



Welcome back, **Valentin Albillo**. You last visited: Yesterday, 10:28 PM **Current time:** 05-01-2019, 02:05 AM
([User CP](#) — [Log Out](#))

[View New Posts](#) | [View Today's Posts](#) | [Private Messages](#) (Unread 0, Total 145)

[Open Buddy List](#)

[HP Forums](#) / [HP Calculators \(and very old HP Computers\)](#) / [General Forum](#) ▼ / [\[VA\]](#)

SRC#004- Fun with Sexagesimal Trigs



[VA] SRC#004- Fun with Sexagesimal Trigs

[Threaded Mode](#) | [Linear Mode](#)

02-11-2019, 06:13 PM

Post: #1



Valentin Albillo

Senior Member

Posts: 347

Joined: Feb 2015

Warning Level: 0%

[VA] SRC#004- Fun with Sexagesimal Trigs

Hi, all:

Welcome to my brand new **SRC#004**, this time commemorating that this is my **300th** post here so if I'm not mistaken I'll be granted the description "*Senior Member*" from now on. What a treat ! 😊

Tired of trigs in *radians* ? Wanna see some trigs in *sexagesimal degrees* ? Read on !

Details:

If you're in the mood for a nice helping of awesome "*sexagesimal*" results go and try your hand at *accurately evaluating* the following trig expressions, either manually or else by writing a short bit of **RPN/RPL/71BASIC/HPPL** code for your preferred **HP** calculator (*not Excel, Matlab, Mathematica, Python, Java, Haskell, C/C++, Wolfram Alpha, etc., there are many other forums/threads for that*), namely:

$$A = (\sqrt{3} + \tan 1^\circ) (\sqrt{3} + \tan 2^\circ) \dots (\sqrt{3} + \tan 29^\circ)$$

$$B = \frac{1}{\sin 45^\circ \sin 46^\circ} + \frac{1}{\sin 47^\circ \sin 48^\circ} + \dots + \frac{1}{\sin 133^\circ \sin 134^\circ}$$

$$C = \left(1 - \frac{\cos 61^\circ}{\cos 1^\circ}\right) \left(1 - \frac{\cos 62^\circ}{\cos 2^\circ}\right) \dots \left(1 - \frac{\cos 119^\circ}{\cos 59^\circ}\right)$$

$$D = \left(1 - \frac{1}{\tan 1^\circ}\right) \left(1 - \frac{1}{\tan 2^\circ}\right) \dots \left(1 - \frac{1}{\tan 44^\circ}\right)$$

$$E = \left(2 \cos 2^{2^0} - \frac{1}{\cos 2^{2^0}} \right) \left(2 \cos 2^{3^0} - \frac{1}{\cos 2^{3^0}} \right) \dots \left(2 \cos 2^{25^0} - \frac{1}{\cos 2^{25^0}} \right)$$

Once you've succeeded in accurately evaluating them you should attempt to **identify** the results that aren't immediately obvious (i.e.: if you get something like $1.7320508..$ you should identify it as $\sqrt{3}$), which will be useful to gauge the accuracy obtained by comparing what you got with the exact results, benchmark-like.

You must *not* use anything other than your intuition and the help of your trusty HP calculator for the identification (in particular give the Internet a miss) but those of you using HP calcs with **CAS** might want to check if your CAS will produce the *exact* results at once or if at least it will simplify them to something much .. well, simpler !

Let's see your code, results and comments (try not to spoil early the fun for others), I will post mine within a few days.

V.

.



02-11-2019, 09:55 PM

Post: #2



J-F Garnier

Senior Member

Posts: 302

Joined: Dec 2013

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

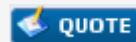
Valentin Albillo Wrote: →

(02-11-2019 06:13 PM)

You must *not* use anything other than your intuition and the help of your trusty HP calculator for the identification...

So the use of your great [IDENTIFY program](#) is perfectly legal ? :-)

J-F



02-11-2019, 11:26 PM

Post: #3



J-F Garnier

Senior Member

Posts: 302

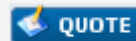
Joined: Dec 2013

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

Amazing expressions !

Programming (especially on the 71) is immediate - no need to post it - , but the identification of the B value stopped me for a while.

J-F



02-11-2019, 11:51 PM (This post was last modified: 02-12-2019 03:37 AM by Gerson W. Barbosa.)

Post: #4

Gerson W. Barbosa

Posts: 1,135

Joined: Dec 2013



Senior Member

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

J-F Garnier Wrote: →

(02-11-2019 11:26 PM)

Amazing expressions !

Programming (especially on the 71) is immediate - no need to post it -

Agreed. Same on the 50g.

Gerson.

PS.: Not a spoiler, just a reminder to myself...

ALBINVASINCDLBE++++- >51.9999999999



02-12-2019, 05:37 AM (This post was last modified: 02-14-2019 04:39 AM by Albert Chan.)

Post: #5

Albert Chan

Senior Member

Posts: 624

Joined: Jul 2018

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

J-F Garnier Wrote: →

(02-11-2019 11:26 PM)

the identification of the B value stopped me for a while.

Here is a good way to estimate B.

Middle term angle is around 90°, any angles bigger than that can be used to fill the "holes":

Example, $1/(\sin(133^\circ)\sin(134^\circ)) = 1/(\sin(47^\circ)\sin(46^\circ))$

So, $B = 1/(\sin(45^\circ)\sin(46^\circ)) + 1/(\sin(46^\circ)\sin(47^\circ)) + \dots + 1/(\sin(89^\circ)\sin(90^\circ))$

I remember derivative of $\tan(x)$ is $\sec(x)^2 = 1/\cos(x)^2$

So, change all the sines to cosines, angles goes from 0° to 45°

The x's are in degree, so need to scale the area, like this:

$\text{sum} \sim (180/\text{Pi}) * \text{integrate}(\sec(x)^2, x, 0, \text{Pi}/4) = (180/\text{Pi}) * (1 - 0) = 180/\text{Pi}$

Actual sum is not exactly like this, but should be close.

My guess for true sum is $1/\sin(1^\circ)$, but to prove it is hard ...



02-12-2019, 10:05 AM (This post was last modified: 02-12-2019 10:37 AM by J-F Garnier.)

Post: #6



J-F Garnier

Senior Member

Posts: 302

Joined: Dec 2013

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

J-F Garnier Wrote: →

(02-11-2019 11:26 PM)

Programming (especially on the 71) is immediate - no need to post it -

Well, *straightforward* programming is indeed immediate and enough to get an idea of the results, but it is possible to do slightly better "in *accurately* evaluating them"

Let's see on value A.

Straightforward programming:

```
>A=1 @ FOR I=1 TO 29 @ A=A*(SQR(3)+TAN(I)) @ NEXT I @ DISP A
536870912.011
```

The rounding error on SQR(3), accumulated on 29 successive iterations introduces a systematic bias.

We can reduce this effect by factoring SQR(3) and evaluating SQR(3)^29 directly:

```
>A=1 @ FOR I=1 TO 29 @ A=A*(1+TAN(I)/SQR(3)) @ NEXT I @ A=A*3^14*SQR(3) @ DISP A
536870912.002
```

This gives us more confidence in identifying it to a certain simple integer expression.

J-F



02-12-2019, 11:50 AM (This post was last modified: 02-12-2019 11:50 AM by Gerson W. Barbosa.)

Post: #7



Gerson W. Barbosa

Senior Member

Posts: 1,135

Joined: Dec 2013

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

J-F Garnier Wrote: →

(02-12-2019 10:05 AM)

The rounding error on SQR(3), accumulated on 29 successive iterations introduces a systematic bias.

We can reduce this effect by factoring SQR(3) and evaluating SQR(3)^29 directly:

```
>A=1 @ FOR I=1 TO 29 @ A=A*(1+TAN(I)/SQR(3)) @ NEXT I @ A=A*3^14*SQR(3) @ DISP
A
536870912.002
```

This gives us more confidence in identifying it to a certain simple integer expression.

Right in between:

```
<< 1 1 29
FOR i i DUP + SIN LASTARG COS 1 + / 3 √ + *
NEXT
>>
```

EVAL ->

```
536870912.006
```



02-12-2019, 02:14 PM

Post: #8

Posts: 624

Albert Chan 

Joined: Jul 2018

Senior Member

RE: [VA] SRC#004- Fun with Sexagesimal TrigsProve $A = 2^{29} = 536870912$:Checking the edges, using shorthand $t\# = \tan(\#\circ)$:

$$\tan(A^\circ+B^\circ) = (tA + tB) / (1 - tA tB)$$

$$\begin{aligned} &(t60 + tA) (t60 + tB), \text{ where } A+B=30^\circ \\ &= t60^2 + t60*(tA + tB) + tA tB \\ &= t60^2 + t60*t30*(1 - tA tB) + tA tB \\ &= 3 + 1 - tA tB + tA tB \\ &= 2^2 \end{aligned}$$

$$\begin{aligned} t15 &= \tan(45^\circ - 30^\circ) \\ &= (t45 - t30) / (1 + t45 t30) \\ &= (1 - 1/\sqrt{3}) / (1 + 1/\sqrt{3}) \\ &= (\sqrt{3} - 1) / (\sqrt{3} + 1) \\ &= (\sqrt{3} - 1)^2 / (3 - 1) \\ &= (3 + 1 - 2*\sqrt{3}) / 2 \end{aligned}$$

$$\rightarrow t15 = 2 - \sqrt{3}$$

$$\rightarrow \text{center} = t60 + t15 = 2$$

14 pairs of edges and 1 center, all can considered value of 2, thus $A = 2^{29}$ 

02-12-2019, 03:23 PM

Post: #9

**Gerson W. Barbosa** 

Senior Member

Posts: 1,135

Joined: Dec 2013

RE: [VA] SRC#004- Fun with Sexagesimal Trigs**Albert Chan Wrote:** →

(02-12-2019 02:14 PM)

Prove $A = 2^{29} = 536870912$:Checking the edges, using shorthand $t\# = \tan(\#\circ)$:

$$\tan(A^\circ+B^\circ) = (tA + tB) / (1 - tA tB)$$

$$\begin{aligned} &(t60 + tA) (t60 + tB), \text{ where } A+B=30^\circ \\ &= t60^2 + t60*(tA + tB) + tA tB \\ &= t60^2 + t60*t30*(1 - tA tB) + tA tB \\ &= 3 + 1 - tA tB + tA tB \\ &= 2^2 \end{aligned}$$

$$\begin{aligned} t15 &= \tan(45^\circ - 30^\circ) \\ &= (t45 - t30) / (1 + t45 t30) \\ &= (1 - 1/\sqrt{3}) / (1 + 1/\sqrt{3}) \\ &= (\sqrt{3} - 1) / (\sqrt{3} + 1) \\ &= (\sqrt{3} - 1)^2 / (3 - 1) \\ &= (3 + 1 - 2*\sqrt{3}) / 2 \end{aligned}$$

-> $t_{15} = 2 - \sqrt{3}$
 -> center = $t_{60} + t_{15} = 2$

14 pairs of edges and 1 center, all can considered value of 2, thus $A = 2^{29}$

A is the only one I had trouble with because at first I had wrongly programmed it as a sum instead of a product. When I notice my mistake and finally got the correct value, I simply computed its base-2 logarithm in order to check it was a power of two. But only because I had identified D already. For whatever reason I decided to check the product of the first and last terms:

$(1 - 1/\tan 1^\circ)(1 - 1/\tan 44^\circ) = 1.99999999997$

That's 2 to me, so I just did $2^{22} = 4194304$ which "matched" 4194303.99965, the value my D program returned, and I gave it no further thought.

I do appreciate, however, your efforts in going deeper into the problem and providing a proof.

Gerson.



02-12-2019, 04:00 PM

Post: #10

Albert Chan

Senior Member

Posts: 624

Joined: Jul 2018

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

J-F Garnier Wrote: →

(02-12-2019 10:05 AM)

We can reduce this effect by factoring $\text{SQR}(3)$ and evaluating $\text{SQR}(3)^{29}$ directly:

```
>A=1 @ FOR I=1 TO 29 @ A=A*(1+TAN(I)/SQR(3)) @ NEXT I @ A=A*3^14*SQR(3) @ DISP
A
536870912.002
```

$3^{14.5}$ may be more accurate than $3^{14} * \sqrt{3}$, avoided 1 rounding.

Example, on my HP-12C:

```
3 [Enter] 14.5 [X^Y] => 8,284,345.319
3 [Enter] 14 [X^Y] 3 [√X] x => 8,284,345.321
```

It may be more accurate to do corrections to A:

```
lua> function t(a) return math.tan(math.rad(a)) end
lua> sqrt3 = math.sqrt(3)
```

```
lua> a = 1
lua> for i=1,29 do a = a * (1 + t(i) / sqrt3) end
lua> a * 3^14.5
536870912.0000006
```

```
lua> a = 3^14.5
lua> for i=1,29 do a = a + a * t(i) / sqrt3 end
lua> a
536870912
```



02-12-2019, 04:11 PM

Post: #11

**J-F Garnier**

Senior Member

Posts: 302

Joined: Dec 2013

RE: [VA] SRC#004- Fun with Sexagesimal Trigs**Albert Chan Wrote:** →

(02-12-2019 02:14 PM)

Prove $A = 2^{29} = 536870912$:Checking the edges, using shorthand $t\# = \tan(\#\circ)$:

$$\tan(A^\circ+B^\circ) = (tA + tB) / (1 - tA tB)$$

$$\begin{aligned} &(t60 + tA) (t60 + tB), \text{ where } A+B=30^\circ \\ &= t60^2 + t60*(tA + tB) + tA tB \\ &= t60^2 + t60*t30*(1 - tA tB) + tA tB \\ &= 3 + 1 - tA tB + tA tB \\ &= 2^2 \end{aligned}$$

...

14 pairs of edges and 1 center, all can considered value of 2, thus $A = 2^{29}$

Great!

With the same method (and notations), it's easy to prove $D = 2^{22}$:

$$\tan(A^\circ+B^\circ) = (tA + tB) / (1 - tA tB)$$

$$\begin{aligned} &(1 - 1/tA) (1 - 1/tB), \text{ where } A+B=45^\circ \\ &= 1 - (1/tA + 1/tB) + 1/(tA tB) \\ &= 1 - (tA + tB) / (tA tB) + 1 / (tA tB) \\ &= 1 - t45 (1 - tA tB) / (tA tB) + 1 / (tA tB) \\ &= 1 - (1 / (tA tB) - 1) + 1 / (tA tB) \\ &= 2 \end{aligned}$$

J-F



02-12-2019, 04:42 PM (This post was last modified: 02-12-2019 05:28 PM by Albert Chan.)

Post: #12

Albert Chan

Senior Member

Posts: 624

Joined: Jul 2018

RE: [VA] SRC#004- Fun with Sexagesimal Trigs**J-F Garnier Wrote:** →

(02-12-2019 04:11 PM)

$$\tan(A^\circ+B^\circ) = (tA + tB) / (1 - tA tB)$$

$$\begin{aligned} &(1 - 1/tA) (1 - 1/tB), \text{ where } A+B=45^\circ \\ &= 1 - (1/tA + 1/tB) + 1/(tA tB) \\ &= 1 - (tA + tB) / (tA tB) + 1 / (tA tB) \\ &= 1 - t45 (1 - tA tB) / (tA tB) + 1 / (tA tB) \\ &= 1 - (1 / (tA tB) - 1) + 1 / (tA tB) \\ &= 2 \end{aligned}$$

Neat! Perhaps this version is more elegant:

$$t_{45} = 1 = (t_A + t_B) / (1 - t_A t_B), \text{ where } A+B=45^\circ$$

$$\rightarrow t_A t_B = 1 - t_A - t_B$$

$$X = (1 - 1/t_A) (1 - 1/t_B)$$

$$t_A t_B X = (t_A - 1) (t_B - 1) = (t_A t_B + 1 - t_A - t_B) = 2 t_A t_B$$

$$X = 2$$

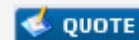
Bonus: For $A+B=45^\circ$, we now have more accurate way to do $1 - t_A - t_B$:

Example, on my Casio FX-115MS:

$$1 - t_{0.01} - t_{44.99} = 1.74472016000e-4 \text{ (with all internal digits)}$$

$$1 - t_{44.99} - t_{0.01} = 1.74472015029e-4 \text{ (slightly better)}$$

$$t_{0.01} * t_{44.99} = 1.74472014117e-4 \text{ (last digit should rounded to 8)}$$



02-12-2019, 06:39 PM (This post was last modified: 02-12-2019 06:41 PM by Albert Chan.)

Post: #13

Albert Chan

Senior Member

Posts: 624

Joined: Jul 2018

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

Prove $C = 1$:

Using shorthand $c(\#) = c\# = \cos(\#^\circ)$, $s(\#) = s\# = \sin(\#^\circ)$

$$C = (1 - c_{61}/c_1)(1 - c_{62}/c_2) \dots (1 - c_{119}/c_{59})$$

$$\text{center} = (1 + c_{90}/c_{30}) = (1 + 0/c_{30}) = 1$$

Check the edges, each pair P had the form:

$$= (1 - c(90-x)/c(30-x)) * (1 - c(90+x)/c(30+x))$$

$$= (1 - s_x / c(30-x)) * (1 + s_x / c(30+x))$$

To simplify P, we need these identities:

$$\mathbf{\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)}$$

$$\mathbf{\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)}$$

Removing the annoying denominator, let $k = c(30-x) * c(30+x)$

k P

$$= (c(30+x) - s_x) * (c(30-x) + s_x)$$

$$= k + s_x^2 + s_x * (c(30+x) - c(30-x))$$

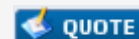
$$= k + s_x^2 + s_x * (-2 s(30) s_x)$$

$$= k + s_x^2 - s_x^2$$

$$= k$$

$$\rightarrow P = 1$$

All factors can considered value of 1, thus $C = 1$



02-12-2019, 07:58 PM

Post: #14

Albert Chan

Senior Member

Posts: 624

Joined: Jul 2018

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

Prove $E = -1$:

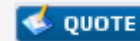
This one is easy. Just do the first term, using shorthand $c(\#) = c\# = \cos(\#^\circ)$

$$(2c4 - 1/c4) = (2c4^2 - 1) / c4 = c8 / c4$$

Numerator and denominator for everything cancelled out, except $c(2^26)/c(4)$

$$c(2^26) = c(2^26 \bmod 360) = c184 = -c4$$

$$\rightarrow E = -c4/c4 = -1$$



02-14-2019, 04:12 AM (This post was last modified: 02-14-2019 04:21 AM by Albert Chan.)

Post: #15

Albert Chan

Senior Member

Posts: 624

Joined: Jul 2018

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

Albert Chan Wrote: →

(02-12-2019 05:37 AM)

$$B = 1/(\sin(45^\circ)\sin(46^\circ)) + 1/(\sin(46^\circ)\sin(47^\circ)) + \dots + 1/(\sin(89^\circ)\sin(90^\circ))$$

I got the prove !!!

Prove $B = 1/\sin(1^\circ)$, using shorthand $c(\#) = c\# = \cos(\#^\circ)$, $s(\#) = s\# = \sin(\#^\circ)$:

Identities required:

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\cos(2A) = 2\cos(A)^2 - 1$$

Replace sine with cosines, $B = 1/(c0c1) + 1/(c1c2) + \dots + 1/(c44c45)$

Sum 2 terms: $1/(c0c1) + 1/(c1c2)$

$$= (c2+c0)/(c0c1c2)$$

$$= (2*c1c1) / (c1c2)$$

$$= 2*c1/c2$$

Sum 3 terms: $2*c1/c2 + 1/(c2c3)$

$$= (2*c1c3 + 1)/(c2c3)$$

$$= (c2 + c4 + 1)/(c2c3)$$

$$= (c2 + 2*c2^2)/(c2c3)$$

$$= (1 + 2*c2)/c3$$

Sum 4 terms: $(1 + 2*c2)/c3 + 1/(c3c4)$

$$= (c4 + 2*c2c4 + 1)/(c3c4)$$

$$= (2*c2^2 + 2*c2c4)/(c3c4)$$

$$= 2*c2*(c2+c4)/(c3c4)$$

$$= 2*c2*(2*c1*c3)/(c3c4)$$

$$= 4*c1c2/c4$$

Sum 5 terms: $4*c1c2/c4 + 1/(c4c5)$

$$= (4*c1c2c5 + 1)/(c4c5)$$

$$= (2*c2*(c4+c6) + 1)/(c4c5)$$

$$= (2*c2c4 + 2*c2c6 + 1)/(c4c5)$$

$$\begin{aligned}
 &= (2*c2c4 + c8 + c4 + 1)/(c4c5) \\
 &= (2*c2c4 + 2*c4^2 + c4)/(c4c5) \\
 &= (1 + 2*c2 + 2*c4)/c5
 \end{aligned}$$

Finally see a pattern! It can be proved by induction (not shown):

$$B = (1 + 2*c2 + 2*c4 + \dots + 2*c44) / c45$$

Sines and Cosines of Angles in Arithmetic Progression formulas:

$$\begin{aligned}
 \sin(x) + \sin(x+y) + \dots + \sin(x+(n-1)*y) &= \sin(x+(n-1)y/2)*\sin(ny/2)/\sin(y/2) \\
 \cos(x) + \cos(x+y) + \dots + \cos(x+(n-1)*y) &= \cos(x+(n-1)y/2)*\sin(ny/2)/\sin(y/2)
 \end{aligned}$$

$$\begin{aligned}
 B &= (1 + 2*(c2 + c4 + \dots + c44)) / c45, \text{ with } x=y=2^\circ, n=22 \\
 &= (1 + 2*(c23 s22 / s1)) / c45 \\
 &= (1 + 2*(c23 c68 / s1)) / c45 \\
 &= (1 + (c91 + c45) / s1) / c45 \\
 &= (1 - s1/s1 + c45/s1) / c45 \\
 &= 1/\sin(1^\circ)
 \end{aligned}$$



02-14-2019, 05:33 AM

Post: #16



Carsen 
Member

Posts: 163
Joined: Jan 2017

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

Solutions with RPL. These programs are designed to not be flexible. Only to solve the problem.

Code:

Spoilers for A

<<

Code:

Spoilers for B

<<

Code:

Spoilers for C

<<

Code:

Spoilers for D

<<

Code:

Spoilers for E

<<

Fun programming.



02-14-2019, 06:08 AM

Post: #17

Albert Chan

Senior Member

Posts: 624

Joined: Jul 2018

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

Albert Chan Wrote: →

(02-14-2019 04:12 AM)

$$B = (1 + 2*c2 + 2*c4 + \dots + 2*c44) / c45$$

Another way, using $2 \sin(A) \cos(B) = \sin(A+B) + \sin(A-B)$

$$\begin{aligned}
 B \sin(1^\circ) &= (s_1 + 2*s_1c_2 + 2*s_1c_4 + \dots + 2*s_1c_{44}) / c_{45} \\
 &= (s_1 + (-s_1 + s_3) + (-s_3 + s_5) + \dots + (-s_{43} + s_{45})) / c_{45} \\
 &= s_{45} / c_{45} \\
 &= 1
 \end{aligned}$$



02-14-2019, 03:23 PM (This post was last modified: 02-14-2019 03:33 PM by Albert Chan.)

Post: #18

Albert Chan

Senior Member

Posts: 624

Joined: Jul 2018

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

Got a very elegant and general proof that $B = 1/\sin(1^\circ)$:

$$1/(c_0c_1) + 1/(c_1c_2) + \dots + 1/(c_{n-1}c_n) = \tan(n^\circ) / \sin(1^\circ), \text{ assumed } n < 90$$

Proof above by induction:

For $n=1$, $1/(c_0c_1) = 1/c_1 = \tan(1^\circ) / \sin(1^\circ)$

Assume this work for $n=k$, add 1 more term, we got:

$$\tan(k^\circ) / \sin(1^\circ) + 1/(c_k c_{k+1}) = (s_k c_{k+1} + s_1) / (s_1 c_k c_{k+1})$$

Since $s_1 = s((k+1) - k) = s(k+1) c_k - c(k+1) s_k$, we got:

$$\text{Sum of } k+1 \text{ terms} = (s(k+1) c_k) / (s_1 c_k c_{k+1}) = \tan((k+1)^\circ) / \sin(1^\circ)$$

QED.

-> With 45 terms, $B = \tan(45^\circ) / \sin(1^\circ) = 1/\sin(1^\circ)$ 😊



02-24-2019, 08:17 PM (This post was last modified: 02-24-2019 09:15 PM by Valentin Albillo.)

Post: #19



Valentin Albillo

Senior Member

Posts: 347

Joined: Feb 2015

Warning Level: 0%

RE: [VA] SRC#004- Fun with Sexagesimal Trigs

Hi, all:

A short proof of the evaluation of B:

$$B = \frac{1}{\sin 45^\circ \sin 46^\circ} + \frac{1}{\sin 47^\circ \sin 48^\circ} + \dots + \frac{1}{\sin 133^\circ \sin 134^\circ} = \frac{1}{\sin 1^\circ}$$

1) First, we multiply both sides by $\sin 1^\circ$:

$$\frac{\sin 1^\circ}{\sin 45^\circ \sin 46^\circ} + \frac{\sin 1^\circ}{\sin 47^\circ \sin 48^\circ} + \dots + \frac{\sin 1^\circ}{\sin 133^\circ \sin 134^\circ} = \frac{\sin 1^\circ}{\sin 1^\circ} = 1$$

2) Now we use the identity

$$\frac{\sin((k+1)^\circ - k^\circ)}{\sin k^\circ \sin(k+1)^\circ} = \cot k^\circ - \cot(k+1)^\circ$$

which transforms the left-hand side into this:

$$\cot 45^\circ - \cot 46^\circ + \cot 47^\circ - \cot 48^\circ + \dots + \cot 133^\circ - \cot 134^\circ$$

3) Then we reorder the terms in the sum like this:

$$\cot 45^\circ - (\cot 46^\circ + \cot 134^\circ) + (\cot 47^\circ + \cot 133^\circ) - \dots + (\cot 89^\circ + \cot 91^\circ) - \cot 90^\circ$$

4) All the terms inside the parentheses cancel out because they feature supplementary angles ($\cot N^\circ + \cot(180^\circ - N^\circ) = 0$), so the expression reduces to:

$$\cot 45^\circ - \cot 90^\circ = 1 - 0 = 1$$

Q.E.D. 😊

Thanks for your interest and have a nice weekend.

V
.



<< Next Oldest | Next Newest >>



[View a Printable Version](#)

[Send this Thread to a Friend](#)

[Subscribe to this thread](#)

User(s) browsing this thread: [Valentin Albillo*](#)

[Contact Us](#) | [The Museum of HP Calculators](#) | [Return to Top](#) | [Return to Content](#) | [Lite \(Archive\) Mode](#) | [RSS Syndication](#)

Forum software: [MyBB](#), © 2002-2019 [MyBB Group](#).