HP Forum Archive 14

A *very* didactic little quiz

Message #1 Posted by ***Valentin Albillo*** *on 16 July 2004, 9:59 a.m.*

Hi all,

Here's a *very didactic* little problem I posed my 12-year old daughter to let her understand a key mathematical concept. She readily took her SHARP PC-1350, computed furiously for a while, and was *amazed* no end by the result.

You are to try your hand at it with your favorite HP calc, but first you should take a risk and make a decision on what would be your choice, *without* any external (electronic or mechanical) help, based on your own internal calculations and judgment. Then, take your trusty HP calc and find out (most any HP calc will do, even the HP-01, but built-in unit conversions would be handy if you're not used to metric units).

The little quiz

> [This is a purely mathematical, ideal problem so physical laws and limits do not apply, no tricks or word games here. Also no cents are rounded, ever; all amounts are kept to as many decimal places as necessary]

Imagine for a moment that you're at year 0, and someone makes this peculiar proposal to you. You must choose the option you deem will result in the greatest amount of money for you, between these two, namely:

- **(a)** From the instant you accept this one choice, you're given $1.00, which will be placed in a bank account at 5% compound interest, to be accrued yearly, i.e:, after 1 year you'll have $1.05, after two years you'll have a little over $1.10 and so on. At the end of a specified time period you'll be given the resulting balance from your $1.00 initial capital, plus accrued interests.

  or

- **(b)** From the instant you accept this other choice, a cylinder of *solid gold*, with a diameter equal to that of the Earth, will start to grow in length *at the speed of light* (i.e.: the cylinder's top will reach to the Moon in just over 1 second, to the Sun in 8 minutes, to Sirius in under 9 years, and to Betelgeuse in some 1,400 years). At the end of the specified time period, you'll be given the monetary value corresponding to the cylinder's resulting mass of gold.

Now, taking the diameter of the Earth as 12,700 Km, the speed of light as 300,000 Km per second, the density of gold as 19 grams per cubic centimeter, the price of gold as $10.00 per gram (all of them realistic values), and further assuming a year has exactly 365 days (no leap years considered) and the agreed time period is to be 2,000 years ...

What would be the better choice ? How much money would you get by the year 2000 ?

"Just imagine [*tell your children, colleagues or students*]: on the one side a humble, silent bank account at a modest 5% interest, gathering dust for 2,000 years, taking one full year to earn a puny $0.05, two years for a trifle over $0.10 earning. On the other side, a massive, planet-wide cylinder of *solid gold*, growing to the stars at the speed of light, a humongous gold rod which will have reached to Alpha-Centaury by the time the account has accrued 22 cents, and will be a full 2,000 light-years in length by the end of the deal !"

Which option would you [*they*] choose ? Wanna bet ? :-)

Best regards from V.

Edited: 16 July 2004, 10:07 a.m.

Re: A *very* didactic little quiz

Message #2 Posted by ***Ron Ross*** *on 16 July 2004, 10:21 a.m.,*
in response to message #1 by Valentin Albillo

And that is why laws have been passed on the banks behalf about inactive accounts. At least in this country and many others.

Re: A *very* didactic little quiz

Message #3 Posted by ***Tizedes Csaba [Hungary]*** *on 16 July 2004, 10:28 a.m.,*
in response to message #1 by Valentin Albillo

Hi,

there is no question: Everybody will choice the gold-cylinder... :)

But after 2000 year, the gold's value will be 4.55E41$, and the bank account will be 2.39E42$. (Calc'd by my 15C :))

Csaba

Ps. (to Valentin): If nobody will answering this response, I will go to Commodore Forum!!! ;) (It's just a joke, dont't fear...)

Edited: 16 July 2004, 1:52 p.m. after one or more responses were posted

Good illustration

*Message #4 Posted by **Gene** on 16 July 2004, 11:02 a.m.,*
in response to message #3 by Tizedes Csaba [Hungary]

I plan to use it when I introduce compound interest in my class this fall.

Gene

Re: A *very* didactic little quiz

*Message #5 Posted by **John Nelson** on 16 July 2004, 12:30 p.m.,*
in response to message #3 by Tizedes Csaba [Hungary]

I got a different answer...

Gold = 1.26E+29

Money = 2.39E+42

I would have thought the gold would be more valuable at the start.

Here is my work.

```
Gold
==============
V=Bh
B=(3.14 * Radius^2)
B=      126676869.8
Grows as 300,000 km/sec for 2000 years, or 63,072,000,000 seconds
h=(300,000 * 63,072,000,000)
h=      1.89216E+16


V=      2.39693E+24     Km
        2.39693E+29     Cm
        1.26154E+28     Grams
        1.26154E+29     $Value


Money
```

```
===========
P=1
r= .05
n = 20


FV = P(1 + r)^n
FV =    2.3911E+42
```

Some observations

Message #6 Posted by ***John Nelson*** *on 16 July 2004, 1:26 p.m.,*
in response to message #5 by John Nelson

Some observations on the compounding of the money. Look at what the annual interest is in various years...

```
Year     Interest
============================
100      $6.26
204      1000.92
252      10410.80
299      103129.08
346      1.02M
360      2.02M
393      10.11M
440      100.24M
488      1042.69M


2000     1.13862009743503E+41
```

Wow!! I wonder if there is that much money even in the world?

Re: A *very* didactic little quiz

Message #7 Posted by ***Eamonn*** *on 16 July 2004, 1:37 p.m.,*
in response to message #5 by John Nelson

Hi John,

Your calculation of the volume of the gold cylinder in km^3 is correct. However, to convert from km^3 to cc, you need to multiply by 1e15. This can be seen as follows:

(1 km)^3 = (1e3 m)^3 = (1e5 cm)^3 = 1e15 cc.

This means that the gold cylinder has a volume of 2.39693E+39 cc after 2000 years.

Next, since gold is $10.00/gram and there are 19 grams of gold per cc, this means that the gold is $190.00 per cc. So the total value of the gold cylinder is:

2.39693E+39 * 190 = $4.554 E+41

This is the same result as obtained by Csaba.

Best Regards,

Eamonn.

Re: A *very* didactic little quiz

*Message #8 Posted by **hugh steers** on 16 July 2004, 12:06 p.m.,*
in response to message #1 by Valentin Albillo

i would have the gold value. since this is defined to be the value of the gold at the end of period. for the money, inflation would weaken the value. eg at 2% inflation it would be worth something like (1.05*.98)^2000 = 6.7e24 whilst the gold would then be something like (1.02)^2000*10 = $1.6e18/gram, so multiply your gold value by 1e18.

so there.

Re: Bad decision

*Message #9 Posted by **Ron Ross** on 16 July 2004, 12:14 p.m.,*
in response to message #8 by hugh steers

Not so much due to your math logic as to the value of gold being relative to its scarcity. I don't think gold would be considered a valuable metal with that much of it around. May even become a liability if someone should want you to remove it off their property!

Re: Bad decision

*Message #10 Posted by **OJM** on 16 July 2004, 2:02 p.m.,*
in response to message #9 by Ron Ross

Yep. Ignoring the fact a 2000-light-year-long, Earth-diameter column of gold would crush the planet (of course this is just a theoretical excercise), so much available gold would redefine the element from a precious metal to a common nuisance, less valuable than dirt.

Go for the cash.

Of course, the same could also be said for the money. Having that many dollars around in today's economy would dramatically affect the money supply, driving down the relative value of that currency if you started spending any significant portion of it.

(I.e. Give everyone a million dollars, then everyone would be a millionaire, but then nearly everything would cost a million bucks.)

Re: Bad decision

*Message #11 Posted by **hugh steers** on 16 July 2004, 4:27 p.m.,*
in response to message #10 by OJM

hang on, there.

the choice is about what cash value to accept. there is no gold in the gold choice. only the value of the gold at that time. as i understand it.

Re: Bad decision

*Message #12 Posted by **unspellable** on 16 July 2004, 5:19 p.m.,*
in response to message #10 by OJM

Heinlein said that $100 at 5% for 400 years would amount to umpity zillion dollars and be worth nothing due to inflation. I'd go with Heinlein.

Re: Bad decision

*Message #13 Posted by **Reid** on 17 July 2004, 12:54 p.m.,*
in response to message #10 by OJM

As for the money supply thing, you'd still be the richest man around. Even if everything did suddenly cost millions of dollars, you'd still possess around 99% of the money.

Re: A *very* didactic little quiz

*Message #14 Posted by **Bill Platt** on 16 July 2004, 3:06 p.m.,*
in response to message #1 by Valentin Albillo

Finanlly, I try to reply to a V A quiz!

I liked the 30s formula method for the gold problem---just typed it in:

```
2000*365.25*24*3600*((12,700E5/4)^2pi(300,000E5))*19*10
= 1.139....E41



{oops!  I can see by the formula that I put the "4' in
the wrong place--should be:


2000*365.25*24*3600*(12,700E5^2/4)pi(300,000E5))*19*10
= 4.56...E41


}


for the second one, I thought I'd just have fun with a loop:


32sii


LBL X
INPUT i
INPUT A
LBL Y
RCL A
1.05
X
STO A
1
STO - i
RCL i
X>0?
GTO Y
RTN


RETURNS 2.3911 E42


So, I guess it looks like the 5% interest is mighty fine.
```

Or, to put it another way, gold does not gain value--it merely continues to buy the same *real* things for the same amount of gold, through the centuries. I.E. an ounce of gold will buy a side of beef (or such) now and in the time of the Roman Empire.

Edited: 16 July 2004, 3:15 p.m.

Re: A *very* didactic little quiz

*Message #15 Posted by **John Nelson** on 16 July 2004, 6:27 p.m.,*
in response to message #14 by Bill Platt

Bill,

In your formula...

2000*365.25*24*3600*(12,700E5^2/4)pi(300,000E5))*19*10 = 4.56...E41

... with the part (12,700E5^2/4)pi ... why did you divide it by 4??

Also, why 365.25?? Leap year?

Re: A *very* didactic little quiz

*Message #16 Posted by **bill platt** on 21 July 2004, 11:00 a.m.,*
in response to message #15 by John Nelson

Hi John,

Area = Pi * r^2, or Pi * D^2 / 4

By way of example:

if R=2, then r^2 = 4, therefore D = 4 And D^2 =16. D^2 / R^2 = 4.

365.25 is leap year----it adds up over 2000 years. In fact, I probably should have included the leap seconds, but I don't remember the formula.

Re: A *very* didactic little quiz

*Message #17 Posted by **Dave Shaffer** on 23 July 2004, 5:02 p.m.,* *in response to message #16 by bill platt*

Bill,

"In fact, I probably should have included the leap seconds, but I don't remember the formula"

There isn't a formula. Leap seconds are added when necessary (to keep stepped atomic time with 0.7 seconds of UTC - the time kept by the rotating Earth). Although we can predict reasonably well for a few years what is likely to happen at this level, over hundreds to thousands of years the Earth will do its own thing.

There is now great debate over whether there hsould be a new definition of time. i.e. change the official rate so that civil time more nearly matches astronomical time, without steps (= leap seconds). There are many pros and cons.

Re: A *very* didactic little quiz - a didactic answer

*Message #18 Posted by **Dave Shaffer** on 16 July 2004, 4:59 p.m.,*
in response to message #1 by Valentin Albillo

While you probably have to do the numbers here to find out if 2000 years is enough, the overall answer is that:

A power law (with positive exponent) will always beat a (positively sloped) linear function in the long run.

For you graphics fans, consider two plots: one line (the linear function) increases upwards to the right at a constant slope, while the power law increases upwards to the right at an ever increasing slope. If you go far enough to the right (in time or x, whichever you are calling the "x"-axis), the power law's increasing value will sooner or later exceed the climbing linear function.

Growth

*Message #19 Posted by **Tizedes Csaba [Hungary]** on 16 July 2004, 5:22 p.m.,*
in response to message #18 by Dave Shaffer

```
...yes, if we examining the derivatives: n is the time in years:

GOLD(n)=227.7E36*n
BANK(n)=1.05^n

and:
```

```
dGOLD(n)/dn=227.7E36
dBANK(n)/dn=n*1.05^(n-1)


where this two derivative is equal, the BANK(n) is more growing
than GOLD(n):


227.7E36=n*1.05^(n-1) --> n=1659 year.


Good seems the power of BANK(n) is 2000-1659=341 year is enough
to cath up the GOLD(n). The difference and ratio is very big:


GOLD(1659)=377.8E39
BANK(1659)=142.2E33
```

Csaba

Re: A *very* didactic little quiz

Message #20 Posted by ***unspellable*** *on 16 July 2004, 5:15 p.m.,*
in response to message #1 by Valentin Albillo

Is Betelgeuse really that far away? It's the only star we have a picture of with enough resolution to show any surface detail. I would think it would have to be much closer for that, even though it's pretty big as stars go.

Re: A *very* didactic little quiz

Message #21 Posted by ***Valentin Albillo*** *on 19 July 2004, 9:18 a.m.,*
in response to message #20 by unspellable

You're right, seems that many web sites list Betelgeuse's distance erroneously, for whatever the reason-

This site states 430+-100 light-years is the correct distance, as well as lots of interesting data on Betelgeuse.

My originally posted distance (1,400 light years) was taken from this site and other sites list the distance as 520 light-years, which more or less agrees with the arguably correct value, within tolerance.

As for seeing its 'surface', it all depends on what you consider a 'surface'. Betelgeuse is some 650 times the diameter of our Sun (so 650^3 = 274,625,000 times our Sun's volume) but only 12-17 times its mass, so Betelgeuse's average density is incredibly low (use your HP calculator now), and you'll probably find that the alleged most-external 'surface' which the Hubble did photograph would be considered the most tenuous gas (nearly vacuum) if we were to try to 'land' there.

Best regards from V.

How many people...

Message #22 Posted by ***Tizedes Csaba [Hungary]*** *on 16 July 2004, 5:37 p.m.,*
in response to message #1 by Valentin Albillo

...lived on the Earth since of dawn of man??? Not in present! All of people! Because if I want to distribute this money...

Cs.

Re: How many people...

Message #23 Posted by ***Richard Garner*** *on 16 July 2004, 11:59 p.m.,*
in response to message #22 by Tizedes Csaba [Hungary]

I remember seeing or hearing that at present 75 percent of all of man kind is alive today. If that is true then the total population of the Earth in human kind from past to present is about 8 Billion.

Re: A *very* didactic little quiz

Message #24 Posted by ***John Nelson*** *on 16 July 2004, 6:31 p.m.,*
in response to message #1 by Valentin Albillo

Ok... so now lets look at something else. In what year (ingnore seconds please) will will the dollar invested at 5% exceed the value of gold.

I already know the answer, but just curious as to how others may solve this part of the problem.

Re: A *very* didactic little quiz

Message #25 Posted by ***Tizedes Csaba [Hungary]*** *on 16 July 2004, 7:42 p.m.,*
in response to message #24 by John Nelson

```
I hope, I understand well the problem:
```

```
(eq1): 1.05^n=1.05*A*n, where A=227.7E36$/yr


-----------------------------------------------------------
1.) Approximation of n:


(eq1): 1.05=(1.05*A)^(1/n)*(n)^(1/n)


if n is big, (n)^(1/n) is close to 1, then we wrote:


(1.05)^(n)=1.05*A, so n=1+ln(A)/ln(1.05)=1811
-----------------------------------------------------------
2.) Solution with iteration:


take log() of (eq1) with 1.05 base:


n=1+(ln(A)+ln(n))/ln(1.05)


Let's start this iteration with n=1811, you will get:


step No      n
   0       1811
   1       1964.98
   2       1965.64
   3       1966.65
   4       1966.67  --> this is the solution.


At this n: DOLLAR=470.2E39, and GOLD=447.8E39
-----------------------------------------------------------
```

Csaba

Re: A *very* didactic little quiz

Message #26 Posted by ***Eamonn*** *on 16 July 2004, 9:53 p.m.,* *in response to message #25 by Tizedes Csaba [Hungary]*

Hi Csaba,

I think the equation that needs to be solved should be

`1.05^n = A*n`

not

`1.05^n=1.05*A*n`

Where A is 2.2771e38, as before.

Solving this second equation gives the solution n = 1965.657.

At this time, the value of both the gold and the interest bearing account is $4.476e41.

Best Regards,

Eamonn.

Eamonn's answer to John Nelson's question is correct

*Message #27 Posted by **Karl Schneider** on 17 July 2004, 1:43 a.m.,*
in response to message #26 by Eamonn

Dave Shaffer posted,

> Quote:
>
> A power law (with positive exponent) will always beat a (positively sloped) linear function in the long run.

In Valentin's quiz, the cylinder of gold grows in value at a constant rate "A" of $2.277082607 x 10^38 per year.

The savings acount grows at a exponential rate of 5% per year.

Given a large-enough input variable, the exponential function will surpass the linear function in magnitude.

So, as Eamonn stated,

1.05^n - A*n = 0 is the equation to solve.

This can be programmed on any unit with SOLVE, particularly easily on a 15C/34C or 32S/32SII/33S.

My 15C yields n = 1965.65664 (years), as Eamonn got.

Edited: 17 July 2004, 1:50 a.m.

Re: Eamonn's answer to John Nelson's question is correct

Message #28 Posted by ***Tizedes Csaba [Hungary]*** *on 17 July 2004, 4:02 a.m.,*
in response to message #27 by Karl Schneider

"In what year will the dollar invested at 5% exceed the value of gold."

This is my problem: I don't understand correctly English grammatical forms. I know it's simple, but my brain can't to understand...

In Hungarian language we don't use too many prepositions. The Hungarian uses inflections.

But the original problem correctly solveable (without SOLVE) the approximated and iterated methods!

Csaba

Ps.:

```
We will get:


-----------------------------------------------------------
1.) Approximation:


1.05^n=A --> n=1810
-----------------------------------------------------------
2.) Iteration:


n=(ln(A)+ln(n))/ln(1.05)


No of steps       n
```

```
    0           1810
    1           1963.97
    2           1965.64
    3           1965.66 --> the solution


And we don't to use SOLVE.
-----------------------------------------------------------
3.) Graphically:


The eq. is: 1.05^n=A*n, the log() of it (base 1.05):


n=(ln(A)+ln(n))/ln(1.05)


this is an linear function:


n=m*ln(n)+b, where m=1/ln(1.05)=20.50 and
                   b=ln(A)/ln(1.05)=1810


to the simplifiest drawing make it to:


1/m*n-b/m=ln(n), so this is:


############################
#                          #
#  ln(1.05)*n+ln(A)=ln(n)  #
#                          #
############################


Scatch the two sides of this equation to the paper, and the
intersection is the root. (Its enough in n=1800...2000)


With this method absolutly scathing I have got n=1965,
and I use ln(n)=7.5 approx in this interval (not important,
it's just a scatch...) I wasn't used ruler and graph paper.
-----------------------------------------------------------
```

Cs.

Edited: 17 July 2004, 4:40 a.m.

SOLVE and iteration

Message #29 Posted by ***Karl Schneider*** *on 17 July 2004, 2:47 p.m.,*
in response to message #28 by Tizedes Csaba [Hungary]

Csaba posted,

> Quote:
>
> ```
> 2.) Iteration:
> n=(ln(A)+ln(n))/ln(1.05)
> No of steps       n
>     0           1810
>     1           1963.97
>     2           1965.64
>     3           1965.66 --> the solution
> And we don't to use SOLVE.
> ```

Indeed, it is fortunate that the simple equation lent itself to a transformation n = f(n), and that direct iterative "recycling" of f(n) as "next n" converged quickly to the answer. However, that will generally not be the case.

SOLVE does a fine job of automating an intelligent single-variable iterative root-finding process. That (and INTEGRATE) was impressive and pioneering about the 34C back in 1979. One of the HP Journal articles from 1980 about the 34C, available on the MoHPC DVD/CD set, describes the effort.

Re: Eamonn's answer to John Nelson's question is correct

Message #30 Posted by ***John Nelson*** *on 17 July 2004, 12:41 p.m.,*
in response to message #27 by Karl Schneider

I agree, although I did mine a bit different....

I created a small program and found the following...

Year = 1965 Day = 239 Hour = 16 Minute = 10 Second = 33

Re: Eamonn's answer to John Nelson's question is correct

Message #31 Posted by ***Tizedes Csaba [Hungary]*** *on 17 July 2004, 5:05 p.m.,*
in response to message #30 by John Nelson

I want to reply with a joke, I hope it will be clear:

An veterinarian was taken ill, and went to a doctor. The doctor asked him: "What is your complaint?" The veterinarian said: "Ahh, that is not a big deal!!!"

So, I think with SOLVE this is too simply to solve this or like this problem. :)

When the all of calc's with SOLVE will be wrong, we will back to brain-work. This is the important, not button pushing.

So, I am use SOLVE too, day by day...!

Csaba

Ps.: If the ABS(df/dn)<=h<1 then the iteration is converged, in an [a,b] interval, where df/dn is available.

Edited: 17 July 2004, 5:06 p.m.

Further Challenge

Message #32 Posted by ***Eamonn*** *on 19 July 2004, 10:13 p.m.,*
in response to message #24 by John Nelson

John,

> Quote:
>
> In what year (ingnore seconds please) will will the dollar invested at 5% exceed the value of gold.

While most of us focused on finding an answer to this question that was somewhere around the year 1965, there is of course another answer - At the very start of year 0, ie. at t=0.000, the account value is $1.00, while the value of the gold is $0.00.

This prompted me to come up with the following questions:

1) At what time does the value of the gold cylinder, growing towards infinity at the speed of light, first exceed the value of the interest bearing account?

2) At this time, how tall is the cylinder?

The second question is an interesting question to ponder from a physics point of view - I'm curious to see what answers people may come up with.

Regards,

Eamonn.

Re: Further Challenge

Message #33 Posted by ***Valentin Albillo*** *on 20 July 2004, 5:52 a.m.,*
in response to message #32 by Eamonn

Hi, Eamonn:

Eamon posted:

"1) At what time does the value of the gold cylinder, growing towards infinity at the speed of light, first exceed the value of the interest bearing account?"

Strictly speaking, it will exceed the value of the interest bearing account the moment it surpasses $1.00, exactly, because as per the conditions stated in my original posting, the interest is accrued *yearly* so that until a full year elapses, you're stuck with *exactly* $1.00. The interest is not accrued continuously, only at the end of each yearly period.

So, the answer to your question is a simple division, and it comes out as:

```
t = 1/((12700/2)^2*Pi*300000*19*(100000)^3*10)


   = 0.00000000000000000000000000000013849299939578841 seconds.
```

Even considering continuous growth for the bank account, this is such an small time that the net balance is still so near the original $1.00 as not to make a difference to the digits shown for t.

"2) At this time, how tall is the cylinder?"

That's a simple multiplication, *t * 300000* , the answer being:

```
Height = 0.0000000000000000000000000415478998187365 Km.
```

```
        = 0.0000415478998187365 attometers
```

which is certainly a small height, yet quite large when measured in Planck's length units (1.6161 * 10^-35 m)

"The second question is an interesting question to ponder from a physics point of view."

I guess so.

Thanks for your interest and keen observation and

Best regards from V.

Edited: 20 July 2004, 5:56 a.m.

Re: Even Further Challenge,

Message #34 Posted by ***Veli-Pekka Nousiainen*** *on 20 July 2004, 8:47 a.m.,*
in response to message #33 by Valentin Albillo

```
"1) At what time does the value of the gold cylinder,
growing towards infinity at the speed of light,
first exceed the value of the interest bearing account?"
X
the answer is:
  t = 1/((12700/2)^2*Pi*300000*19*(100000)^3*10)
    = 0.00000000000000000000000000001384929993957884l seconds.


Even considering continuous growth for the bank account,
this is such an small time that the net balance
is still so near the original $1.00
as not to make a difference to the digits shown for t.
*******************************************
VPN: If the interest would be non-discrete
VPN: (eg. continuous instead of at the end of the year)
VPN: How many digits after the "five" do you need to get 6 figures at the end
VPN: 1,05nnn...nnn654321
*******************************************
"2) At this time, how tall is the cylinder?"
That's a simple multiplication, t * 300000 , the answer being:
Height = 0.00000000000000000000000415478998187365 Km.
       = 0.0000415478998187365 attometers
```

```
which is certainly a small height, yet quite large when measured in Planck's length units (1.6161 * 10^-35 m)
*******************************************
VPN: How many gold atoms would that mass be?
VPN: How many grams vs oz (mass)?
VPN: Give the oz like inches eg. a/2^n parts, what is n?
VPN: example: 5/1048576 oz = 5/2^10 oz => n=10
VPN: the numerator (a) size is your choise, but let say 3 digits is OK
* END *
```

Edited: 20 July 2004, 12:51 p.m.

Re: Further Challenge

Message #35 Posted by ***Eamonn*** *on 20 July 2004, 3:54 p.m.,*
in response to message #33 by Valentin Albillo

Hi Valentin,

> Quote:
>
> ---
>
> So, the answer to your question is a simple division, and it comes out as:
>
> t = 1/((12700/2)^2*Pi*300000*19*(100000)^3*10)
>
> = 0.0000000000000000000000000000001384929993957841 seconds.
>
> ---

This is correct - at least to the 16 significant digits I was able to verify. I'm going to have to get a PC-1475 to verify that last digit :-)

> Quote:
>
> ---
>
> "2) At this time, how tall is the cylinder?"
>
> That's a simple multiplication, t * 300000 , the answer being:
>
> Height = 0.0000000000000000000000000415478998187365 Km.
>
> ---

For notational convenience, I'll express this answer as the equivalent 4.15478998...e-23 meters. From a theoretical point of view, which is what we are dealing with, this is entirely correct (at least to the 16 significant figures I can verify).

As you stated, this is quite large when expressed in units of Planck's length, but it is quite small when compared to the diameter (height) of a gold atom (1.3e-10 meters). An alternate answer would be that since the cylinder is made of gold, then the height would necessarily have a minimum of the diameter of one gold atom.

I was wondering how such a gold 'cylinder', worth all of $1.00, would physically manifest itself. As far as I can tell, at this time, it could consist of of gold atoms (1.6e20 of them), spread out across an area of 1.26677e14 m^2. Assuming an even distribution of atoms, there would be 1.27e6 atoms per m^2. If these are evenly distributed in a square lattice, then the spacing between each atom is 0.875 mm.

However, since this quiz is theoretical in nature, I accept your answer to my second question as the correct height of such a gold cylinder with a value $1.00.

Thanks for the original quiz and best regards,

Eamonn.

Re: Further Challenge

Message #36 Posted by ***W Rice*** *on 21 July 2004, 2:37 p.m.,*
in response to message #35 by Eamonn

Eamonn wrote:

I was wondering how such a gold 'cylinder', worth all of
$1.00, would physically manifest itself. As far as I can
tell, at this time, it could consist of of gold atoms
(1.6e20 of them), spread out across an area of 1.26677e14
m^2. Assuming an even distribution of atoms, there would be
1.27e6 atoms per m^2. If these are evenly distributed in
square lattice, then the spacing between each atom is 0.875 mm

----

I think the interatomic spacing you used (0.875 mm) is more akin to a dusting of gold atoms than solid gold. Here is my solution, assuming each ring of atoms grows outwards from the centre as they are deposited:

```
density of gold: 19g cm^-3
molecular weight: 197 g/mol
Avagadro's number: 6.02 exp23 particles/mol


1 cc = 19g / (197 g/mol) * 6.02exp23 atoms/mol
     = 5.81 exp 22 atoms
A line of atoms 1cm in length= (5.81exp22)^-3 = 38.7 exp6 atoms
One square m of gold atoms:
(38.7 exp6)^2 atoms/cm^2 * 100^2 cm^2/m^2
= 1.50 exp 19 atoms m^-2


$1 of gold is 0.1 g /(197 g/mol) * 6.02exp23 = 3.06exp20 atoms


Area required = 3.06exp20/1.50exp19 = 20.4 m^2
```

Which corresponds to a circle of gold atoms with diameter 2.55 m.

Regards,
Bill

Re: Further Challenge

Message #37 Posted by ***Eamonn*** *on 22 July 2004, 1:34 p.m.,*
in response to message #36 by W Rice

Hi Bill,

Your calculations look correct to me. I made an error when calculating the number of atoms in $1.00 of gold - 3.06e20 is correct.

I completely with you when you point out that with such a large inter-atomic spacing you are far from having a solid gold cylinder. Thanks for the alternative analysis of what size gold 'cylinder' you would have under your assumptions.

Best Regards,

Eamonn.

Re: A *very* didactic little quiz

Message #38 Posted by ***Gordon Dyer*** *on 17 July 2004, 9:48 a.m.,*
in response to message #1 by Valentin Albillo

I solved this in Excel in just a few minutes with a nice graph and log scale to go with it.
the answer needs 14 decimal places to be accurate:
1$ @ 5% Interest = $2.3911022046137500E+42 Gold @ light speed = $4.5541652123333300E+41

I would go for the gold and cash it in after 1 second. I'd rather have enough money alive than a legacy which bankrupts the earth after I'm dead!!!

Re: A *very* didactic little quiz

Message #39 Posted by ***GE (France)*** *on 17 July 2004, 5:41 p.m.,*
in response to message #38 by Gordon Dyer

Nice puzzle, Valentin (and lots of replies).

There is something to take into account : risk. "What if" the process stops before the end (or the deal is totally negated) ? Without additional data on the reliability of banks in the next 2000 years and of your strange gold generation process, I'd take the gold, as it is better than cash for over 90% of the time. Additionnaly, cash is better only at the 'bad' end of time, when the probability of default is highest.

On another subject, I believe the equation A^x=B*x is equivalent to exp(x)/x=C (with a suitable 'simple' formula for C=f(A,B)). Now my question : in Maple, solving the second form brings an expression using the funtion "LambertW()". Does anyone know what this is ? Pointers welcome.

Re: A *very* didactic little quiz

Message #40 Posted by ***Valentin Albillo*** *on 19 July 2004, 5:41 a.m.,*
in response to message #39 by GE (France)

Hi, GE:

GE posted:

"Nice puzzle, Valentin (and lots of replies)"

Yes, thanks for your kind words, I'm very happy people were insterested, as I feel it makes a pretty fine example of *didactic-yet-entertaining* mathematics, if I may coin a term.

"There is something to take into account : risk. "

Pleeeease, GE ! :-) It's only a *pure-math* quiz, I said so much in my original post. No inflation, risk, physical feasibility, speed-of-light unattainable for non-zero mass objects, not so much gold in the Universe, the Earth would be crushed, gold would be worth nothing, the bank teller would run out with the money, etc, etc, etc, etc, etc, etc, etc, etc ! :-) We do have a very descriptive Spanish term for this, but it's very *graphic* and I don't know how to properly translate it. I'm sure you do have some equivalent in French ;-)

"In Maple, solving the second form brings an expression using the funtion "LambertW()". Does anyone know what this is ? Pointers welcome. "

The Lambert-W function (known in Mathematica as "ProductLog") is an extremely interesting function, that appears everywhere and it's been unofficially nominated for the next "standard elementary function", similar to trigonometric and exponential functions, so it's not that impossible that we'll have calculators including it as a standard function in the near future. The advantage to consider it a standard function is that you can then express a large number of roots to trascendental equations, integrals, and ODEs, to name a few, in closed form in terms of LambertW (instead of having non-closed, infinite series expansions or sums), which is exactly what we do achieve by using sin, cos, exp, log, etc.. I've certainly used it a lot, to express roots of equations in terms of it (for instance, the solution to ***x^x = a*** can be expressed in terms of LambertW), like this:

```
    x^x = a  ->  x = e^W(ln(a))


    e.g.: x^x = Pi  -> x = e^W(ln(Pi) =  1.85410596792


HP-71B code: first define


    10 DEF FNW(X)=FNROOT(-1,10,FVAR*EXP(FVAR)-X)


then:


    > X=EXP(FNW(LN(PI)))
    > X
          1.85410596792
    > X^X
          3.14159265358
```

As for your equation, ***e^x/x = C***, the solution is:

```
     e^x/x = C  ->  x = -W(-1/C)
```

HP-71B code, assuming the previous definition of FNW(X):

```
     e.g: e^x/x = 5  -> x = -W(-1/5)


     >X=-FNF(-1/5)
     >X
```

```
          0.259171101818
     >EXP(X)/X
          5
```

The best, most comprehensive and interesting document I know about LambertW is available from here, in PDF format. Just go down the page to the **Some interesting math-related papers** section and click the first link, "On the Lambert-W function".

Best regards from V.

Edited: 19 July 2004, 8:36 a.m.

Re: A *very* didactic little quiz

Message #41 Posted by ***GE (France)*** *on 19 July 2004, 5:23 p.m.,*
in response to message #40 by Valentin Albillo

Thank you ! Excellent reading.

How about another didactic quiz?

Message #42 Posted by ***John Nelson*** *on 18 July 2004, 3:10 p.m.,*
in response to message #1 by Valentin Albillo

I just wanted to say I thought this little quiz was really fun. I remember seeing something similar to it when I was earning my MBA a few years back to show the power of compounding.

My enjoyment not only came from playing with my HP calc, but also seeing the many different ways that this problem could be solved.

So I guess my challenge point is this... who will post the next didactic challenge for all of us? I can't wait.

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