simple calculations.
Well, I cheated a bit since I used the observation I made in my previous message :-) so the execution time is reduced to less than two minutes including keystrokes. Not bad for this machine....

Code:
```
Program for the 32S/32SII:
LBL V
RCL X
COS
STO C
```

J-F

01-04-2021, 05:00 PM   **Post: #9**

Albert Chan 🔒
Senior Member

RE: [VA] SRC #008 - 2021 is here !

Code:
```
For #2, it seems constant 2.021 can be changed, and still gives same integral result.
Why ?


Let k = 2.021, c = 2^(-1/k)

I = ∫ [(ln(|sin(x)|^k/2 + |cos(x)+c*sin(x)|^k) / (k*sin(x)*cos(x))] dx  , x = 0 .. pi
```

01-04-2021, 05:59 PM (This post was last modified: 01-04-2021 06:41 PM by StephenG1CMZ.)   **Post: #10**

StephenG1CMZ 🔒
Senior Member

RE: [VA] SRC #008 - 2021 is here !

On the HP Prime, I get a result for number 2 if I use the constant 2.021 (2.021 approximately 2).
But if that "." is meant to be a thousands separator rather than decimal, representing the year 2021 rather than the year 2 🙂, I get undef (in home mode).
(And to get the answers others are giving, remember to change log into Ln)

Stephen Lewkowicz (G1CMZ)
ANDROID HP Prime App broken offline on some mobiles

01-04-2021, 06:16 PM (This post was last modified: 01-04-2021 06:17 PM by Albert Chan.)   **Post: #11**

Albert Chan 🔒
Senior Member

RE: [VA] SRC #008 - 2021 is here !

Code:

```
Post #10, we transform I by using substitution: y = pi/2-x, dy = -dx

We can continue substitutions: t = tan(y), dt = (1+t^2) dy
This shift integration limit from y = 0 .. pi/2, to t = 0 .. infinity

One more substitution, t = (1-u)/u, dt  = (-1/u^2) du
This shift back integration limit back to finite: u = 0 .. 1

This remove all use of trigonometric functions, or constant PI :)
```

reference: http://fmnt.info/blog/20180818_infinite-integrals.html

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01-04-2021, 07:56 PM   **Post: #12**

Gene 🔓   Posts: 1,117
Moderator   Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !

For problem 1, I don't understand the Y^X function showing up in the answers... ?

A + B + C = 2021 (or however many #'s you propose in your solution)

Then A x B x C = Big number.

505 + 505 + 505 + 505 + 1 works, but the product would be 505 x 505 x 505 x 505 x 1 or 6.5038 x 10^10.

Biggest number I can see is a portion of the factorial.

For example 63! = (roughly) 1.98 x 10^87. The sum of 63, 62, ... down to 2 is 2015. So with a little adjustment, one could get 2021 as a sum and something in the 10^80-something as a product.

Think I've missed something :-) but don't see where. Wouldn't be the first time!

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01-04-2021, 08:24 PM (This post was last modified: 01-04-2021 08:26 PM by ijabbott.)   **Post: #13**

ijabbott 🔓   Posts: 941
Senior Member   Joined: Jul 2015

RE: [VA] SRC #008 - 2021 is here !

Here's my quick and dirty RPL solution for #1:

Code:

— Ian Abbott

EMAIL   PM   FIND   QUOTE   REPORT

01-04-2021, 09:08 PM   **Post: #14**

Albert Chan 🔓   Posts: 1,342
Senior Member   Joined: Jul 2018

RE: [VA] SRC #008 - 2021 is here !

> **Gene Wrote:** ➡   (01-04-2021 07:56 PM)
>
> Biggest number I can see is a portion of the factorial.

No, we would like as many equal numbers as possible.
Say, we partition number N into n parts.

AM-GM inequality:

$$\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} \geq \sqrt[n]{x_1\, x_2\, x_3\, \cdots\, x_n}$$

LHS = N/n = constant
RHS = LHS, i.e. maximized products, when $x_1 = x_2 = x_3 = \cdots = x_n$

01-04-2021, 09:24 PM   Post: #15

Gene 🔓
Moderator

Posts: 1,117
Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !

Ah, now I see it more clearly.

For example... 2 + 2 + 2... + 2 + 21 (suppose there are one thousand 2's there).

Sum is 2021.

Product is 2^1000 x 21 which is huge... and still not the biggest.

ty

01-04-2021, 10:27 PM (This post was last modified: 01-04-2021 10:56 PM by ijabbott.)   Post: #16

ijabbott 🔓
Senior Member

Posts: 941
Joined: Jul 2015

RE: [VA] SRC #008 - 2021 is here !

Some clues:

Code:

```




* Splitting a 1 out of the number makes things worse unless (unless the number is 1).
  E.g. 2 -> 1+1 -> 1*1 = 1 which is less than 2.
```

Conclusion:

Code:

```




* If N > 3 and N ≡ 0 (mod 3) then only split out 3s.
* If N > 3 and N ≡ 1 (mod 3) then split out a single 4 (or 2 2s) and only split out 3s for any remaining part.
```

For example:

Code:

```



7 -> 4+3 -> 4*3 = 12 (or 7 -> 2+2+3 -> 2*2*3 = 12)
8 -> 2+3+3 -> 2*3*3 = 18
```

— Ian Abbott

01-04-2021, 10:36 PM   Post: #17

telemachos 🔓
Junior Member

Posts: 16
Joined: Apr 2017

RE: [VA] SRC #008 - 2021 is here !

As to No. 1, my candidate for greatest product exceeds my candidate for second-greatest by 12.5%. I'll explain later. I obtained these results without coding, but I knew the route to a solution many years ago.

Valentin, while we wait for your solutions, would you be willing to tell us how you came to choose the number 5, in gold, as your avatar/icon? I ask because the image was a puzzle until yesterday, when I saw that number on a postcard.

01-05-2021, 01:52 AM   Post: #18

Valentin Albillo   Posts: 709
Senior Member   Joined: Feb 2015
                                                                       Warning Level: 0%

RE: [VA] SRC #008 - 2021 is here !

.
Hi, **telemachos**:

> **telemachos Wrote:** ➡   (01-04-2021 10:36 PM)
>
> Valentin, while we wait for your solutions, **would you be willing to tell us how you came to choose the number 5, in gold, as your avatar/icon?**
>
> I ask because the image was a puzzle until yesterday, when I saw that number on a postcard.

Well, being *very* fond of both the number **5** and fine art, I've liked **Charles Demuth**'s 1928 painting "***I Saw the Figure 5 in Gold***" since I saw it for the first time many decades ago, and I thought it would make a very nice, classy, artistic avatar for me in the *MoHPC*'s forums, which it does.

Full info on the painting **here**

Thanks for your interest in both my *SRC #008* and my avatar ! 🙂

Best regards.
V.

Find All My HP-related Materials here: **Valentin Albillo's HP Collection**

01-05-2021, 05:39 AM (This post was last modified: 02-16-2021 03:41 AM by robve.)   Post: #19

robve   Posts: 115
Member   Joined: Sep 2020

RE: [VA] SRC #008 - 2021 is here !

My solution for #1 is in my earlier post with the non-CAS program listing, followed by the full number printed with HP Prime CAS. But if you really can't accept the number printed with HP Prime CAS, then below is a simple non-CAS program to produce the full decimal result as a list of digits:

```
EXPORT POWER3()
BEGIN
  L0:=MAKELIST(0,I,1,322);
  L0(SIZE(L0)):=2;
  FOR N FROM 1 TO 673 DO
    L1:=L0;
    FOR T FROM 1 TO 2 DO
      C:=0;
      FOR I FROM SIZE(L0) DOWNTO 1 DO
        D:=L0[I]+L1[I]+C;
        IF D>9 THEN
          D:=D-10;
          C:=1;
        ELSE
          C:=0;
        END;
        L0[I]:=D;
      END;
    END;
  END;
  PRINT(L0);
END;
```

HP 71B,Prime G2;Ti Nspire CXII CAS;Casio fx-CG50,fx-115ES+2;Sharp PC-G850VS,E500S,1475,1450,1360,1350,2500,1262,1500A

01-05-2021, 08:32 PM (This post was last modified: 01-05-2021 08:33 PM by Valentin Albillo.)   Post: #20

Valentin Albillo   Posts: 709
Senior Member   Joined: Feb 2015
                                                                       Warning Level: 0%

RE: [VA] SRC #008 - 2021 is here !

Hi all:

Thanks for your interest in this *SRC #008*, many more replies than I expected, many correct solutions and last but not least, *A-Chan* finally saw the light and actually *refrained* from posting solutions using everything but *HP* calcs in my threads, and has posted **HP-71B** code ! *Shocking !* 🙂

To express my gratitude, and as we say in Spain *"No hay dos sin tres"*, I've edited my original post to include a ***third*** 2021-related question, namely this one:

- *It is trivially easy to express the fraction **4/2021** as the sum of 4 numbers of the form **1/N** where N is a positive integer, namely:*

    4/2021   =   1/2021 + 1/2021 + 1/2021 + 1/2021

  *See if you can do it with just **3** such numbers.*

As I've added this third *"teaser"* I'll delay posting my own solutions to all three for a few days, to allow you to ponder it and post your own solution.

Thanks again and best regards.
V.

Find All My HP-related Materials here: **Valentin Albillo's HP Collection**

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01-05-2021, 09:07 PM   **Post: #21**

Maximilian Hohmann   Posts: 834
Senior Member   Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !

Hello!

> **Valentin Albillo Wrote:** ⇒   (01-05-2021 08:32 PM)
>
> ... and last but not least, *A-Chan* finally saw the light and actually *refrained* from posting solutions using everything but *HP* calcs...

This may be due to the fact that most older HP calcs (which are the ones I - as well as many others - prefer) can't handle the 3-digit exponents of puzzle #1 :-)

Regards
Max

EMAIL	PM	FIND		QUOTE	REPORT

01-05-2021, 09:42 PM (This post was last modified: 01-06-2021 01:45 AM by Albert Chan.)   **Post: #22**

Albert Chan   Posts: 1,342
Senior Member   Joined: Jul 2018

RE: [VA] SRC #008 - 2021 is here !

> **Valentin Albillo Wrote:** ⇒   (01-05-2021 08:32 PM)
>
> - *It is trivially easy to express the fraction **4/2021** as the sum of 4 numbers of the form **1/N** where N is a positive integer, namely:*
>
>   *4/2021   =   1/2021 + 1/2021 + 1/2021 + 1/2021*
>
>   *See if you can do it with just **3** such numbers.*

Code:

```
2021 = 43 * 47
```

EMAIL	PM	FIND		QUOTE	REPORT

01-06-2021, 12:57 AM   **Post: #23**

Gerson W. Barbosa   Posts: 1,378
Senior Member   Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !

I chose #2 only and I chose this one not because it's hard but because it's easy. :-)

Incidentally I got the same result when solving this equation for *x* on my HP-50g:

$$e^{3x + x \cdot e^{-x}} = 2021 + \frac{2^{3+\sqrt{2}}}{30 + 3^3 \cdot 2^{\sqrt{2}}}$$

Not nearly as beautiful as Valentín's integral, but at least easier to key in.

01-06-2021, 03:20 PM   **Post: #24**

Albert Chan   Posts: 1,342
Senior Member   Joined: Jul 2018

RE: [VA] SRC #008 - 2021 is here !

> **Albert Chan Wrote:** ⇒   (01-04-2021 06:16 PM)
>
> Post #10, we transform I by using substitution: y = pi/2-x, dy = -dx
>
> We can continue substitutions: t = tan(y), dt = (1+t^2) dy
> This shift integration limit from y = 0 .. pi/2, to t = 0 .. infinity
>
> One more substitution, t = (1-u)/u, dt = (-1/u^2) du
> This shift back integration limit back to finite: u = 0 .. 1

Another way is made use of symmetry.

Code:

```
I = ∫ f(y) dy, y = -pi/2 .. pi/2
  = ∫ [f(y) + f(-y)] dy, y = 0 .. pi/2         // Fold
  = ∫ g(y) dy, y = 0 .. pi/2
  = ∫ [g(y) + g(pi/2-y)] dy, y = 0 .. pi/4     // Fold again

Now, let t = tan(y), dt = (1+t^2) dy
This shift integration limit from y = 0 .. pi/4, to t = 0 .. 1

Integrand looks more complicated, but it actually converge faster !
```

01-06-2021, 04:54 PM (This post was last modified: 01-06-2021 04:55 PM by Gene.)   **Post: #25**

Gene   Posts: 1,117
Moderator   Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !
Couple of solutions:

4/2021 = 1/506 + 1/348128 + 1/16362016
4/2021 = 1/506 + 1/348458 + 1/15664771

Using Egyptian fractions. HP-41C program in the Test Stat rom. Program originally in the PPCCJ. Form is a series of 1/N fractions where each N must be different.

01-06-2021, 09:06 PM   **Post: #26**

Nihotte(lma)   Posts: 66
Member   Joined: Mar 2020

RE: [VA] SRC #008 - 2021 is here !

> **Nihotte(lma) Wrote:** ⇒   (01-02-2021 06:09 PM)
>
> **Code:**
>
> ```
> 1) at 07.09 PM 2021/01/02
> 2) at 10.31 PM 2021/01/02
> .
> .
> .
> .
> .
> .
> .
> .
> ```

I'm coming back on the second point. I didn't want to stay on a half-failure...

Code:

```
2) at 08.06 PM 2021/01/06
   .
   .
   .
   .
   .
   .
   .
   .
   .
```

01-07-2021, 12:49 AM

Albert Chan 🔒   Posts: 1,342
Senior Member   Joined: Jul 2018

RE: [VA] SRC #008 - 2021 is here !

Another way to calculate I, simpler and faster 🙂

Code:

```
Let f(t) = ln((0.5+|t+c|^k)/(0.5+|t-c|^k)), where c = 2^(-1/k)

k*I = ∫(f(t)/t, t = 0 .. inf)
    = (∫f(t)/t, t = 0 .. c) + (∫f(t)/t, t = c .. inf)
    = (∫f(t)/t, t = 0 .. c) + (∫f(1/t)/t, t = 0 .. 1/c)
```

01-07-2021, 11:37 PM (This post was last modified: 01-09-2021 03:38 AM by Valentin Albillo.)

Valentin Albillo 🔒   Posts: 709
Senior Member   Joined: Feb 2015
                                                                     Warning Level: 0%

RE: [VA] SRC #008 - 2021 is here !

Hi all:

First of all, thanks to all of you for the warm welcome to my **SRC #008** and the many contributions by **Nihotte(lma)**, **Vincent Weber**, **J-F Garnier**, **robve**, **Dwight Sturrock**, **Albert Chan**, **StephenG1CMZ**, **Albert Chan** *(again)*, **Gene**, **ijabbott**, **telemachos**, **Maximilian Hohmann**, **Gerson W. Barbosa** and last but not least, **Albert Chan**, thanks a lot to all of you for your interest and your solutions and quite varied approaches 🙂

These are **my original solutions**:

1) Let's partition **2021** into a set of positive integer numbers that add up to 2021. Find the set of such numbers whose product is *maximum*, and output that maximum in all its full glory.

It's easy to see that only using just the terms **2** and **3** will produce the maximum product, because:

- if a term = 5 then we can replace it by *2+3*, which contributes the same to the sum, 5, but increases the product as *2*3 = 6 > 5*.
  The same is true for terms >5 (e.g.: *6 = 3+3* but *3*3 = 9 > 6*, *7 = 3+2+2* but *3*2*2 = 12 >7*, etc.)

- if a term = 4 then it can be replaced by *2+2*, which contributes the same for both the sum and the product.

- if a term = 1 then *1+2* can be replaced by *3*, as *1*2 = 2 < 3*, and *1+3* can be replaced by *2+2*, as *1*3 = 3 < 2*2 = 4*.

Thus only terms **2** and **3** will need to be considered, and furthermore as three 2's can be replaced by two 3's, which have the same sum *(6)* but $2*2*2 = 8 < 3*3 = 9$, then at most <u>two</u> 2's can be part of the solution. As **2021** $= 3 * 673 + 2$, then the maximum product will be the product of *673 3's* and *one 2*, i.e: **Max. P = 2*3^{673}**.

In general, for integer $N = 3 * q + r$, where q is the *quotient* and r is the *reminder*, the maximum products are:

- if r = 0: Max. P = 3^q *(no 2's, all are 3's)*
- if r = 1: Max. P = $2^2 * 3^{q-1}$ *(two 2's, the rest are 3's)*
- if r = 2: Max. P = $2 * 3^q$ *(one 2, the rest are 3's)*

For *real N* all summands must be equal because of the *Arithmetic-Geometric Means* inequality, and they must be as close to $e = 2.718+$ as possible, which is the reason behind the need for a majority of *3's* in the integer *N* case, as *3* is the integer closer to e.

Also, as I am so adamant in *HP* calcs being used to solve the questions I propose, you may wonder where do *HP* calcs intervene in my solution ? Well, to fulfill the *"output that maximum in all its full glory"* part of the task, that's where, and to that effect I've concocted this *5-line (220-byte)* program for the **HP-71B**, which exactly computes the 322 digits of the max. product **P = 2*3^{673}** in half a second using the excellent J-F

Garnier's *Emu71* emulator:

```
1   DESTROY ALL @ OPTION BASE 0 @ DIM A(36) @ K=10^9 @ A(0)=2*3^13 @ P=0
2   FOR J=1 TO 110 @ MAT A=(729)*A @ FOR I=0 TO P @ A(I+1)=A(I+1)+A(I) DIV K
3   A(I)=MOD(A(I),K) @ NEXT I @ P=P+SGN(A(P+1)) @ NEXT J @ J=0
4   DISP STR$(A(P)); @ FOR I=P-1 TO 0 STEP -1 @ J=J+1 @ IF J=5 THEN DISP @ J=0
5   A$=STR$(A(I)) @ DISP RPT$("0",9-LEN(A$));A$; @ NEXT I @ DISP


    Line 1      initializes
    Lines 2-3   compute the 322-digit product
    Lines 4-5   output the result

  >RUN

     2532995521886826292328149655127793939711079
    6485699809049268130708906002579675557880841 32
    3830523234581366754082537819748828642552123 14
    2645059118075006593388245733455569632621982 32
    7471866288083832732135816196262336795334877 06
    0602534949818961126645208857166304838990291 42
    0039165446449570767915207217592406716047397 81
    810307846
```

2) Numerically evaluate as accurately as possible this nice definite integral using your favorite *HP* calc (*per standard notation, the | ... | vertical bars mean "absolute value")*:

$$\int_0^\pi \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\frac{1}{2}}\sin\theta|^{2.021}\right)}{2.021\ \cos\theta\ \sin\theta}\,d\theta$$

and for extra points, see if you can *symbolically* recognize the resulting value. You'll need a sufficiently accurate value to do it, though ... 🙂

The funny thing about this integral is that *its value remains the **same*** if you replace *2.021* by **any** positive real value !!

This means that the integral's value is the same for *2.021* or *0.2021*, *20.21*, *2021*, *Pi*, *e*, the *Golden Ratio*, your age, your phone number, etc. So it doesn't actually matter whether the *"."* in *2.021* is taken as the *decimal point* or as the *thousands separator*, as one of you wondered 🙂

Let's compute it using **Free42 Decimal** for extra accuracy. We'll need this short *32-step (60-byte)* program which defines a generalized version of the function to integrate (you can specify *2.021* or any other constant):

```
LBL "SRC82"    Y^X         1/X          ABS
MVAR "X"       2           X<>Y         RCL "X"
MVAR "Z"       /           STO ST T     STOx ST T
RCL "Z"        X<>Y        x            Y^X
SIN            2           RCL "Z"      +
ENTER          RCL "X"     COS          LN
ABS            1/X         STOx ST T    RCL/ ST Y
RCL "X"        Y^X         +            END
```

and now, using the built-in *Integrate* application (the *"S"* are *Integral* symbols):

```
[Sf(x)]    -> Select Sf(x) Program
[SRC82]    -> Set Vars; Select Svar
              [X][Z]

  2.021 [X] -> X=2.021
        [Z] -> [LLIM][ULIM][ACC]    [S]

    0 [LLIM] -> LLIM=0
 [PI] [ULIM] -> ULIM=3.14159265359
 1E-6 [ACC] -> ACC=0.000001
        [S] -> Integrating -> 2.46739926409
  [+][SHOW] -> 2.46740110027233423194...
```

All that remains now is to *symbolically recognize* this value, for example using my **IDENTIFY** program (listing, description and examples here), and the result is **Pi^2/4**, which evaluates to **2.46740110027233**965470... so we've got 15 correct digits. You may vary the *2.021* value above and re-compute the integral, to check that indeed you get the same result.

3) Decompose **4/2021** into a sum of three fractions of the form **1/K** where *K* is a positive integer.

As we saw in **(1)** above, **2021** is a number of the form $N = 3*m + 2$ and for such numbers we have the algebraic identity:

4/(3*m + 2) = 1/(3*m + 2) + 1/(m + 1) + 1 /((m + 1) * (3*m + 2))

so we'll use this trivial program to ask for *N* (which must be of the form $3*m + 2$) and output the resulting denominators:

```
1   DESTROY ALL @ INPUT N
2   IF MOD(N,3)#2 THEN DISP "Not of form 3*m+2" @ END
3   M=N DIV 3 @ DISP N;M+1;N*(M+1)
```

```
                    >RUN
                    ? 2021
                                2021        674        1362154
```

so: **4/2021 = 1/2021 + 1/674 + 1/1362154**, and of course many other solutions are possible as well.

As another example, using the program for **2027**, which is a *prime* number and thus can't be factorized, we get:

```
                    >RUN
                    ? 2027
                                2027        676        1370252
```

so *4/2027 = 1/2027 + 1/676 + 1/1370252*, and so on and so forth.

Note that as the first summand fraction is *1/N*, the remaining two add up to *3/N*, thus we also have a decomposition of *3/N* into *two 1/K* fractions.

Thanks again for your interest and contributions.

Best regards.
V.

Edited to include a slightly optimized version of the BASIC code (30 bytes shorter)

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[PM] [WWW] [FIND]   [EDIT] [X] [QUOTE] [REPORT]

Albert Chan 🔓   Posts: 1,342
Senior Member   Joined: Jul 2018

RE: [VA] SRC #008 - 2021 is here !

> **Valentin Albillo Wrote:** ⇒   (01-07-2021 11:37 PM)
>
> $$\int_0^{\pi} \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\frac{1}{2}}\ \sin\theta|^{2.021}\right)}{2.021\ \cos\theta\ \sin\theta}\, d\theta$$
>
> The funny thing about this integral is that *its value remains the **same*** if you replace *2.021* by **any** positive real value !!

Can you explain why ? ???😕

With the help of HP Prime emulator, here is the proof for k = 2:

I = ∫(ln(sin(x)^2/2 + (cos(x)+sin(x)/sqrt(2))^2) / sin(2x), x = 0. .. pi)
  = ∫(ln(1+sin(2x)/sqrt(2)) / sin(2x), x = 0. .. pi)

I = ∫(ln(1+sin(2y)/sqrt(2)) / sin(2y), y = -pi/2. .. pi/2)   -- y = pi/2-x, dy=-dx
  = ∫(2*atanh(sin(2y)/sqrt(2)) / sin(2y), y = 0. .. pi/2)   -- ln((1+x)/(1-x)) = 2*atanh(x)

I = ∫(atanh(sin(z)/sqrt(2)) / sin(z), z = 0. .. pi)   -- z = 2*y, dz=2*dy
  = ∫(2*atanh(sin(z)/sqrt(2)) / sin(z), z = 0. .. pi/2)   -- sin(z) symmetry around pi/2

I = ∫(2*atanh(sqrt(2)*t/(1+t^2))/t, t = 0. .. 1)   -- t = tan(y/2), dt=(1+t^2)/2*dx

CAS> series(atanh(√2*t/(1+t^2))/(√2),t,0,18,polynom)

t -1/3*t^3-1/5*t^5+1/7*t^7+1/9*t^9 -1/11*t^11-1/13*t^13+1/15*t^15+1/17*t^17

2*atanh(√(2)*t/(1+t*t))/t
= √(8) * (1 - t^2/3 - t^4/5 + t^6/7 + t^8/9 - t^10/11 - t^12/13 + t^14/15 + t^16/17 - ...)

Integrate term by term, t from 0 to 1:

I = √(8) * (1 - 1/3² - 1/5² + 1/7² + 1/9² - 1/11² - 1/13² + 1/15² + 1/17² - ...)

Cas> s1 := simplify(sum(1/(2k-1)^2, k=1..inf))   // pi^2/8
Cas> s2 := simplify(sum(1/(8k-1)^2 + 1/(8k+1)^2, k=1..inf)) // (√(2)*pi^2+2*pi^2-32)/32
Cas> simplify(sqrt(8) * (2 - s1 + 2*s2))   // **I = pi^2/4**

I can proof s1 by hand, but not s2 (any help appreciated) 😕

[EMAIL] [PM] [FIND]   [QUOTE] [REPORT]

Albert Chan 🔓   Posts: 1,342
Senior Member   Joined: Jul 2018

RE: [VA] SRC #008 - 2021 is here !

Let f(t) = ln((0.5+|t+c|^k)/(0.5+|t-c|^k)), where c = 2^(-1/k)

k*I = ∫(f(t)/t, t = 0 .. inf)
= (∫f(t)/t, t = 0 .. c) + (∫f(t)/t, t = c .. inf)
= (∫f(t)/t, t = 0 .. c) + (∫f(1/t)/t, t = 0 .. 1/c)

Code:

```
I should have gone further, and scale both limits, from 0 to 1:

k*I = (∫f(c*t)/t, t = 0 .. 1) + (∫f(c/t)/t, t = 0 .. 1) = (∫g(t), t = 0 .. 1)

What is nice is c = 2^(-1/k) is not needed in the code !
Another hint that value of k does not matter ...

Combined parts into 1 integral, the code also run faster.
```

For k=1, g(t) is simple enough that Wolfram Alpha can proof it (after a few seconds) 🙂

I = ∫(ln((2+t)/(2-t)*(2t+1))/t, t = 0 .. 1) = pi^2/4

EMAIL   PM   FIND   QUOTE   REPORT

01-08-2021, 11:04 AM   **Post: #31**

J-F Garnier   Posts: 506
Senior Member   Joined: Dec 2013

RE: [VA] SRC #008 - 2021 is here !

Valentin Albillo Wrote: ⇒   (01-07-2021 11:37 PM)

... I've concocted this unoptimized *5-line (250-byte)* program for the **HP-71B**, which exactly computes the 322 digits of the max. product **P = 2*3^{673}** in half a second using the **excellent J-F Garnier's *Emu71*** emulator

Thanks :-)
Emu71/DOS is more than 20 years old but still going strong !

Albert Chan Wrote: ⇒   (01-08-2021 12:40 AM)

Valentin Albillo Wrote: ⇒   (01-07-2021 11:37 PM)

$$\int_0^\pi \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\frac{1}{2}}\sin\theta|^{2.021}\right)}{2.021\,\cos\theta\,\sin\theta}\,d\theta$$

The funny thing about this integral is that *its value remains the **same*** if you replace *2.021* by **any** positive real value !!

Can you explain why ? ☹

Yes, please Valentin; I was expecting to get an explanation or at least some indications.

I used this effect to actually compute the integral on the HP-32S with the value k=2 to replace the exponentiation operations with simple squaring, for a much faster computation.

Also I computed the integral in two parts, from 0 to pi/2 and pi/2 to pi, since the pi/2 value causes a 0/0 limit of the function. The HP algorithm is robust when this kind of limit is at the edges, but not always when it is within the integration range especially when using high target accuracy (I used accuracy= 1E-10).

J-F

EMAIL   PM   WWW   FIND   QUOTE   REPORT

01-08-2021, 03:32 PM   **Post: #32**

EdS2   Posts: 285
Member   Joined: Apr 2014

RE: [VA] SRC #008 - 2021 is here !

I second these calls for an explanation of the inner workings of this ingenious and unexpected integral - but I have another request too: how, Valentin, did you discover or construct this integral?

EMAIL   PM   FIND   QUOTE   REPORT

01-12-2021, 05:27 AM   **Post: #33**

Albert Chan   Posts: 1,342

RE: [VA] SRC #008 - 2021 is here !

Re-posting this from Spence function thread, per Valentin Albillo's request.

> **Albert Chan Wrote:** ⇒   (01-11-2021 08:16 PM)
>
> > **Valentin Albillo Wrote:** ⇒   (01-07-2021 11:37 PM)
> >
> > $$\int_0^\pi \frac{\log\left(\frac{|\sin\theta|^{2.021}}{2} + |\cos\theta + \sqrt[2.021]{\frac{1}{2}}\,\sin\theta\,|^{2.021}\right)}{2.021\,\cos\theta\,\sin\theta}\,d\theta$$
> >
> > The funny thing about this integral is that *its value remains the **same*** if you replace *2.021* by *any* positive real value !!
>
> I think Spence's function (Dilogarithms) is the key to proof I = pi^2/4, for any k
>
> After transformation, we have I = ∫(g(t) dt, t = 0 .. 1)
>
> $$\exp(g(t)\cdot k\,t) = \frac{(1+(1+t)^k)\cdot(1+(\frac{1}{t}+1)^k)}{(1+(1-t)^k)\cdot(1+(\frac{1}{t}-1)^k)} = \frac{(1+(1+t)^k)\cdot(t^k+(1+t)^k)}{(1+(1-t)^k)\cdot(t^k+(1-t)^k)}$$
>
> For now, assume k is positive integer → numerator is polynomial of t, with degree 2k.
>
> Let W = exp((2n+1)/k*pi*i), n = 0 .. k-1. In other words, W = roots of x^k = -1
>
> roots of numerator   , t = W-1, 1/(W-1)
> roots of denominator, t = 1-W, 1/(W+1)
>
> Consider just 1 of W (we have k of them), factor it out, scaled away k, and integrate:
>
> $$\int_0^1 \ln\left(\frac{(1-\frac{t}{w-1})\cdot(1-t(w-1))}{(1-\frac{t}{1-w})\cdot(1-t(w+1))}\right)\frac{dt}{t} = -Li_2(\frac{1}{w-1}) + Li_2(\frac{1}{1-w}) - Li_2(w-1) + Li_2(w+1)$$
>
> I = average of all the k pieces, dropped imaginary parts (since I is real)
>
> For example, for k = 7, these are the integral pieces.
>
> ```
> >>> from mpmath import *
> >>> Li2 = lambda x: polylog(2, x)
> >>> k = 7
> >>> W = [exp((2*n+1)/k*pi*1j) for n in range(k)]
> >>> for w in W: print -Li2(1/(w-1)) + Li2(1/(1-w)) - Li2(w-1) + Li2(w+1)
> ...
> (2.46740110027234 + 5.0951007958138j)
> (2.46740110027234 + 2.21291211873152j)
> (2.46740110027234 + 0.940187304494618j)
> (2.46740110027234 + 1.22410719457408e-16j)
> (2.46740110027234 - 0.940187304494618j)
> (2.46740110027234 - 2.21291211873152j)
> (2.46740110027234 - 5.0951007958138j)
> ```
>
> All the real parts have the same size = pi^2/4 🙂
>
> What happened if k is not integer ? Lets try k = 2.021
> Amazingly, for any <u>non-zero real k</u>, real part of the integral piece still = pi^2/4
>
> ```
> >>> w = exp(pi*1j/2.021)
> >>> print -Li2(1/(w-1)) + Li2(1/(1-w)) - Li2(w-1) + Li2(w+1)
> (2.46740110027234 + 1.85770404643526j)
> ```
>
> Note: this only show how integral evaluated to the same value, for different k's
> It is still not a proof, but much closer than what I had before ...

[EMAIL] [PM] [FIND]   [QUOTE] [REPORT]

01-14-2021, 03:42 AM   **Post: #34**

Valentin Albillo 🔒   Posts: 709
Senior Member   Joined: Feb 2015
                                                  Warning Level: 0%

RE: [VA] SRC #008 - 2021 is here !

Hi, all:

Thanks for your interest in my *SRC #008*, some comments to your posts and some answers to your questions:

> **Nihotte(lma) Wrote:**
>
> I have tested several configurations but it seems to me that 673x3 + 2 gives the best result With a result of 2021 and **2,5329955 x 10^321**
> ... I'm coming back on the second point. I didn't want to stay on a half-failure... and for the whole integral it gives : [...] **2.467401102**

You gave the first correct solution to *Case 1* but forgot to also give the exact 322-digit result as requested. Not bad ! And your value for the integral using the **HP-15C** is correct as well, congratulations.

> **J-F Garnier Wrote:**
>
> Using your great "Identifying Constants" program, I found the symbolic value, a nice surprise, that lead me to another conclusion about the expression to integrate :-)

Hehe, you got it, both the symbolic value ($Pi^2/4$) and the fact that the integral's value doesn't depend on the constant being *2.021* and thus you could use 2 instead to speed up the numerical integration using the *HP32S* no less. And thanks a lot for your kind comments and the link to my *Article*.

> **robve Wrote:**
>
> Thanks for posting. A good motivation to write some code on HP Prime. You did not mention that RPN is required...

Indeed, the *Prime* is an HP calculator alright. And your fine solution is the very first one to give the full 322-digit correct value for *Case 1*.

> **Albert Chan Wrote:**
>
> *(many, many things spread over many, many posts)*

Thanks for your abundant, relentless and interesting approaches both numerical and theoretical, Albert, and most especially for using HP calculators to obtain them, as I requested. At least you did for your very first posts ...

> **StephenG1CMZ Wrote:**
>
> But if that "." is meant to be a thousands separator rather than decimal, representing the year 2021 rather than the year 2 , I get undef ...

As you may have seen, the dot may be taken as the *decimal* separator <u>or</u> the *thousands* separator and it willl make no difference to the result 😃

> **ijabbott Wrote:**
>
> Some clues: Splitting a 1 [...]

Your detailed explanation mimics my original solution and is quite didactic for people who would like to understand the logic behind the result, well done.

> **Maximilian Hohmann Wrote:**
>
> This may be due to the fact that most older HP calcs (which are the ones I - as well as many others - prefer) can't handle the 3-digit exponents of puzzle #1 :-)

Well, that shouldn't be a problem, many of my former *Challenges* and *Short & Sweet Math Challenges* did require multiprecision values. Remember for instance my *"Sweet & Short Math Challenges #15: April 1st Spring Special"*, where I posted a 9-line program for the **HP-71B** which computed and printed in full the exact solution to the *"Take 5"* sub-challenge, namely $2^{65536}-3$, which is a **19,729**-digit integer. There were **RPL** programs doing likewise, too.

> **Gerson W. Barbosa Wrote:**
>
> I chose #2 only and I chose this one not because it's hard but because it's easy. :-) [...] Not nearly as beautiful as Valentín's integral, but at least easier to key in.

Hehe, Gerson, you're playing tricks here, your (very nice, by the way) expression evaluates to **2.46740110027**18... while the correct value is $Pi^2/4 =$
2.4674011002723... (Very) close but no cigar ! 😃

> **Gene Wrote:**
>
> Couple of solutions: [...] Using Egyptian fractions. HP-41C program in the Test Stat rom.

Very correct, if different from my original solution. And a very useful reference, thanks.

> **Albert Chan Wrote:**
>
> Can you explain why ?

> **J-F Garnier Wrote:**
>
> Yes, please Valentin; I was expecting to get an explanation or at least **some indications**.

> **EdS2 Wrote:**
>
> I second these calls for an explanation of the inner workings of this ingenious and unexpected integral

Thanks for your interest and well, my goal with these *SRC*'s and *Challenges* is to show some interesting HP-calc programming techniques (mostly for the classic models), as well as some interesting, little-known and even *unexpected* math topics, hopefully for the enjoyment of (at least some) *MoHPC* forum's readers. Also, if I can provide technical details or proofs which aren't too lengthy and/or complicated, I'll usually do it.

For instance, this is not the first time I post a definite integral allegedly dependent on some parameter but actually being independent of it, such as the two integrals featured in my *Short & Sweet Math Challenge #18: April 1st, 2007 Spring Special*, namely:

```
        / Inf
        |
  I1 =  |  1/((1+x²)*(1+x^4.012007))  .dx
```

```
              |
              /  0

and
              /  Pi/2
              |
    I2  =  |  1/(1+Tan(x)^4.012007) .dx
              |
              /  0
```

which actually are one and the same upon a simple change of variables and their value is Pi/4. The proof of their being independent of the parameter (the *4.012007* value) is short enough that I did post my own proof of the fact in *Message #54* in that thread.

However, for the present integral the proof is not that easy or short, actually it's quite long and would require a lot of time and cumbersome math *"typesetting"* to post it here, which regrettably is beyond the scope of these *SRC*'s. I'll give however a link to *"some indications"* for a very similar integral, which you can find here.

The proof for the similar integral I posted (I don't have a link to the proof, sorry) essentially goes along the same lines. Also, if you think that an integral with a parameter whose value is independent of it is something quite unexpected, wait till you see the next integral I'll post in a future *SRC*, it's *10x* as much ! 🙂

Best regards to all and thanks for participating.
V.

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01-14-2021, 03:59 PM   Post: #35

Vincent Weber 👤   Posts: 178
Member   Joined: May 2015

RE: [VA] SRC #008 - 2021 is here !

Hi Valentin,

And not one word about my attempt to give a general, mathematical (albeit certainly flawed in many ways) solution for #1 ? Ok, ok, I didn't try to compute it for utmost precision with an HP calculator... but I gave it some thoughts. Just teasing 😀

Cheers,

Vincent
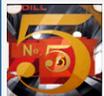
01-14-2021, 08:51 PM   Post: #3(

Valentin Albillo 👤   Posts: 709
Senior Member   Joined: Feb 2015
                                                             Warning Level: 0%

RE: [VA] SRC #008 - 2021 is here !

.
Hi, Vincent:

> **Vincent Weber Wrote:** ➡   (01-14-2021 03:59 PM)
>
> And not one word about my attempt to give a general, mathematical (albeit certainly flawed in many ways) solution for #1 ?

Matter of fact I couldn't read your message. I don't know what browser or app you used to format and post it, but this is how it does appear in my browser and yes, that infinitely long message is the one you posted ...

... . As you may see, it dwarfs all other messages in the thread, in fact it extends much beyond the limits of the browser's window and forces me to zoom a lot to be able to read all other messages which are scaled down to unreadable small text, while yours can't be scrolled horizontally any further but is cropped instead so I can't read it properly either, it's truncated.

I considered asking you to remove it and re-post it again properly formatted but I didn't want to ruffle any feathers and preferred to say nothing about the matter though this will probably affect the future creation of the thread's PDF as well, I'll have to edit it somehow.

I *do* appreciate your continued interest in my productions, Vincent, but next time you'd like to post something in one of my threads, please make sure to format it properly, everyone else does. 🙂

Thanks and regards.
V.

Find All My HP-related Materials here: **Valentin Albillo's HP Collection**

01-15-2021, 12:18 AM   **Post: #37**

Vincent Weber 🔒   Posts: 178
Member   Joined: May 2015

RE: [VA] SRC #008 - 2021 is here !
Hi Valentin,
My bad ! I posted this using my phone, not realizing how bad the formatting would be.
I have deleted the original post.
Now that the challenge is over, please find below my original text without the code tags, for your kind consideration.

Many thanks and best regards,

Vincent

First, let's fix the number of numbers the sum is composed of.
Let's try the simplest case - 2 numbers. There are x and 2021-x.
The product is maximum when the derivative of x*(2021-x) is 0, so when 2021-x-x=0, or x=2021/2. This is not an integer, ok, but then I got the intuition that the product is maximized when the numbers are as close to each other as possible.
So let's try with 3 numbers. They are x,y,and (2021-x-y).
The product is maximized when both x and y partial derivatives are zero, so when 2021-2x-y=0 and 2021-x-2y=0. This simple system leads to x=y=2021/3. Again, non-integer, but confirms the intution that the numbers need to be equal !
So I was bald engouh to take this intuition for granted for any number of numbers 🙂
So now, what this number of numbers should be ?
Well if x is this number, then 2021/x is each equal number contributing to the sum, and the product is (2021/x)^x, which is also equal to e^(x.ln(2021/x)).
Composition derivation leads to a derivative of
(ln(2021/x)+x.-2021/x^2*x/2021)*...= (Ln(2021/x)-1)*... ("..." being the original product function which is never going to be zero).

Which leads to an optimal number of numbers of x=2021/e = 743.48... non integer of course.So each number should ideally be equal to e=2.718...
the closest integer that comes to mind is obviously 3. You can fit 673 times 3 in 2021. 3×673=2019, remains 2. As for the product,
3^673*2=2.53E321, which is, well, quite a lot 🙂 maybe not as much as you expected though , Valentin ? 🙂
I could write a stupid brute force algorithm for the fast free42, trying every single possibility, but I am too lazy for that 🙂

01-15-2021, 12:49 AM (This post was last modified: 01-15-2021 11:16 AM by Albert Chan.)   **Post: #38**

Albert Chan 🔒   Posts: 1,342
Senior Member   Joined: Jul 2018

RE: [VA] SRC #008 - 2021 is here !

> **Vincent Weber Wrote:** ➡   (01-15-2021 12:18 AM)
>
> Well if x is this number, then 2021/x is each equal number contributing to the sum, and the product is (2021/x)^x ...

Instead of letting x = number of products, it may be better letting x = base.

P = x ^ (2021/x)
ln(P) = (2021/x) * ln(x)
ln(P)/2021 = ln(x)/x

We like to maximize P, so find the local extremum.

0 = (x*(1/x) - ln(x)*1) / x^2   → 1 = ln(x)   → x = e

> **Quote:**
>
> e=2.718... the closest integer that comes to mind is obviously 3

The safer way is not going for closest integer, but actually check value of ln(x)/x

3^2 > 2^3
2 ln(3) > 3 ln(2)
ln(3)/3 > ln(2)/2 = ln(4)/4

This showed ln(x)/x maximized when x=e, and we should pack as many 3's as possible.
(with the exception of N mod 3 = 1, since 3+1 > 3×1)

$2021 = 3*673 + 2 \rightarrow P = 3^{673} * 2$

01-15-2021, 02:50 AM   **Post: #39**

Valentin Albillo 🔒   Posts: 709
Senior Member   Joined: Feb 2015
                                                          Warning Level: 0%

RE: [VA] SRC #008 - 2021 is here !

.
Hi, Vincent:

> **Vincent Weber Wrote:** ⇒   (01-15-2021 12:18 AM)
> My bad ! I posted this using my phone, not realizing how bad the formatting would be. I have deleted the original post.

That sort of explains it. Thanks for deleting the post, that will make generating a properly formatted PDF much easier.

> **Quote:**
> Now that the challenge is over, please find below my original text without the code tags, for your kind consideration.

But of course, my pleasure. Let's see:

> **Quote:**
> First, let's fix the number of numbers the sum is composed of. Let's try the simplest case - 2 numbers. There are x and 2021-x. The product is maximum when the derivative of x*(2021-x) is 0, so when 2021-x-x=0, or x=2021/2. This is not an integer, ok, but then I got the intuition that the product is maximized when the numbers are as close to each other as possible.

Totally correct, and a logical intuition.

> **Quote:**
> So let's try with 3 numbers. They are x,y,and (2021-x-y).
> The product is maximized when both x and y partial derivatives are zero, so when 2021-2x-y=0 and 2021-x-2y=0. This simple system leads to x=y=2021/3. Again, non-integer, but confirms the intuition that the numbers need to be equal !

Again, quite correct and indeed reinforces said intuition.

> **Quote:**
> So I was bald engouh to take this intuition for granted for any number of numbers 🙂

Very plausible, the previous cases are motivation enough to hypothesize that if the intuition was correct for the cases *N=2* and *N=3* it's reasonable to assume that it might also be true for *general N* and see where that leads us. So far so good.

> **Quote:**
> So now, what this number of numbers should be ? Well if x is this number, then 2021/x is each equal number contributing to the sum, and the product is (2021/x)^x, which is also equal to e^(x.ln(2021/x)). Composition derivation leads to a derivative of
>
> (ln(2021/x)+x.-2021/x^2*x/2021)*...= (Ln(2021/x)-1)*... ("..." being the original product function which is never going to be zero).
>
> Which leads to an optimal number of numbers of x=2021/e = 743.48... non integer of course.

Correct reasoning, *nihil obstat*.

> **Quote:**
> So each number should ideally be equal to e=2.718... the closest integer that comes to mind is obviously **3.** You can fit 673 times 3 in 2021. 3×673=2019, remains **2**. As for the product, **3^673*2=2.53E321**, which is, well, quite a lot 🙂 maybe not as much as you expected though , Valentin ? 🙂

The reasoning is crystal-clear and the conclusion, *correct*. And I *did* expect exactly as much from you, Vincent 🙂

Congratulations are in order and I'm glad I finally got to see your original message, it's been an interesting read for me and I'm sure that many other readers of this thread will find it very interesting (and enlightening!) as well.

Thanks again for your fine contribution, it sure helps a lot to achieve my stated main goal for these *SRC's*. Hope you enjoyed it all and I fully expect to see you participate in the next one ! 🙂

Best regards.
V.

Find All My HP-related Materials here: **Valentin Albillo's HP Collection**

01-15-2021, 08:24 AM (This post was last modified: 01-15-2021 08:25 AM by Vincent Weber.)   **Post: #40**

Vincent Weber 👤
Member

Posts: 178
Joined: May 2015

RE: [VA] SRC #008 - 2021 is here !

Many thanks, Valentin, for your kind words ! Much appreciated.

Many thanks also to Albert, for rightfully pointing that rounding to the closest integer is not good enough, that indeed needs to be tested.

Best regards,

Vincent

[EMAIL] [PM] [FIND]   [QUOTE] [REPORT]

01-15-2021, 09:56 PM   **Post: #41**

Nihotte(lma) 👤
Member

Posts: 66
Joined: Mar 2020

RE: [VA] SRC #008 - 2021 is here !

> **Nihotte(lma) Wrote:** ➡   (01-02-2021 06:09 PM)
>
> **Code:**
> ```
> 1) at 07.09 PM 2021/01/02
> 2) at 10.31 PM 2021/01/02
>
> 2) at 08.06 PM 2021/01/06
> ```

I push this post to come back on several details that I have taken the time to explore a bit since.

First point :
It seems to me that ultimately the best way to get an error 8 by wanting to calculate an integral is still to run an f SOLVE on a range of a function where there is no 0. This time, I therefore suppose that the anomaly was about 12 inches (30 centimeters) above the calculator. I think that it is indeed preferable to launch an integral calculation by f INTEGR because I never reproduced the error thus!

Second point :
I finally managed to calculate the integral brilliantly using the HP35s. It is indeed simple as long as you use the calculator correctly. The technique consists of not simply reproducing a program in the spirit of the HP15C.
I mean: the HP15C don't need an intermediate storage register to calculate a function on which we can then evaluate an integral. On the other hand, this technique is perhaps less easy with the help of the HP35s which asks to know the derivative variable.
In this case, avoid reproducing my way of proceeding and avoid using STO A followed by one or more RCL A. Only RCL A is required, in fact!
This is all the more correct, since it is easy to enter the equation via EQU and to launch the calculation of the integral from the equation present on the screen. You just need to provide the name of the derivation variable (i.e. A) when the HP35s asks for it and everything is fine!

On the other hand, it seems that the computation of integrals remains more efficient by taking into account the limit of the functions evaluated. It is therefore more judicious, according to my observations, to evaluate 0 -> 90, then 180 -> 90 (or, as Albert Chan pointed out to me 0 -> -90) rather than 0 -> 180 directly because the calculation runs up against the limits around 90.

Last point :
Eventually the HP35s gave me a clean result using RAD directly with π based values instead of DEG and starting values in degrees.

Thanks again to Valentin Albillo who offered us this challenge which turned out to be rich in lessons and questions for all those who took part!

[EMAIL] [PM] [FIND]   [QUOTE] [REPORT]

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