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[VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

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03-21-2019, 04:08 AM

Post: #1



Valentin Albillo
Senior Member

Posts: 376
Joined: Feb 2015
Warning Level: 0%

[VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Hi, all ! Happy Spring to all of you !

... and once again to commemorate its arrival, here you are, a brand new **Short & Sweet Math Challenge #24 "Spring Special 5-tier"** to give you all a chance to put to good use both your favourite **HP calculator** and your programming ingenuity (*NOT* your *Google Search* proficiency). Try all **5 tiers** below and see what's your top one !

Rules:

- Any **HP calc** of your choice may be used but I'll suggest a *Minimum Recommended Model (MRM)* for each tier, which is the simplest model I deem capable of solving it more or less comfortably.
- Using anything other than a **physical** or **emulated HP calculator** is strictly disallowed. Also **no VBA, Excel, Pascal, C/C#/C++, Java, Python, Haskell**, etc. code, please go elsewhere for that. You must write your code in a language supported in some HP calc (i.e.: **RPN, RPL, 71B BASIC/FORTH**, etc).
- Googling the solutions is lame beyond belief and, frankly, if you do you'll be but a *sore loser* in my eyes 😊

Tier 1: Noob

[MRM: **HP-11C** and up]

Let's begin with something affordable. As it happens, I have **2** HP-**71B** and a **41C** in my collection and I love them dearly so I'll pay them a little homage here: we'll call **Homage Number** to any *10-digit* positive integers which are *multiples of 271*, *divisible by 41* and further their digits are *all distinct*.

The Challenge:

Write a program that takes no inputs but simply finds out and outputs just **how many Homage Numbers** there are. No need to output any of them, just count'em and do it fast !

Your code should be as *fast* and *short* as possible, in that order. I'll post my original code (a 2-liner) and results for the **HP-71B** .

Tier 2: Beginner

[MRM: **HP-29C** and up]

Consider the function **S_B(N)** which returns the *sum* of the *base-B* digits of an integer **N**. For instance:

$$\begin{aligned}
 S_2(2019) &= S_2(11111100011_2) = 1+1+1+1+1+1+0+0+0+1+1 = 8 \\
 S_5(2019) &= S_5(31034_5) = 3+1+0+3+4 = 11 \\
 S_{10}(2019) &= 2+0+1+9 = 12 \\
 S_{16}(2019) &= S_{16}(7E3_{16}) = 7+E (=14) + 3 = 24 \\
 S_{36}(2019) &= S_{36}(1K3_{36}) = 1+K (=20) + 3 = 24
 \end{aligned}$$

The Challenge:

Write a program that accepts a base **B** (2 to 36) and outputs in order those **prime** numbers **N** such that **S_B(N)** is **composite** and *distinct* from the previous ones.

For instance, this is what your program should generate for bases **B = 6, 8 and 16 (hexadecimal)**:

B = 6: 11, 19, 23, ... 179, ...
B = 8: 11, 13, 23, ... 191, ...
B = 16: 19, 23, 29, ... 223, ...

Once verified that your code reproduces the above sample results, go on and generate the corresponding sequences for base **B = 31** first and then for base **B = 7**.

What results do you get? Which is the *smallest* (first) prime in each sequence? How many elements can you generate for each sequence? What about other bases?

Again, your code should be as *fast* and *short* as possible, in that order. I'll post my original code (a 6-liner) and results for the **HP-71B**.

Tier 3: Intermediate

[MRM: **HP-25** and up]

Consider the real numbers **77.4019...** and **231.4859...**, which are sums of *distinct* non-negative integer powers of **e** ($= \exp(1) = 2.71828\dots$):

$$\begin{aligned} 77.4019\dots &= e^1 + e^3 + e^4 \\ 231.4859\dots &= e^0 + e^2 + e^3 + e^4 + e^5. \end{aligned}$$

Those positive real numbers that are either powers of **e** or sums of distinct powers of **e** form an increasing sequence whose first term is **1** (i.e.: e^0) and matter of fact we have that **77.4019..** is the **26th** term in the sequence and **231.4859...** is the **61th** term.

The Challenge:

Generalizing to powers of an arbitrary real number $P \geq e$, write a program or function which accepts as input both **P** and an index **k** and returns the corresponding **kth** term in the sequence (in the example above we would have $MyFunction(e, 26) = 77.4019\dots$ and $MyFunction(e, 61) = 231.4859\dots$). Your code should be as short and fast as possible.

Now use your program/function to find the **1,000,000th** term and the **3,141,593th** term when $P = e$ as well as the **1,234,567th** term and the **2,718,282th** term when $P = \pi$. Also, just for show, use it to list the **first 10 terms** or so for each sequence.

I'll post both a *1-line* user-defined function for the **HP-71B** and an equivalent *24-step RPN* program for the **HP-25** (which should work with little or no change in *all RPN-based* HP calcs).

Tier 4: Advanced

[MRM: **HP-11C** and up]

Consider the **n**-point dataset (x_j, y_j) where $x_j = 1, 2, 3, 4, 5, 6, \dots, n$ (the natural numbers) and $y_j = 2, 3, 5, 7, 11, 13, \dots, p_n$ (the prime numbers), and the $(n-1)^{st}$ degree *polynomial fit* to this dataset of the form:

$$P(x) = a_0 + a_1(x-1) + a_2(x-1)(x-2) + \dots + a_{n-1}(x-1)(x-2)(x-3)\dots(x-(n-1))$$

The Challenge:

Write a program that takes no inputs but computes and outputs the limit of the **sum of the coefficients a_0, a_1, \dots, a_{n-1}** when **n** tends to *infinity*. Your program must be as short and fast as possible and must compute the limit to the **10-12 digits** maximum accuracy of your calc, give or take a few *ulps*.

I'll post a *4-line, 168-byte* program for the **HP-71B** which computes and outputs the limit in ~ 0.2 sec (*Emu71*) but a fast *RPN* version for the **HP-11C** and up is also perfectly possible.

Tier 5: Guru

[MRM: **HP-11C** and up]

Surely you're well aware of the elementary trigonometric function **sin(x)**, you know, the wavy one. Now consider a related function, which henceforth I'll call **cin(x)** which has the defining property that **cin(cin(cin(x))) = sin(x)**.

The Challenge:

Write a program or function which accepts an argument **x** in the range $[-\pi, \pi]$ and outputs the corresponding value of **cin(x)**. The faster and shorter the better but you should strive for *maximum accuracy* (at least 8-10 correct digits in

the *whole* range, give or take a few *ulps*).

Once written, use it to **tabulate** $\text{cin}(x)$ for $x = 0.0, 0.2, 0.4, \dots, 1.0$ and also to **compute** $\text{cin}(\pi/2)$, $\text{cin}(-0.71)$, $\text{cin}(2.019)$, and with the experience gained do likewise with another similar function $\text{tin}(x)$ which has the property that $\text{tin}(\text{tin}(x)) = \sin(x)$.

I'll give a short program for the **HP-71B** which achieves about 10 correct digits for any argument in $[-\pi, \pi]$.

Hint: You can check that you get adequately accurate values of $\text{cin}(x)$ by simply computing $\text{cin}(\text{cin}(\text{cin}(x)))$ and comparing the results with $\text{sin}(x)$. Likewise with $\text{tin}(x)$.

Finally, the usual caveat:

- Please **do NOT include CODE panels** in your replies to this thread, as it makes it difficult for me to generate the online PDF document which will include the whole thread. I expect you'll kindly comply with this requirement but otherwise I'll remove from the final PDF document any replies featuring **CODE** panels. Thank you.

I'll post my original solutions in a week or so but meanwhile let's see what you can do. 😊

That's all. Hope you'll enjoy it !

V.



03-21-2019, 02:21 PM

Post: #2

Albert Chan 🧑

Senior Member

Posts: 627

Joined: Jul 2018

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Homage number must be divisible by $271 * 41 * 9 = 99999$

Since $1e5 \bmod 99999 = 1$, Homage number had another property:

top 5 digits + bottom 5 digits = 99999

Example, since $10234 + 89765 = 99999$, smallest Homage number is 10234 89765

Top digit > 0 , thus 9 cases to choose.

2nd digit cannot be the first, or 9 - first, thus $10 - 2 = 8$ cases ...

Total Homage numbers = $9 * 8 * 6 * 4 * 2 = 3456$



03-21-2019, 02:30 PM (This post was last modified: 03-21-2019 03:50 PM by Paul Dale.)

Post: #3



Paul Dale 🧑

Senior Member

Posts: 1,467

Joined: Dec 2013

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

I'm not sure why but tier 3 and tier 4 look like the easiest.

Spoilers towards the end.

Anyway, a 24 step program for tier 3 for the HP 25:

```
01 CLREG
02 1
03 x<>y
04 x=0?
05 GTO 24
06 2
07 /
08 INT
09 LASTx
10 FRAC
11 x=0?
12 GTO 17
13 Rv
14 x<>y
15 STO+ 0
```

```

16 GTO 19
17 Rv
18 x<>y
19 1
20 e^x
21 *
22 x<>y
23 GTO 04
24 RCL 0

```

For the Pi flavour, change steps 19 and 20 to: Pi and NOP.

The 1,000,000th term is 278,394,443.2 and the 3,141,593rd term is 1,601,007,657. Both in several seconds on a real 25.

The 1,234,567th term for Pi is 9,091,632,462 and the 2,718,282nd term is 30,446,503,22x (x being beyond the accuracy of the 25). Again, fairly quickly.

The key observation being that:

$$\sum_{i=0}^n e^i = \frac{e^{n+1} - 1}{e - 1} < e^{n+1}$$

which means that the position expressed in binary determines the powers used. Note that $2^1 + 2^3 + 2^4 = 26$ and $2^0 + 2^2 + 2^3 + 2^4 + 2^5 = 61$ and compare to the initial examples.

Larger bases also have this property. I didn't try to prove that e is the smallest base for which this holds true, which is good because the smallest base is, unsurprisingly, two.

Pauli



03-22-2019, 10:04 AM

Post: #4



J-F Garnier
Senior Member

Posts: 304
Joined: Dec 2013

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

(I noticed that the SSMC24 was Valentin's post #314 at the time of writing :-)

Tier 5: Guru

In summary : Write a program or function cin(x) which has the defining property that cin(cin(cin(x))) = sin(x).

Once written, use it to tabulate cin(x) for x = 0.0, 0.2, 0.4, ..., 1.0

From the properties of sin we can restrict the search of cin(x) in the interval [0,pi/2].

The problem implicitly assumes that the cin(x) function is unique, otherwise it would make no sense to discuss the cin(x) values.

If we understand 'function' in the mathematic sense of an analytical, non-pathologic function, this may be true.

But if we understand 'function' in the computer science sense of a procedure that takes one argument and returns one result, there are very likely many cin 'functions' such as cin(cin(cin(x)))=sin(x)

One such solution for the HP71 is below:

```

10 ! SSMC24A
20 DEF FNC(X)
30 IF X=0 THEN Y=0 @ GOTO 70
40 X=ABS(X)
50 Y=LN(X)
60 IF Y<2 THEN Y=1+EXP(X) ELSE Y=SIN(LN(LN(X-1)-1))
70 FNC=Y
80 END DEF
85 !
90 FOR X=0 TO 1 STEP .1
100 PRINT FNC(FNC(FNC(X)));SIN(X)
110 NEXT X

>RUN

```

0 0
 9.98334166508E-2 9.98334166468E-2
 .198669330795 .198669330795
 .295520206664 .295520206661
 .389418342308 .389418342309
 .479425538604 .479425538604
 .564642473395 .564642473395
 .644217687238 .644217687238
 .717356090899 .7173560909
 .783326909628 .783326909627
 .841470984808 .841470984808

I don't take this solution too seriously :-)
 Still searching for a better one...

J-F



03-23-2019, 02:40 AM

Post: #5



Valentin Albillo
 Senior Member

Posts: 376
 Joined: Feb 2015
 Warning Level: 0%

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Hi J-F, long time no see !

J-F Garnier Wrote: →

(03-22-2019 10:04 AM)

(I noticed that the SSMC24 was Valentin's post #314 at the time of writing :-)

Yes, that was intentional on my part. Notice also that Paul Dale's post just before yours is his post #1444, i.e. 38^2 , which has the maximum number of 4's (three) in which a square can end.

Quote:

The problem implicitly assumes that the $\text{cin}(x)$ function is unique, otherwise it would make no sense to discuss the $\text{cin}(x)$ values. If we understand 'function' in the mathematic sense of an analytical, non-pathologic function, this may be true.

Yes, that's the kind of "function" I intended.

Quote:

But if we understand 'function' in the computer science sense of a procedure that takes one argument and returns one result, there are very likely many cin 'functions' such as $\text{cin}(\text{cin}(\text{cin}(x))) = \sin(x)$
 One such solution for the HP71 is below ...

Quite clever of you to find the trick, but certainly that's not what I intended as a solution. The trick will work for any function (not just $\sin(x)$) and for any number of function compositions (not just 3), but it's of no mathematical value or interest whatsoever.

Also, you overdid it a little, there's no need of *EXP's* or *LN's* to make it work, simply *adding* and later *subtracting* a suitable *constant* would do just as well (and faster). Finally, your line

```
30 IF X=0 THEN Y=0 @ GOTO 70
```

can be shortened to

```
30 IF X=0 THEN END
```

because if a multi-line user-defined function ends without assigning the return value then it simply returns 0 by default. And `IF X=0` can be replaced by `IF NOT X`

Quote:

I don't take this solution too seriously :-)

Indeed you shouldn't, neither do I ... 😊

Quote:

Still searching for a better one...

Good luck with that, I'm sure you'll succeed and I'm eager to see what you come up with.

Very glad to see you post here, much appreciated. Thanks for your continued interest in my *S&SMC's* and have a nice weekend.

V.

*P.S.: What would it take to lure you into releasing a version of **Emu71** which would run on a **32-bit** or 64-bit Windows OS, or at least on Android ? Begging ? Bribing ? Taking some relative hostage ? Just plain ol' money ? You tell me, please, it's really affecting my productivity !! 🙄*



03-23-2019, 09:53 AM

Post: #6



J-F Garnier
Senior Member

Posts: 304
Joined: Dec 2013

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Valentin Albillo Wrote: →

(03-23-2019 02:40 AM)

Hi J-F, long time no see !

...

Very glad to see you post here, much appreciated. Thanks for your continued interest in my *S&SMC's* and have a nice weekend.

I'm still following your various challenges, even participated to the the [last one](#).
And when I don't participate, it may just mean I have nothing interesting to contribute.

Quote:**Quote:**

Still searching for a better one...

Good luck with that, I'm sure you'll succeed and I'm eager to see what you come up with.

I investigated a few ideas, without success up to now.

Quote:

*P.S.: What would it take to lure you into releasing a version of **Emu71** which would run on a **32-bit** or 64-bit Windows OS, or at least on Android ?*

It's a bit OT - well maybe not so much since the HP71 and Emu71 are your favourite tools for your challenges. In short, the answer is that I don't have the competences to port my Emu71/Dos to Windows, MacOS, Linux, not to mention iOS, Android. However, Emu71/DOS is open source...
And you know for sure that there is already an excellent [Emu71](#) running on Windows (32/64 bits), and also an [HP71 emulator](#) for Android.

J-F



03-23-2019, 06:03 PM

Post: #7

Albert Chan
Senior Member

Posts: 627
Joined: Jul 2018

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Perhaps $\text{cin}(x)$ need a bit more restriction, not just $\text{cin}(\text{cin}(\text{cin}(x))) = \sin(x)$

$$\sin(\text{cin}(x)) = \text{cin}(\sin(x))$$

$\text{cin}(x)$ should be an odd function, shape like $\sin(x)$, with value between $\sin(x)$ and x

For domain $[-\pi/2, \pi/2]$, **$\text{cin}(x) \approx (1 + x^2/9) \sin(x)$**

Example:

$\sin(\frac{1}{2}) = 0.4794255386$
 $\text{cin}(\text{cin}(\text{cin}(\frac{1}{2}))) = 0.4791650974$, relative error $\sim -0.05\%$

$\sin(\text{cin}(\frac{1}{2})) \approx 0.473044$
 $\text{cin}(\sin(\frac{1}{2})) \approx 0.473050$



03-23-2019, 11:49 PM

Post: #8

rprosperi
Senior Member

Posts: 3,386
Joined: Dec 2013

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Valentin Albillo Wrote: →

(03-23-2019 02:40 AM)

*P.S.: What would it take to lure you into releasing a version of **Emu71** which would run on a **32-bit** or **64-bit** Windows OS, or at least on Android ? Begging ? Bribing ? Taking some relative hostage ? Just plain ol' money ? You tell me, please, it's really affecting my productivity !! 🙄*

Valentin - Emu71/DOS runs great on Win7/8/10 (including x64) operating under DOSBOX, though of course not at the full speed it would run if running as native code.

So, if by productivity you mean having to change to a DOS machine to run Emu71, then the above would save a lot of time and hassle, however if raw Emu71 execution speed is the issue, this may not be a big increase in speed, if it increases at all.

As for using Christoph's EMU71/Win, you can remove the "run at actual speed" option and have much higher speed, but I don't know how it compares to native Emu71/DOS on your machine.

If you send me a sample program with relatively long run-time, I'd be happy to time it running on my PC under both Emu71/DOS/DOSBOX and Emu71/Win at it's max speed. This is an older PC with a XEON 3GHz CPU running Win7x64, so not the fastest, but it at least provides some relative performance numbers.

--Bob Prosperi



03-24-2019, 11:08 PM

Post: #9



J-F Garnier
Senior Member

Posts: 304
Joined: Dec 2013

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

J-F Garnier Wrote: →

(03-22-2019 10:04 AM)

Tier 5: Guru

In summary : Write a program or function $\text{cin}(x)$ which has the defining property that $\text{cin}(\text{cin}(\text{cin}(x))) = \sin(x)$. Once written, use it to tabulate $\text{cin}(x)$ for $x = 0.0, 0.2, 0.4, \dots, 1.0$

...

Still searching for a better one...

Albert Chan Wrote: →

(03-23-2019 06:03 PM)

$\text{cin}(x)$ should be an odd function, shape like $\sin(x)$, with value between $\sin(x)$ and x

For domain $[-\pi/2, \pi/2]$, **$\text{cin}(x) \approx (1 + x^2/9) \sin(x)$**

Example:

$\sin(\frac{1}{2}) = 0.4794255386$
 $\text{cin}(\text{cin}(\text{cin}(\frac{1}{2}))) = 0.4791650974$, relative error $\sim -0.05\%$

$\sin(\text{cin}(\frac{1}{2})) \approx 0.473044$
 $\text{cin}(\sin(\frac{1}{2})) \approx 0.473050$

Since $\text{cin}(x)$ is an odd function, its Taylor expansion can be written as $\text{cin}(x) = x + a \cdot x^3 + b \cdot x^5 + \dots$

So my approach was to build the Taylor expansion of $\text{cin}(\text{cin}(\text{cin}(x)))$ and identify it to the well known sinus expansion $\sin(x) = x - x^3/3! + x^5/5! - \dots$

The x^3 term is easy to calculate and is just $-(1/3!)/3 = -1/18$, in agreement with Albert's approximation $(-1/3! + 1/9)$. The best I could calculate (by hand) was the x^5 term. Then I search for the x^7 term by try-and-error to minimize the error.

Here is my best approximation and results showing $\text{cin}(\text{cin}(\text{cin}(x)))$ and $\sin(x)$:

```
10 ! SSMC24
30 A=-1/18 @ B=-7/1080 @ C=-.0015
40 DEF FNC(X)=X+A*X^3+B*X^5+C*X^7
50 FOR X=.1 TO 1 STEP .1
60 PRINT X;FNC(FNC(FNC(X)));SIN(X)
70 NEXT X
```

> RUN

```
.1 9.98334166706E-2 9.98334166468E-2
.2 .198669334313 .198669330795
.3 .295520279883 .295520206661
.4 .38941902196 .389418342309
.5 .479429531682 .479425538604
.6 .564659801456 .564642473395
.7 .644278060315 .644217687238
.8 .717533814261 .7173560909
.9 .783784102052 .783326909627
1 .842523170608 .841470984808
```

Another approach?

J-F



03-25-2019, 11:19 AM (This post was last modified: 03-27-2019 11:40 AM by Oulan.)

Post: #10

Oulan

Member

Posts: 50

Joined: Dec 2013

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Extending the approach of JF, you can use the following HP prime program

```
-----
#pragma mode( separator(., 😊 integer(h32) )
#cas
reduc(l,n):= BEGIN
LOCAL m;
m:=MAKELIST(0,z,0,n);
FOR Z FROM 0 TO n DO
m[n+1-Z]:=[SIZE(l)-Z];
END;
RETURN m;
END;
fff():= BEGIN
LOCAL l,m,n,z,p,q,r;
LOCAL a15,a13,a11,a9,a7,a5,a3;
PURGE(a15,a13,a11,a9,a7,a5,a3);
z:={a15,0,a13,0,a11,0,a9,0,a7,0,a5,0,a3,0,1,0};
n:=15;
p:=poly2symb(z,x);
r:=poly2symb(z,y);
FOR Y FROM 1 TO 2 DO
q:=(p|x=r);
l:=symb2poly(q,y);
l:=reduc(l,n);
p:=poly2symb(l,x);
END;
m:=MAKELIST(0,Z,0,n);
FOR Z FROM 0 TO n DO
IF (Z MOD 2) = 1 THEN
m[n+1-Z]:=(-1)^FLOOR(Z/2)/(Z!);
END;
END;
PRINT(l);
PRINT(m);
a3:=eval(solve(l[n-2]=m[n-2],a3)[1]);
// was a3:=solve(l[n-2]=m[n-2],a3);
a5:=eval(solve(l[n-4]=m[n-4],a5)[1]);
a7:=eval(solve(l[n-6]=m[n-6],a7)[1]);
a9:=eval(solve(l[n-8]=m[n-8],a9)[1]);
a11:=eval(solve(l[n-10]=m[n-10],a11)[1]);
```

```

a13:=eval(solve(l[n-12]=m[n-12],a13)[1]);
a15:=eval(solve(l[n-14]=m[n-14],a15)[1]);
RETURN [a3,a5,a7,a9,a11,a13,a15];
END;
#end
-----

```

to compute the coefficients of the taylor serie of CIN(x).
Use it in CAS mode : " k:=fff() 'enter' ", then " k 'enter' "

Be careful this program will take some times on a real Prime.

But the converging is very slow, coefficient up to x^{15} follows but give only 10^{-6} error near 1

```

-1/18 -7/1080 -643/408240 -13583/29393280 -29957/215550720 -24277937/648499737600
-6382646731/953294614272000

```

Btw can someone explain the warning displayed when solving for the coefficients ?

"Warning, ^ is ambiguous on non square matrices. Use .^ to apply ^ element by element."

I don't see any matrices solving here ... ok I saw the problem see listing. Sometimes list of list are not displayed with all brackets.

Anyway, there should be a better approach to solve this nice challenge

EDIT new version of program, avoid computing useless power

```

#pragma mode( separator(., 😊 integer(h32) )
#cas
mult(a,b):= BEGIN
LOCAL n,p,j,k;
n:=SIZE(a);
p:=MAKELIST(0,j,1,n);
FOR j FROM 1 TO n DO
FOR k FROM 1 TO n+1-j DO
p[j+k-1]+=a[j]*b[k]; // test removed Thanks Albert
END;
END;
RETURN simplify(p);
END;

fff2(n):= BEGIN
LOCAL cin,ccin,si;
LOCAL p,q,xn,s;
LOCAL lvar,lexpr;
LOCAL a3,a5,a7,a9,a11,a13,a15,a17;
LOCAL a19,a21,a23,a25,a27,a29,a31,a33;
LOCAL a35,a37,a39,a41,a43,a45,a47,a49;
LOCAL vars;
PURGE(a3,a5,a7,a9,a11,a13,a15,a17);
PURGE(a19,a21,a23,a25,a27,a29,a31,a33);
PURGE(a35,a37,a39,a41,a43,a45,a47,a49);
PURGE(x,y);
vars:={1,a3,a5,a7,a9,a11,a13,a15,a17,a19,a21,a23,a25,a27,a29,a31,a33,a35,a37,a39,a41,a43,a45,a47,a49};
cin:=MAKELIST(IFTE(p MOD 2,vars[(p+1)/2],0),p,0,n);
ccin:=cin;
FOR q FROM 1 TO 2 DO
s:=MAKELIST(0,p,0,n);
s[1]:=ccin[1];
xn:=cin;
FOR p FROM 2 TO n+1 DO
s:=s+ccin[p]*xn;
IF p<=n THEN xn:=mult(xn,cin);END;
END;
ccin:=s;
END;
si:=MAKELIST(IFTE(p MOD 2,((-1)^FLOOR(p/2))/(p!),0),p,0,n);
lvar:=MAKELIST(cin[p],p,4,n+1,2);
lexpr:=MAKELIST(ccin[p]=si[p],p,4,n+1,2);
s:=solve(lexpr,lvar);
RETURN s[1];
END;

```

#end

use with $ff2(2*n+1)$ n from 2 to 24 ($fff2(29)$ start to be long on a real G2), $fff2(49)$ take few seconds on a virtual one.

03-25-2019, 06:27 PM (This post was last modified: 03-25-2019 06:28 PM by Oulan.)

Post: #11

Oulan

Member

Posts: 50

Joined: Dec 2013

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Perhaps applying some series acceleration techniques such as Romberg-Richardson or Aitken could help ...

P.S. at least try to compute on an HP hand-held device those Taylor coefficient 😊



03-25-2019, 11:01 PM

Post: #12

**Gerson W. Barbosa**

Senior Member

Posts: 1,155

Joined: Dec 2013

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

hp 33s

```

C0001 LBL D
C0002 ENTER
C0003 ENTER
C0004 2
C0005 /
C0006 COS
C0007 SQRT
C0008 *
C0009 RTN

```

0.2 XEQ C XEQ C XEQ C -> 1.985105083E-01

0.5 XEQ C XEQ C XEQ C -> 4.7756191243E-1

1.0 XEQ C XEQ C XEQ C -> 8.4122242185E-1

At least this is a tiny program...



03-25-2019, 11:53 PM

Post: #13

Albert Chan

Senior Member

Posts: 627

Joined: Jul 2018

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"Calculate $\text{cin}(x)$ taylor coefficients with XCas. Only need adjust vars for more terms.

vars := [a3, a5, a7, a9, a11, a13, a15];

n := 2*len(vars) + 1;

y := x + sum(vars(k) * x^(2*k+1), k, 1, (n-1)/2);

cin(x0) := subst(y, x=x0);

solve(coeff(taylor(cin(taylor(cin(cin(x))), x, n)) - sin(x), x, n, polynom), x) = 0, vars)

→ [-1/18, -7/1080, -643/408240, -13583/29393280 ...]



03-28-2019, 02:38 AM

Post: #14

**Valentin Albillo**

Senior Member

Posts: 376

Joined: Feb 2015

Warning Level: 0%

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Hi, all:

First of all, thanks to all 5 of you who contributed to this thread so far, namely **Albert Chan, Paul Dale, J-F Garnier, Oulan** and **Gerson W. Barbosa**, your efforts and interest are much appreciated (and also thanks for not using *CODE* sections as I requested).

A week has elapsed and now I'll post my original solutions and comments to the different tiers discussed, one at a time, beginning with **Tier 1**:

Tier 1 - The Challenge:

We'll call **Homage Number** to any 10-digit positive integers which are multiples of **271**, divisible by **41** and further their digits are all distinct. Write a program that takes no inputs but simply finds out and outputs just **how many Homage Numbers there are**.

My original solution:

This tier was expressly created to be really **easy** so anyone interested could write code to solve it without much trouble. Matter of fact, as **Albert Chan** realized, it can be solved by hand with just a little thinking. The key is to realize that 41 and 271 are coprime, so every *Homage* number should be divisible by both and thus by their product, which is $41 * 271 = 11,111$.

Also, being a 10-digit number and having all its digits distinct means that its digits are 0, 1, 2, 3, ..., 9 in some order and thus their sum is $1 + 2 + 3 + \dots + 9 = 45$, which is divisible by **9** so each *Homage* number has to be divisible by 9 too. As 9 is coprime to 41 and 271, each *Homage* number N must be divisible by their product, i.e., by $41 * 271 * 9 = 99,999$.

Now let's split N into two 5-digit parts, **A, B**, like this: $N = 100,000*A + B$, which must be a multiple of 99,999, so subtracting $99,999*A$ from it the resulting value: $100,000*A + B - 99,999*A = A + B$ must be a multiple of 99,999 too and, as both A and B are 5-digit long, i.e., less than 100,000, that multiple must be 99,999 itself. Now, considering their individual digits we have:

$$A + B = abcde + uvxyz = 99999$$

and thus all 5 pairs of digits must comply with $a + u = b + v = \dots = e + z = 9$.

Now, there are **5!** permutations of the 5 pairs, so **120** permutations in all, but each of the 5 pairs has **2** possible orderings, say (a,u) and (u,a) , so $2^5 = 32$ variations for each of the 120 permutations and thus there are $120 * 32 = 3,840$ potential *Homage* numbers in all.

However, only 9 out of 10 begin with a non-zero digit (numbers beginning with a 0 aren't 10-digits numbers) and so finally there are $3,840 * 9/10 = 3,456$ **Homage Numbers**.

What if we can't or won't engage on such math reasoning ? Well, that's where our trusty **HP calc** will take away all the drudgery and work out the solution by itself, doing all the work for us in mere seconds and saving our neurons for better endeavours. In my case, this little *2-liner* for the **HP-71B** (fits in just 1 line too) will scan the whole range at steps of 99,999, increasing the count each time the corresponding number happens to have all its 10 digits different:

```
1 DESTROY ALL @ C=0 @ D=99999 @ FOR N=D*CEIL(10^9/D) TO 10^10-1 STEP D
2 C=C+NOT SPAN("0123456789",STR$(N)) @ NEXT N @ DISP C

>RUN
3456
```

so there are **3,456 Homage Numbers** in all.

That's it. *Affordable*, as promised. In the next days I'll post my solutions for the subsequent tiers.

V.



03-28-2019, 01:10 PM

Post: #15

Werner
Senior Member

Posts: 349
Joined: Dec 2013

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Valentin Albillo Wrote: → (03-28-2019 02:38 AM)

Hi, all:

A week has elapsed and now I'll post my original solutions and comments to the different tiers discussed, one at a

time, beginning with **Tier 1**:

[...]

In the next days I'll post my solutions for the subsequent tiers.

Does that mean I'll have to wait 4 more days for the solution of tier 5? Dang.

Werner

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03-28-2019, 02:51 PM (This post was last modified: 03-28-2019 03:07 PM by Albert Chan.)

Post: #16

Albert Chan 

Senior Member

Posts: 627

Joined: Jul 2018

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Homage number 99999 divisibility trick can be used for other numbers.

Example, to do *both* mod 7 and mod 13 at the same time, for $X = 20\ 190\ 328$

$$7 * 13 = 91$$

$$100 \pmod{91} \equiv 9$$

$$1000 \pmod{91} \equiv 90 \equiv -1$$

$$X \pmod{91} \equiv 20 - 190 + 328 \equiv 158 \equiv 9 + 58 \equiv 67$$

$$X \pmod{7} \equiv 67 - 63 \equiv 4$$

$$X \pmod{13} \equiv 67 - 65 \equiv 2$$

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03-29-2019, 03:55 AM

Post: #17



Valentin Albillo 

Senior Member

Posts: 376

Joined: Feb 2015

Warning Level: 0%

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Hi, all:

Let's continue with my original solutions, today it's time for:

Tier 3 - The Challenge:

Those positive real numbers that are either powers or sums of distinct powers of an arbitrary real number P form an increasing sequence whose first term is 1 (i.e.: P^0). Write a program which accepts as input both P and an index k and returns the corresponding k^{th} term in the sequence.

Use your program to find the **1,000,000th** term and the **3,141,593th** term when $P = e$ as well as the **1,234,567th** term and the **2,718,282th** term when $P = \pi$.

My original solutions:

Though the concept used in this challenge will work for powers of any number $P \geq 2$, whether integer or real, I purposefully used a sequence of *real* numbers which were either powers or sums of distinct powers of e instead of powers of an *integer* to avoid giving as examples some integer sequences that people would immediately search for in *OEIS*.

For instance, my preliminary (not posted) examples were based on the integer sequences for $P = 3$ and $P = 61$, namely:

1 3 4 9 10 12 13 27 28 30 31 36 37 39 40 81 ...

1 61 62 3721 3722 **3782** 3783 226981 226982 227042 ...

but I decided to use instead $P = e$ and $P = \pi$, which generate sequences of reals not present in *OEIS*. That said, the key fact is that the elements which are either powers or sum of distinct powers of a base P naturally *map* to the elements which are either powers or sum of distinct powers of base 2 , which of course are all the integers $1, 2, 3, \dots$,

i.e. precisely the *indexes* for the elements in the base- P sequence. Thus we only need to find the *base-2* expression for a given *index* and then interpret that base- 2 expression as a number in base P , which we then convert to the usual base 10. For example:

- to find the 6^{th} element in the sequence for $P = 61$:

the index 6 in base $2 = 110_2 \rightarrow 110_{61} = 61^2 + 61^1 = 3721 + 61 = 3782$ in base 10

My original solution for the **HP-71B** is this *68-byte 1-liner*:

```
1 DEF FNE(N,K) @ M=0 @ P=1 @ REPEAT @ M=M+P*MOD(N,2) @ P=P*K @ N=N DIV 2 @ UNTIL NOT N @
FNE=M
```

and to compute the particular elements asked for in the challenge, simply:

```
>FNE(1000000,EXP(1))
```

```
278394444.173 { = e^6 + e^9 + e^14 + e^16 + e^17 + e^18 + e^19 }
```

```
>FNE(3141593,EXP(1))
```

```
1601007663.31
```

```
>FNE(1234567,PI)
```

```
9091632437.43 { = Pi^0 + Pi^1 + Pi^2 + Pi^7 + Pi^9 + Pi^10 + Pi^12 + Pi^14 + Pi^15 + Pi^17 + Pi^20
}
```

```
>FNE(2718282,PI)
```

```
30446503139.5
```

It's worth mentioning that for $P = 8$ and $P = 16$ there's an even *simpler* solution for the **HP-71B** right from the *command line*. For instance, to find the 123^{th} element in the sequence of powers or sum of distinct powers of 8 , simply execute this from the command line:

```
>BVAL(BSTR$(123,2),8)
```

```
299529
```

which of course agrees with the *1-liner*: `FNE(123,8) -> 299529.`

Also worth mentioning is the fact that my solution also works for $P < 2$, even for $P = 1$, $P = 0$ and $P < 0$ but then the resulting sequence is no longer in increasing order as is the case for $P \geq 2$. For instance:

$P = 2$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...	<i>increasing, Ok</i>
$P = 1.9$	1, 1.9, 2.9, 3.61, ..., <u>13.369</u> , <u>13.0321</u> , ...	<i>not increasing</i>
$P = Phi$	1, 1.6180, <u>2.6180</u> , <u>2.6180</u> , 3.6180, ...	<i>not increasing, repetitions</i>
$P = 1$	1, 1, 2, 1, 2, 2, 3, 1, 2, 2, 3, 2, ...	<i>ditto</i>
$P = 0$	1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, ...	<i>ditto</i>
$P = -1$	1, -1, 0, 1, 2, 0, 1, -1, 0, -2, -1, ...	<i>ditto</i>
$P = -2$	1, -2, -1, 4, 5, 2, 3, -8, -7, -10, ...	<i>ditto</i>

Last but not least, this is **my original solution** for the **HP-25**, a simple *24-step* affair:

```
01 STO 0      13 *
02 STO 1      14 RCL 1
03 STO/ 1     15 *
04 CLX       16 +
05 X<>Y      17 RCL 0
06 2         18 STO* 1
07 /         19 Rv
08 INT       20 X<>Y
09 X<>Y      21 X#0
10 LASTX     22 GTO 06
11 FRAC      23 X<>Y
12 2         24 GTO 00
```

For instance:

```
FIX 0
26, ENTER, 3, R/S -> 111
61, ENTER, 3, R/S -> 361
1000000, ENTER, 3, R/S -> 1726672221
```

So much for **Tier 3**, thanks a lot to **Paul Dale** for his interest in this particular tier and for taking the time to create a nice 24-step solution for the **HP-25** as well. In the next days I'll post my solutions for the subsequent tiers.

V.



03-31-2019, 04:51 PM

Post: #18

Juan14
Junior Member

Posts: 24
Joined: Jan 2014

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

For the last challenge.

Let $\text{cin}(x) = \sin(x+A)$, we have:

$\sin(\sin(\sin(x+A)+A)+A) = \sin(x)$ or

$\sin(\sin(x+A)+A) + A = x$

For a given value of x , we can solve the last equation for A (A is a function of x).

Here is the program for the hp 50g:

```
<<
-> x
<<
'SIN(SIN(x+A)+A)+A-x'
'A' 1 ROOT x + SIN ->NUM
>>
```

```
x ____ cin(x)
0 ____ 0.
0.2 ____ 0.19954743606
0.4 ____ 0.39617257453
0.6 ____ 0.586447829132
0.8 ____ 0.761 006258889
1 ____ 0.906981195071
n/2 ____ .835085096711
-0.71 ____ -0.684625012855
2.019 ____ 4.81961624069E-3
```

There are so many ways to define the function $\text{cin}(x)$ in a similar way, that's why I didn't post my solution before, but here it is anyway :-)



03-31-2019, 05:26 PM (This post was last modified: 03-31-2019 08:10 PM by Albert Chan.)

Post: #19

Albert Chan
Senior Member

Posts: 627
Joined: Jul 2018

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Juan14 Wrote: →

(03-31-2019 04:51 PM)

For the last challenge.

Let $\text{cin}(x) = \sin(x+A)$, we have:

$\sin(\sin(\sin(x+A)+A)+A) = \sin(x)$ or

$\sin(\sin(x+A)+A) + A = x$

For a given value of x , we can solve the last equation for A (A is a function of x).

Nice try, but identity $\text{cin}(\text{cin}(\text{cin}(x))) = \sin(x)$ does not hold

Example, with above $\text{cin}(x)$ definition, and $x = 1.0$

$\sin(1) \approx 0.841471$
 $\text{cin}(\text{cin}(\text{cin}(1))) \approx \text{cin}(\text{cin}(0.906981)) \approx \text{cin}(0.844196) \approx 0.796542$

Even worse if $x=\pi/2$

$\sin(\pi/2) = 1$
 $\text{cin}(\text{cin}(\text{cin}(\pi/2))) \approx \text{cin}(\text{cin}(0.835085)) \approx \text{cin}(0.789340) \approx 0.752222$

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04-01-2019, 12:11 AM

Post: #20

Albert Chan 
Senior Member

Posts: 627
Joined: Jul 2018

RE: [VA] Short & Sweet Math Challenge #24: "2019 Spring Special 5-tier"

Valentin Albillo Wrote: →

(03-21-2019 04:08 AM)

Tier 4: Advanced

[MRM: **HP-11C** and up]

Consider the n -point dataset (x_i, y_i) where $x_i = 1, 2, 3, 4, 5, 6, \dots, n$ (the natural numbers) and $y_i = 2, 3, 5, 7, 11, 13, \dots, p_n$ (the prime numbers), and the $(n-1)^{st}$ degree polynomial fit to this dataset of the form:

$$P(x) = a_0 + a_1(x-1) + a_2(x-1)(x-2) + \dots + a_{n-1}(x-1)(x-2)(x-3)\dots(x-(n-1))$$

Getting sum of above coefficients can be done by doing forward difference of the primes:

2 3 5 7 11 13 17 19 23 29 ... ; primes

1 2 2 4 2 4 2 4 6 ... ; Δ

1 0 2 -2 2 -2 2 2 ... ; Δ^2

-1 2 -4 4 -4 4 0 ... ; Δ^3

3 -6 8 -8 8 -4 ... ; Δ^4

...

$$a_k = \Delta^k(0) / k!$$

$$\sum(a_k, k = 0 \text{ to } \text{Inf}) = 2/0! + 1/1! + 1/2! - 1/3! + 3/4! + \dots$$

Sum converge very fast:

2.0

3.0

3.5

3.33333 333333

3.45833 333333

3.38333 333333

3.41527 777778

3.40476 190476

3.40761 408730

3.40696 097884

3.40708 691578

3.40706 684905 ; 6 digits accuracy with 12 primes

3.40706 938140

3.40706 915834

3.40706 916344

3.40706 916625

3.40706 916552

3.40706 916564

3.40706 916563 ; 12 digits accuracy with 19 primes

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