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Sweet & Short Math Challenges #15: April 1st Spring Special

Message #1 Posted by ***Valentin Albillo*** *on 31 Mar 2006, 11:31 a.m.*

Hi all,

Where I am right now, April 1st has just begun, the very start of a brand-new month, so time for a new S&SMC, this time #15, an "April 1st Spring Special".

What's so "special" about it ? Well, for one, you'll have **5 individual, affordable mini-challenges** to test your smarts and programming abilities against instead of a single, harder challenge.

And secondly, this time you won't only be fighting to find your way through the challenges but you'll also experience the agony of a **deadline**, i.e., delivering the goods in a fixed, non-expandable amount of time. Namely this means you won't have a whole week before I post the solutions but just this very weekend.

Next Monday I'll post my original solutions and comments, so if you want to try your hand at it before I spill them out, you'd better hurry up this one time, no excuses. :-)

In all five cases, you must write a program for your chosen HP handheld (preferably, but other brands also acceptable as long as they're handheld but no PDAs!) to help you answer the challenge. Any language (RPN/RPL/BASIC/FORTH/C...) is acceptable as long as it actually runs in your handheld.

Now for the Spring Special:

Take 1

Given the coefficients **a**, **b**, **c**, **d** of the general 4th-degree polynomial equation:

$$x^4 + a*x^3 + b*x^2 + c*x + d = 0$$

you must write a program to compute the sum of the cubes of all its four roots (real and/or complex).

If in RPN/RPL, your program must assume the coefficients have previously been entered into the stack as follows:

```
T: d
Z: c
Y: b
X: a
```

and upon running, it must stop with the sum of the cubes of the four roots in the display. You should try to make your program as short and fast as possible.

For instance, given the equation:

$$x^4 - 2.006*x^3 - 4*x^2 - x + 4.44 = 0$$

your program must output **35.144216216**, which is the sum of the cubes of its four roots, namely:

$$x_1 = 0.848975027766$$
$$x_2 = 3.21371823523$$
$$x_3 = -1.0283466315 +0.754884455486*i$$
$$x_4 = -1.0283466315 -0.754884455486*i$$

As this is S&SMC #15, I'll post my 15-step RPN solution for the HP-15C (which will also run in the HP-34C and other RPN models as well), plus a couple of solutions for the HP-71B, for good measure.

Take 2:

Write a program to find all positive integers up to 10,000 such that they're simultaneously equal to the sum of the 1st powers of two consecutive integers and also to the positive difference of the 2nd powers of those same integers.

For instance, **153** is a solution because:

$$153 = 76^1 + 77^1$$
$$= 77^2 - 76^2$$

where 76 and 77 are consecutive integers.

Your program must find out and output *all* solutions, and be as short and fast as possible.

Take 3:

A cable is intended to tightly surround the Earth (assumed to be a perfect sphere) by the equator so that it touches the ground at all its points. Unfortunately, due to a slight manufacturing error, the cable, which should be some 40 million meters long, is actually a trifle 1 meter longer, so that regrettably, instead of fitting *tightly* as intended, there's some *slack*.

To fix the problem, a man sits comfortably on a small cushion at ground level, picks the cable between his fingers, and raises his arm vertically upwards from the ground till the cable is *perfectly tight* again, with no slack whatsoever.

Assuming that the Earth is a perfect sphere of radius exactly equal to 6,400 Km, and that the cable doesn't stretch at all and has negligible radius, what's the name of the man's brother-in law ?

Note:

> In order to help you find the answer, you are required to write a program to compute the required height to remove the slack for any given extra cable length (measurements in meters), and then run your program using the data for this particular case (1 extra meter).

Take 4:

The subject of *self-reproducing programs*, i.e., programs that upon running do produce their own source code as their output, is highly fascinating and has tons of actual instances in every programming language known to man.

For instance, in standard C you have:

```
main(){char *c="main(){char *c=%c%s%c;printf(c,34,c,34);}";printf(c,34,c,34);}
```

in RPL you have:

```
\<<
  \<< "\<<" SWAP +
      "DUP EVAL" + OBJ\->
  \>>
  DUP EVAL
\>>
```

in Java you have:

```
class S{public static void main(String[]a){String s="class
S{public static void main(String[]a){String s=;char ='';
System.out.println(s.substring(0,52)+c+s+c+s.substring
(52,61)+c+s.substring(61));}}";char c='"';System.out.println
(s.substring(0,52)+c+s+c+s.substring(52,61)+c+s.substring(61));}}
```

and in Pascal you have:

```
program s;const p='program s;const p=';a='a';aa=''';';aaa='a=''';aaaa
='''';aaaaa='begin       write(p,aaaa,p,aa,aaa,a,aa,a,aaa,aaaa,aa,aa,a,a,
aaa,aaa,aaaa,aa,a,a,a,aaa,aaaa,aaaa,aa,a,a,a,a,aaa,aaaaa,aa,aaaaa)
end.';begin      write(p,aaaa,p,aa,aaa,a,aa,a,aaa,aaaa,aa,aa,a,a,aaa,aaa,
aaaa,aa,a,a,a,aaa,aaaa,aaaa,aa,a,a,a,a,aaa,aaaaa,aa,aaaaa)end.
```

Now you must try your hand at writing a self-reproducing program for the HP-71B such that upon running produces as output its own source code. You may use not only statements and functions in the System ROMs but also in external ROMs, such as the Math ROM, the FORTH/Assembler ROM, or even the JPC ROM, to name a few. You should try to optimize for program size, the shorter the better.

Take 5:

We're given a function f(x,y) *recursively* defined as follows:

```
f(0,y) = y+1
f(x,0) = f(x-1,1)
f(x,y) = f(x-1,f(x,y-1))
```

by applying this definition, it's straightforward to compute the function's exact value for various arguments, for instance we readily find the values:

```
f(0,4) = 5
f(1,2) = 4
f(2,3) = 9
f(3,1) = 13
```

you're now asked to write a program that computes and outputs the exact value of f(4,2).

I'll post my original 9-line solution for the HP-71B.

Caveats:

That's all, the usual caveats apply, mainly:

- Please refrain from posting just the solutions, actual code is *mandatory*. Googling solutions and posting them is pretty *lame* and only serves to spoil the challenge for others and makes blatantly public your unability to comply and your serious attitude problem and disrespect for rules.

That said, let's see your answers before I post my solutions next Monday. You have more than 48 hours, so hurry up, you can do it ! :-)

Best regards from V.

Post-Edited 3 Apr 2006 solely to correct improper formatting (extremely long lines).

Edited: 3 Apr 2006, 5:48 a.m. after one or more responses were posted

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

*Message #2 Posted by **Arnaud Amiel** on 31 Mar 2006, 1:15 p.m.,*
in response to message #1 by Valentin Albillo

After trying a little program to solve 2, I stopped and thought. Looks like it is April 1st...

My solutions a bit later.

Arnaud

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

*Message #3 Posted by **Marcus von Cube, Germany** on 2 Apr 2006, 11:39 a.m.,*
in response to message #2 by Arnaud Amiel

> Quote:
>
> After trying a little program to solve 2, I stopped and thought.

So did I, but here is my 42S/41C program anyway:

```
LBL "SSMC152"
STO 00     ' Starting number in X
LBL 00
RCL 00
ENTER
ENTER
ENTER
1
STO+ 00
+
X<>Y
X^2
+/-
RCL 00
X^2
+
```

```
Rv
+
R^
X=Y?
VIEW ST X
GTO 00     ' Stopping at an upper limit is up to the user
END
```

Marcus

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #4 Posted by ***Bram*** *on 31 Mar 2006, 2:35 p.m.,*
in response to message #1 by Valentin Albillo

A program you ask, a program you get:

solution to nr. 2:

```
10000
ENT^
1
LBL 1
R/S (or PSE)
2
+
x<=y?
GTO 1
R/S
```

I'm beginning to understand why you choose this day :-)

groeten,
Bram

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #5 Posted by ***Kiyoshi Akima*** *on 31 Mar 2006, 4:21 p.m.,*
in response to message #1 by Valentin Albillo

It appears Take 2 is the easiest of the bunch. Here's my HP 48 solution:

<< 1 9999 FOR i i 1 DISP 2 STEP >>

A second take on Take 2 doesn't work on the 48 because of RAM limitations---it may work on a 49:

<<'I' DUP 1 99999 2 SEQ >>

As for Take 4, I haven't touched a 71B in years, but I believe this should work:

10 PRINT

Or was that LIST? I'll have to go dig up my manuals...

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #6 Posted by ***Kiyoshi Akima*** *on 1 Apr 2006, 2:58 p.m.,*
in response to message #5 by Kiyoshi Akima

Duh! Not "Print", obviously.

10 LIST

or

10 PLIST

should work, assuming either of these commands are programmable. They are on many but not all BASIC dialects. Unfortunately I don't have a 71B or an emulator at hand so I can't try it out.

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #7 Posted by ***Marcus von Cube, Germany*** *on 2 Apr 2006, 11:41 a.m.,*
in response to message #6 by Kiyoshi Akima

```
10 LIST
```

works fine on my real 71B.

Marcus

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #8 Posted by ***Crawl*** *on 31 Mar 2006, 5:50 p.m.,*
in response to message #1 by Valentin Albillo

Well, the solution to problem 1 was already known to Newton (and I was aware of it before this challenge). But Newton didn't have a programmable calculator, so here's an RPL solution:

<< -> d c b a

<< '-a^3+3*a*b-3*c' EVAL >>

>>

I don't think it gets much shorter or faster than that.

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #9 Posted by ***Crawl*** *on 31 Mar 2006, 5:59 p.m.,*
in response to message #8 by Crawl

I guess these all might be April Fool's challenges?

For problem 2, the condition is that

x + (x + 1) = (x + 1)^2 - x^2

Expanding gives

2x + 1 = 2x + 1

which is always identically fulfilled. So, any number of the form 2x + 1 is a solution. Though we only allow integers..

Program:

```
<< 1 -> a
<< DO
a 1 DISP
a 2 + 'a' STO
UNTIL
```

```
a 9999 >
END
>>
>>
```

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #10 Posted by ***Crawl*** *on 31 Mar 2006, 11:12 p.m.,*
in response to message #9 by Crawl

Challenge 3

It's difficult to describe the nonprogramming side of the solution without a figure, but basically...

Having the rope pulled taut implies that it's tangent to the earth at the two points where it touches; ie., it's at 90 degrees to the earth's radius.

The part of the rope that is pulled tight is like a triangle. There is another triangle underneath it, made by the two radii. Call the angle by which the radii are separated 'y'.

Then it turns out that the horizontal distance of the base of either triangle (since they share that base) is

r * sin (y/2)

Then the length of the bit of wire that is pulled taut is

2 * r * tan (y/2)

and the total length of the wire is

r * (2 * pi - y + 2 * tan(y/2))

that has to equal

2 * pi * r + x

with x being the slack.

Simplifying that equation and solving for y is the first part of the program.

Then we have to find the height of the rope. The height of the first triangle is r * cos(y/2)

The height of the second triangle is

r * sin(y/2) * tan(y/2)

The sum of those, minus the radius of the earth, is the height of the wire.

Putting it together, here's the program:

```
<< 'y' PURGE -> x << '-y+2*tan(y/2)=x/6400000' EVAL 'y' x ROOT '6400000*(cos(y/2)+sin(y/2)*tan(y/2)-1)' EVAL >> >>
```

The answer, for x = 1 meter, is the surprisingly large 121.64 meters.

I don't know if I can guarantee I haven't made a mistake somewhere, but the first question is, is this reasonable? I think I can argue that it is.

If the person were to *pinch* the wire, so that all the slack would be in one little line above a point, then of course the height of that slack would be 0.5 meters. Much less than 121 meters. But as you release the "pinch", the area that's not touching the ground grows and grows, and the height of the lifted line can be much more than the original slack.

As for the "brother in law", I don't know. My first guesses would be that it would have something to do with Goliath, or Atlas, but, oh, well.

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #11 Posted by ***Crawl*** *on 1 Apr 2006, 12:45 a.m.,*
in response to message #10 by Crawl

Challenge 4

I don't have a 71B, so this shall remain unsolved by me.

Challenge 5

Sorry, I didn't do this one on the calculator; I did it by hand.

It's easy to verify that

f(0,y) = y + 1

f(1,y) = y + 2

f(2,y) = 2 * y + 3

f(3, y) = 8 * 2^y - 3

Then

f(4, 0) = f(3,1) = 13

f(4, 1) = f(3, f(4,0)) = f(3, 13) = 65533

f(4,2) = f(3, f(4,1)) = f(3, 65533) = 8 * 2 ^ 65533 - 3

Heh, now I have to find the *exact* value of that? That ain't happening. But this gives an approximate value in scientific notation (in Level 2 * 10 ^ Level 1 format)

<< 2 LOG 65536 * DUP FP DUP UNROT - >>

So I managed to fulfill the requirement of having a program after all.

The answer is

2.0035297704... * 10 ^ 19728

Edited: 1 Apr 2006, 12:52 a.m.

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

*Message #12 Posted by **Werner** on 1 Apr 2006, 12:04 p.m.,*
in response to message #11 by Crawl

Easy enough on a 49, were it not for the built-in limit of 9999 on integer powers..

So, all you have to to is write a 'IBIGPOW' program to compute a^b (a and b integers), up to the limits of the available memory:

```
@ IBIGPOW
@ In : a b
@ Out: a^b
\<< \-> a b
```

```
  \<< b 9999 IDIV2 a SWAP ^
    IF OVER
    THEN a 9999 ^ SWAP
      1 4 ROLL
      START OVER * NEXT
    END
    SWAP DROP
  \>>
\>>
```

Now, I know Valentin asked for results, but you don't seriously want me to post the full number, do you? But the 49 is perfectly capable of computing it. After all, it's only 19729 digits.

Cheers, Werner

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #13 Posted by ***Crawl*** *on 1 Apr 2006, 1:27 p.m.,*
in response to message #12 by Werner

Thanks! I really did learn something new. I didn't know the reason it refused to do it was because of an arbitrary (9999 limit on powers) restriction; I thought it might have been a memory restriction.

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #14 Posted by ***Arnaud Amiel*** *on 1 Apr 2006, 1:54 p.m.,*
in response to message #12 by Werner

Otherwise, I decided to try something else. I took emu49 and used the following program F with 2 and 4 on the stack.

Now, I will see what happens when I wake up tomorrow morning.

```
Program called F
<<
DUP2 ->STR " " + SWAP ->STR + 1 DISP
IF DUP 0 ==
THEN DROP 1 +
ELSE
   IF OVER 0 ==
   THEN SWAP DROP 1 SWAP 1 - F
   ELSE DUP UNROT SWAP 1 - SWAP F SWAP 1 - F
   END
```

```
END
>>
```

Now, don't try this on a real 49 if you want to keep your batteries.

Arnaud

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #15 Posted by ***Crawl*** *on 1 Apr 2006, 4:30 p.m.,*
in response to message #12 by Werner

Now that I know how to do it, I couldn't resist: This should be the *exact* answer. In fact, this reveals that the last couple of digits I reported in the approximate answer were wrong. I should have known that, because 65536 * log(2) only gives seven places after the decimal (because the rest are taken up by the digits to the right of the decimal), so I should have only expected 6 or 7 digits of accuracy in the manitessa.

I'm also posting this because of how impressed I am by the size of numbers this calculator can deal with.

2003529930406846464979072351560255750447825475569751419265016973710
8940595563114530895061308809333481010382343429072631818229493821188
1266886950636476154702916504187191635158796634721944293092798208430
9104855990570159318959639524863372367203002916969592156108764948889
2540908059114570376752085002066715637023661263597471448071117748158
8091413574272096719015183628256061809145885269982614142503012339110
8273603843767876449043205960379124490905707560314035076162562476031
8637931264847037437829549756137709816046144133086921181024859591523
8019533103029216280016056867010565164675056803874152946384224484529
2537361442533614373729088303794601274724958414864915930647252015155
6939226281806916507963810641322753072671439981585088112926289011342
3778270556742108007006528396332215507783121428855167555407334510721
3112427399562982719769150054883905223804357045848197956393157853510
0189920000241419637068135598404640394721940160695176901561197269823
3789001764151719005113346630689814021938348143542638730653955296969
1388024158161859561100640362119796101859534802787167200122604642492
3851113934004643516238675670787452594646709038865477434832178970127
6445552940909202195958575162297333357615955239488529757995402847194
3529913543763705986928913757153740001986394332464890052543106629669

1652434191746913896324765602894151997754777031380647813423095961909
6065459130089018888758808473362595606544488850144733570605881709016
2108499714529568344061979690565469813631162053579369791403236328496
2330464210661362002201757878518574091620504897117818204001872829399
4344618622432800983732376493181478984811945271300744022076568091037
6203999203492023906626264491909167985461515778839060397720759279378
8522412943010174580868622633692847258514030396155585643303854506886
5221311481363840838477826379045960718687672850976347127198889068047
8243230394718650525660978150729861141430305816927924971409161059417
1853522758875044775922183011587807019755357222414000195481020056617
7358978149953232520858975346354700778669040642901676380816174055040
5117670093673202804549339027992491867306539931640720492238474815280
6191669009338057321208163507076343516698696250209690231628593500718
7419057916124153689751480826190484794657173660100589247665544584083
8334790544144817684255327207315586349347605137419779525190365032198
0201087647383686825310251833775339088614261848003740080822381040764
6887847164755294532694766170042446106331123802113458869453220011656
4076327023074292426051582811070387018345324567635625951430032037432
7407808790562836634069650308442258559670392718694611585137933864756
9974856867007982396060439347885086164926030494506174341236582835214
4806726676841807083754862211408236579802961200027441324438432402331
2574035450193524287764308802328508558860899627744581646808578751158
0701474376386797695504999164399828435729041537814343884730348426190
3388841494031366139854257635577105335580206622185577060082551288893
3322264362819848386132395706761914096385338323743437588308592337222
8464428799624560547693242899843265267737837317328806321075321123868
0604674708428051166488709084770291208161104912555598322366244868556
6514026846412096949825905655192161881043412268389962830716548685255
3691485029953967550395493837185340590009618748947399288043249637316
5753803673586710175783994818471798498246948060532081996066183434012
4760966395197780214411997525467040806084993441782562850927265237098
9865153946219300460736450792621297591769829389236701517099209153156
7814439791248475706237804600009918293321306880570046591458387208088
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9267217612833084527393646924458289257138887783905630048248379983969

2029222215486145902373478222682521639957440801727144146179559226175
0838890200741699262383002822862492841826712434057514241885699942723
3160699871298688277182061721445314257494401506613946316919762918150
6579745526236191224848063890033669074365989226349564114665503062965
9601997206362026035219177767406687774635493753188995878662821254697
971020657472327213729181446666594218720034745089428309115351892711 1
4287108376159222380276605327823351661555149369375778466670145717971
9012271178127804502400263847587883393968179629506907988171216906869
2953824852983002347606845411417813911064856023654975422749723100761
5131870024053910510913817843721791422528587432098524957878034683703
3378184214440171386881242499844186181292711985333153825673218704215
3063119774853521467095533462633661086466733229240987984925669110951
6143618601548909740241913509623043612196128165950518666022030715613
6847323646608689050142639139065150639081993788523183650598972991254
0447944342516677429965981184923315155527288327402835268844240875281
1283289980625912673699546247341543333500147231430612750390307397135
2520693381738433229507010490618675394331307847980156551303847581556
8523621801041965025559618193498631591323303609646190599023611268119
6023441843363334594927631946101716652913823717182394299216272538461
7760656945422978770713831988170369645886898118632109769003557358846
2446483570629145305275710127887202796536447972402540544813274839179
4128826423835171949197209797145936887537198729130831738033911016128
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7219213128703558530796694271341639103388275431861364349010094319740
9047331014476299861725424423355612237435715825933382804986243892498
2227807159517627578471094751190334822414120251826887137281931042534
7819612844017647953150505711072297431456991522345164312184865757578
6528197564843508958384722923534559464521215831657751471298708225909
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4545567470314903886022024639948260501762431969305640666366626090207
0488874388989074981528654443818629173829010518208699363826618683039
1527326458128678280660133750009659336462514609172318031293034787742
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7005375421344858745868560472867510654233418938390991105864655951136

4606105515683854121745980180713316361257307961116834386376766730735
4583494789788316330129240800836356825939157113130978030516441716682
5183465736759341980849589479409832925000863897785634946932124734261
0306271374507728615692259662857385790553324064184901845132828463270
9269753830867308409142247659474439973348130810986399417379789657010
6870267341619671965915995885378348229882701256058423655895396903064
7496558414798131099715754204325639577607048510088157829140825077773
8559790129129407309462785944505859412273194812753225152324801503466
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Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #16 Posted by ***Arnaud Amiel*** *on 1 Apr 2006, 2:43 a.m.,*
in response to message #10 by Crawl

I also have been spending a couple of hours trying to find a suitable brother in law... Still searching.

Arnaud

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #17 Posted by ***Marcus von Cube, Germany*** *on 2 Apr 2006, 11:19 a.m.,*
in response to message #10 by Crawl

Take 3

A little complicated, isn't it?

```
C = 2pi*r
C+1m = 2pi*r+1m = 2pi*(r+x) = 2pi*r+2pi*x
=> 1m = 2*pi*x
=> x = 1m / 2pi
```

C: Circumphere
r: Radius of the Earth (or of a tin can, doesn't matter at all)
x: value to increase the radius to get the additional length.
Lifting the wire on one end only needs a value of 2x.

Program in RPN:

```
PI
/
R/S
'Hulk Hogan'
AVIEW
END
```

Marcus

Edited: 2 Apr 2006, 11:24 a.m.

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #18 Posted by ***Marcus von Cube, Germany*** *on 2 Apr 2006, 11:44 a.m.,*
in response to message #8 by Crawl

Quote:

'-a^3+3*a*b-3*c'

It's interesting that the solution doesn't depend on d.

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #19 Posted by ***Crawl*** *on 2 Apr 2006, 3:01 p.m.,*
in response to message #18 by Marcus von Cube, Germany

It's not difficult to see (even without doing much algebra) why the sum of CUBES wouldn't depend on D, though.

Each coefficient of a polynomial in x^n + a * x^(n-1) + ... + d form (with leading coefficient of 1) is a symmetric polynomial of the roots.

You can see that by factoring the polynimal ((x-x1)*(x-x2)*... etc.)

So, for the quartic case, if the roots are x1, x2, x3, x4...

a = -(x1 + x2 + x3 + x4)

and

d = x1 * x2 * x3 * x4

-a^3, by the multinomial theorem, contains a sum of the cubes. It also has some cross terms, which the next few terms in the answer get rid of. But a^3 ONLY has products of three terms taken at a time (again, by the multinomial theorem). So there's no need for d, which has 4 terms taken at a time.

If the problem was to find the sum of the FOURTH powers of the roots, then d would be needed.

My Take on Take 3

Message #20 Posted by ***Michael F. Coyle*** *on 31 Mar 2006, 10:57 p.m.,*
in response to message #1 by Valentin Albillo

This is a variation of the old "how far is the horizon" problem.

Anyway, I get the formula:

'(R+r)*SIN(ACOS(R/(R+r)))-R*ACOS(R/(R+r))-S/2'

= 0

where R=6.4e6 meters, S, the slack = 1 meter, and we are solving for r, height above ground.

I didn't bother to write a program, but just keyed it in and solved for r; I got 121.645 meters (399 feet). I used my trusty 48GX and the Solver.

I don't know the brother-in-law's name but maybe our person was a princess sitting on a 40-story pile of mattresses.

- Michael

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #21 Posted by ***Marcus von Cube, Germany*** *on 3 Apr 2006, 4:57 a.m.,*
in response to message #1 by Valentin Albillo

Hi Valentin,

Take 5 is a battery drainer. I've written a small C program for my PB-2000C, recursive style:

```
/* SSMC15-5 */
main()
{
    int x,y;
    printf("x=");
    scanf("%d",&x);
    printf("y=");
    scanf("%d",&y);
    printf("f(%d,%d)=%d    \n",
           x,y,f(x,y));
}


int f(x,y)
int x,y;
{
    gotoxy(0,0);printf("%5d %5d\n",x,y);
    return x==0 ? y+1
         : y==0 ? f(x-1,1)
         : f(x-1,f(x,y-1));
}
```

I tried it for the solutions you gave:

```
f(0,4) = 5
f(1,2) = 4
f(2,3) = 9
f(3,1) = 13
```

And the results were as expected. But entering x=4 and y=2 at the prompts seems to be an almost endless exercise, so no solution this time.

Marcus

Edited: 3 Apr 2006, 4:58 a.m.

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #22 Posted by ***Klaus*** *on 3 Apr 2006, 5:12 a.m.,*
in response to message #21 by Marcus von Cube, Germany

Hi Marcus,

this reminds me of the infamous Ackermann-function, which can be used as a benchmark for procedure calls.

Greetings, Klaus

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

Message #23 Posted by ***Marcus von Cube, Germany*** *on 3 Apr 2006, 5:46 a.m.,*
in response to message #22 by Klaus

Hi Klaus,

you're perfectly right, it is the Ackermann-Péter function (See http://de.wikipedia.org/wiki/Ackermannfunktion or http://en.wikipedia.org/wiki/Ackermann_function).

The english article lists the results. Valentin is asking for
f(4,2) = 2^{65536} - 3.

I don't see a chance to calculate it with my simple C program because arbitrary precision would be needed.

Marcus

Edited: 3 Apr 2006, 6:08 a.m.

Re: Sweet & Short Math Challenges #15: April 1st Spring Special

*Message #24 Posted by **Arnaud Amiel** on 3 Apr 2006, 7:48 a.m.,*
in response to message #23 by Marcus von Cube, Germany

I posted an RPL version a bit higher which can work in arbitrary precision. I tried to run it on EMU49 see what was the first error I would get. I had to stop before getting any error as my laptop was getting REALLY hot. If someone has a very cool machine it would be good to run it without the first line see what happens...

Arnaud

S&SM#15: My Original Solutions and Comments

*Message #25 Posted by **Valentin Albillo** on 3 Apr 2006, 7:45 a.m.,*
in response to message #1 by Valentin Albillo

Hi all,

Thank you very much to all of you who were interested in this Spring Special challenge and managed to find the time to try and solve its various parts despite the very short deadline, I certainly hope you enjoyed it. Mostly, you did unexpectedly well, essentially solving all of them five, and I must confess it never ceases to amaze me, the extreme ingenuity displayed in both actual solutions and close calls.

These are my original solutions, plus comments:

Take 1:

Though it might seem at first that you'd need to actually compute all four real and/or complex roots in order to be able to compute the sum of their cubes, actually this isn't so, because the asked value is a symmetric function of the roots (i.e.: it remains the same for every permutation of the roots) and so can be computed directly by using *symmetric polynomials*, a technique developed by Newton and Girard.

For this particular Take, the asked sum S as a function of the coefficients is simply:

$$S = -a^3 + 3*a*b - 3*c$$

For example, if we have the equation:

```
x^4 + x^3 + 2*x^2 + x + 1 = 0
```

the coefficients are:

```
a = 1, b = 2, c = 1, d = 1
```

and the asked sum is:

```
S = -1^3 + 3*1*2 - 3*1 = 2
```

Let's check it. The roots are:

```
x1 = i
x2 = -i
x3 = -1/2 + Sqrt(3)/2*i
x4 = -1/2 - Sqrt(3)/2*i
```

and the sum of their cubes comes indeed to 2. This is my HP-15C solution to compute the required sum, assuming the coefficientes are previously placed in the stack like this:

```
T: d
Z: c
Y: b
X: a
```

the corresponding program listing is:

```
01 LBL A
02 CHS
03 X^2
04 LASTX
05  *
06 X<>Y
07 LASTX
08  *
09 GSB 0
10 X<>Y
11 LBL O
12  3
13  *
14  -
15 RTN
```

My **HP-71B** solution (32 bytes) is:

```
1 DESTROY ALL @ INPUT A,B,C @ DISP -A^3+3*A*B-3*C
```

By the way, the HP-71B is powerful enough that even if you didn't realize that the sum could be obtained by using symmetric polynomials and tried for brute force instead (computing the four real and/or complex roots and then adding up their cubes), you can still make do with this 2-liner:

```
1 DESTROY ALL @ OPTION BASE 0 @ DIM A(4) @ COMPLEX R(3),S @ MAT INPUT A
2 MAT R=PROOT(A) @ S=0 @ FOR I=0 TO 3 @ S=S+R(I)^3 @ NEXT I @ DISP S
```

For instance:

```
>RUN
      A(0)? 1,-2,-3,4,-5  [ENTER]  ->  (14,0)
```

i.e.: the sum of the cubes of the four roots is the complex value (14, 0), which is of course the real value **14**

Take 2:

This 2-liner (64 bytes) for the HP-71B will find all solutions quickly:

```
1 FOR I=1 TO 10000 STEP 2 @ A=(I-1)/2 @ B=A+1 @ IF B*B-A*A=I THEN DISP I;
2 NEXT I @ DISP "OK"
```

Let's run it:

```
>RUN
        1 3 5 7 9 11 [...] 9991 9993 9995 9997 9999
```

though from an eminently practical point of view, it can actually be simplified to the following 1-line, 30-byte program:

```
1 FOR I=1 TO 10000 STEP 2 @ DISP I; @ NEXT I @ DISP "OK"
```

which outputs just about the same. Both programs can have their upper limits increased if desired. For the given range, there are *5000* solutions in all

---

Take 3:

A little geometry will duly solve this. Let's call the Earth's radius **r**, the subtended angle between the point of maximum height and the point of tangency we'll call **x**, and let's say the length of the cable not in contact with the Earth is **2a**. The corresponding circular arc length is 2rx. Let **d** be the extra length added to the cable. Then 2a = 2rx + d. Hence a = rx + d/2, and so a/r = x + d/2r.

We also have, tan(x) = a/r. Therefore the equation to solve for the angle x is:

```
tan(x) = x + d/2r
```

Once we've got the angle **x** by solving it, we then have that the required height **h** over the ground is:

```
h = r(sec(x) - 1)
```

For our particular problem d = 1, r = 6,400,000, so we must solve:

```
tan(x) = x + d/2r = x + 1/12,800,000
```

which gives:

```
x = 0.00616549902401 radians
```

and from this, the height h is:

```
h = r(sec(x) - 1) = 6,400,000*(sec(0.00616549902401) - 1)
  = 121.644736 m
```

If the man can stretch his arm to a height in excess of 120 m (360 feet) while comfortably sitting at ground level, he must be Mr. Reed Richards, alias "Mr. Fantastic", so *his brother-in-law's name should be **Johnny Storm**, alias "The Human Torch"*, both members of the well-known Fantastic Four superhero team.

This is my *20-step program* for the **HP-15C** which will return the height for any given extra length. First store the constant 12,800,000 (2r) in R0 and set RAD mode:

```
01 LBL A
02 STO1
03  0
04 SOLVE B
05 COS
06 1/X
07  1
08  -
09 RCL*0
10  2
11  /
12 RTN
```

```
13 LBL B
14 TAN
15 LASTX
16  -
17 RCL 1
18 RCL/0
19  -
20 RTN
```

Running it, we can get for instance:

```
For d =   1 m:    1 GSB A  ->  121.6448 m  (h)
    d = 0.1 m:  0.1 GSB A  ->   26.2080 m
    d =  10 m:   10 GSB A  ->  564.6400 m
```

This is the 2-line, 65-byte **HP-71B** version:

```
1 INPUT "D=";D @ R=6400000
2 DISP "H=";R*(1/COS(FNROOT(0,1,TAN(FVAR)-FVAR-D/(2*R)))-1)
```

Let's run it:

```
>RUN
     D=1
     H= 121.644736
```

As you may see, even a mere 0.1 meter (some 4 inches) of extra length can raise the highest point to a height of more than 26 meters (nearly 80 feet). Mr Fantastic would still be needed.

Take 4:

This *1-line, 1-statement, 5-byte* **HP-71B** program:

```
1 LIST
```

is surely the simplest one which fulfills the conditions. Upon running it, lo and behold:

```
>RUN
        1 LIST
```

it *does* produce its own source code as output !

My, my, aren't we fortunate that HP-71B's BASIC is such a powerful programming language ?

Take 5:

This recursively defined function is no other than the infamous Ackerman function, which among many interesting properties most of which are theoretically important, it has the unexpected characteristic that its value grows extremely quickly, even for very small input, which means a direct naive attempt will most likely overflow your handheld. This difficulty is easily overcome by doing a little research to see what we're trying to compute. We'll proceed to compute f(4,2) step by step:

1. Let's find **f(1, y)**:

   f(1,y) = f(0, f(1, y-1)) = f(1, y-1)+1

   and f(1,0) = f(0,1) = 1+1 = 2

   so we have the *difference equation* (not *differential)*:

   f(1, y) = f(1, y-1)+1 with initial condition f(1,0) = 2

   whose exact solution is: **f(1,y) = y + 2**

2. Let's find **f(2, y)**:

   f(2, y) = f(1, f(2, y-1)) = f(2, y-1)+2

   and f(2,0) = f(1,1) = 1+2 = 3

   so we have the difference equation:

   f(2, y)= f(2, y-1)+2 with initial condition f(2,0) = 3

   whose exact solution is: **f(2,y) = 2*y + 3**

3. Let's find **f(3, y)**:

   f(3, y) = f(2, f(3, y-1)) = 2*f(3, y-1)+3 and f(3,0) = f(2,1) = 2*1+3 = 5

so we have the difference equation:

<u>f(3, y) = 2*f(3, y-1)+3</u> with initial condition <u>f(3,0)=5</u>

whose exact solution is: **f(3, y) = 2^y+3^-3**

4. Now for **f(4,2)**:

   f(4,2) = f(3, f(4, 1)) = f(3, f(3, f(4, 0)) = f(3, f(3, f(3, 1))) = f(3, f(3, 13)) = f(3, 65533) = $\mathbf{2^{65536} - 3}$

   and our program must simply compute this value, *exactly*. This is my 9-line program for the HP-71B which does the job:

```
1 DESTROY ALL @ OPTION BASE 0 @ N=65536 @ M=9 @ DIM A(1) @ K=10^M
2 A(0)=1 @ P=0 @ L=9 @ R=2^L @ FOR J=0 TO N-L STEP L @ MAT A=(R)*A @ GOSUB 8
3 IF NOT MOD(J,27) THEN DIM A(UBND(A,1)+1)
4 NEXT J @ R=MOD(N,L) @ IF R THEN MAT A=(2^R)*A @ GOSUB 8
5 A(0)=A(0)-3 @ J=0 @ PRINT @ PRINT STR$(A(P));
6 FOR I=P-1 TO 0 STEP -1 @ J=J+1 @ IF J=7 THEN PRINT @ J=0
7 A$=STR$(A(I)) @ PRINT RPT$("0",M-LEN(A$));A$; @ NEXT I @ END
8 FOR I=0 TO P @ A(I+1)=A(I+1)+A(I) DIV K @ A(I)=MOD(A(I),K) @ NEXT I
9 P=P+SGN(A(P+1)) @ RETURN
```

   You'll need 20 Kb RAM in your HP-71B or Emu71 to run it. After less than 25 minutes under Emu71 or just one week in a physical HP-71B, it outputs the exact, 19,729-digit result that follows.

   Best regards from V.

```
>RUN


200352993040684646497907235156025575044782547556975141926501697371089405955631145
308950613088093334810103823434290726318182294938211881266886950636476154702916504
187191635158796634721944293092798208430910485599057015931895963952486337236720300
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378295497561377098160461441330869211810248595915238019533103029216280016056867010
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890113423778270556742108007006528396332215507783121428855167555407334510721311242
739956298271976915005488390522380435704584819795639315785351001899200
```

02414196370681355984046403947219401606951769015611972698233789001764151719005113
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92766451163475810119387531965724212147603828477477457170457861041738574791130190
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29083858124784693157203232336018994694376477267218793768264318283826035645206994
68630216048874528424363593558622333506235945002890558581611275341783750455936126
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04447054295735383611136775231699727402929416742044232481138750756313190782721888
64053374694213842169928862940479635305150560788126366206497231257579019598873041
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