

## HP Forum Archive 15

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### Short & Sweet Math Challenges #8: Squares !

Message #1 Posted by [Valentin Albillo](#) on 20 Apr 2005, 10:09 a.m.

Hi all, long time no see:

After many months have elapsed since I posted the last S&SMC, here's a new one which certainly lives up to its title. Should you feel like it, grab your favorite HP calculator(s) (any programmable one will do), then try and solve this

### Challenge:

1. Find a **10-digit square** which consists of just **two different** numbers, from 1 to 9 (0's not allowed). For example, this would be a solution (featuring just the digits 1 and 3):

3113311313

if said number were a square which, alas, it isn't.

The solution is unique. You're expected to produce a **program** for your chosen calculator(s) that upon running will find a solution, also making sure there are no others. If you succeed, you may want to extend your program to also:

2. Find **all** 10-digit squares which consist of at most **three** different numbers, from 1 to 9 (0's not allowed). For example, this would be a solution, (featuring just the digits 3,5, and 7):

5733753373

if it were a square, which of course it isn't. There are 48 solutions in all.

As stated, you must produce a program for your HP calculator(s) that will compute and output the solution(s), the shorter and faster the better. Giving just the solutions or providing programs for machines other than HP calculators is very nice of you, thank you, but you simply forfeited the challenge, you loser ! ;-)

Next Friday I'll post the solutions and a 4-line program for a bare bones HP-71B which promptly finds out the solutions. Meanwhile be a sport and try your best. Remember, it's one thing to boast about your efficiency with RPN or RPL or whatever, and quite another to deliver the goods when challenged ... ;-)

Best regards from V.

*Edited: 22 Apr 2005, 7:20 a.m. after one or more responses were posted*

### My brain hurts

Message #2 Posted by [Gene](#) on 20 Apr 2005, 10:39 a.m.,  
in response to message #1 by [Valentin Albillo](#)

:-)

**C'mon, Gene ! It's quite simple :-)**

Message #3 Posted by **Valentin Albillo** on 20 Apr 2005, 10:48 a.m.,  
in response to message #2 by Gene

In your case I'll gladly make an exception: do post a program for a TI machine, should you feel more comfortable with it.

I know you're a keen defender of TI machines, just as I am of (some) SHARP ones, so this might be your opportunity to show that they're up to the task.

Best regards from V.

**Re: C'mon, Gene ! It's quite simple :-)**

Message #4 Posted by **Gene** on 20 Apr 2005, 10:51 a.m.,  
in response to message #3 by Valentin Albillo

Insults? :-)

I like the old TI machines, but I am an RPN / RPL man!

I can't imagine trying to write a program to do this on a 100-step SR-56. :-)

Of course, that's primarily because I haven't thought through the problem enough yet, I suppose!

Gene

**Re: C'mon, Gene ! It's quite simple :-)**

Message #5 Posted by **Valentin Albillo** on 20 Apr 2005, 12:24 p.m.,  
in response to message #4 by Gene

Hi again, Gene:

Gene posted: "*Insults? :-)*"

No, why on earth would you feel insulted ? Myself, I've stated a number of times that vintage SHARP machines are every bit as good if not better than contemporary HP ones and I wouldn't feel insulted in the least if someone were to offer me using a SHARP instead of an HP. But a TI ... that's a horse (donkey ?) of a different color ... come to think of it, you have a point, my apologies.

*"I like the old TI machines, but I am an RPN / RPL man!"*

I'm \*not\*.

*"I can't imagine trying to write a program to do this on a 100-step SR-56. :-)"*

I don't know about the SR-56, but I've got a quick'n'dirty 41CX version in some 60 steps ... Way to go.

Now stop ranting and do write a solution, man ! :-)

Best regards from V.

### SR-56 memories...

Message #6 Posted by **Marcus von Cube** on 23 Apr 2005, 11:17 a.m.,  
in response to message #4 by Gene

Quote:

I can't imagine trying to write a program to do this on a 100-step SR-56.

My flag (re)setting algorithm will certainly not work! The most complicated problem I've ever tackled on the 56 was the solving of 3 linear equations with 3 unknowns. Because there are only 10 registers on the machine, I had to enter the constants on the right hand side of the equations 3 times (once for each unknown.)

And of course, I had to type in the whole program every time I wanted to use it...

### ... and a link to the HP-20S

Message #7 Posted by **Karl Schneider** on 23 Apr 2005, 1:37 p.m.,  
in response to message #6 by Marcus von Cube

Marcus von Cube posted,

Quote:

The most complicated problem I've ever tackled on the [Texas Instruments TI-]56 was the solving of 3 linear equations with 3 unknowns. Because there are only 10 registers on the machine, I had to enter the constants on the right hand side of the equations 3 times (once for each unknown.)

The HP-20S (discontinued in 2002), also has only 10 storage registers. It includes a built-in loadable keystroke program for solving an exactly-determined 3x3 set of linear equations, using Cramer's Rule.

After the program "3by3" is loaded into program memory, the user enters the 9 matrix coefficients in registers 1-9, and the "upper" result constant into register 0. The remaining two constants are placed in the stack, separated by "INPUT". "XEQ A" calculates the solution.

The program can be used to solve an exactly-determined 2x2 system by entering the system as follows:

```
[a11 a12 0] [b1]
[a21 a22 0] [b2]
[ 0  0 1] [ 0]
```

A matrix can be inverted by solving for  $[1\ 0\ 0]^T$ ,  $[0\ 1\ 0]^T$ , and  $[0\ 0\ 1]^T$  in succession.

Determinants can be calculated using "XEQ D" after loading the "3by3" program.

Kind of a pain in many respects, but it's better than nothing, I suppose...

-- KS

**Re: Short & Sweet Math Challenges #7: Squares !**

Message #8 Posted by [Larry Corrado, USA \(WI\)](#) on 20 Apr 2005, 9:03 p.m.,  
in response to message #1 by Valentin Albillo

Valentin:

Rather than RPN or RPL, I used "whatever." Namely, Java on my Dell. Does that count? ;-)

I love my HP-25, which I used daily for 25 years. My HP-15C is beautiful, with its crisp, hi-contrast display. I love the glowing digits on my 34C... But when it comes to programming, I'll stick to Java or VB.NET.

I look forward to seeing your 4-line solution. Thanks for presenting the challenge.

Ciao, Larry

**Re: Short & Sweet Math Challenges #7: Squares !**

Message #9 Posted by [Arnaud Amiel](#) on 21 Apr 2005, 4:40 a.m.,  
in response to message #1 by Valentin Albillo

This is short but dirty and slow (just over 1 hour) on the 49+. It will not work in RPN though...

```
<< RCLF {} 'RESULTS' STO 10. STWS BIN
1. 8. FOR I #1b I 1. + 9.
  FOR J 1. 1022.
    FOR K DUPDUP NOT ->STR TAIL OBJ-> DROP I * SWAP ->STR TAIL
      OBJ-> DROP J * + DUP
      IF FP THEN DROP ELSE 'RESULTS' STO+ END
    #1b +
  NEXT
NEXT DROP
NEXT STOF
>>
```

And it tells me that 6661661161 is the only answer.

Now I have to find an other more efficient way to do it.

Arnaud

**Erratum**

Message #10 Posted by [Arnaud Amiel](#) on 21 Apr 2005, 5:53 a.m.,  
in response to message #9 by Arnaud Amiel

I forgot to type a SQRT (well a square root sign) just before the IF

Appart from that it works but so slowly... this is the kind of algorithm that would do well in assembly. Oh and I can't use this algorithm to go to step 2.

Arnaud

*Edited: 21 Apr 2005, 6:21 a.m.*

## Re: Short & Sweet Math Challenges #8: Squares !

Message #11 Posted by **Jeff O.** on 21 Apr 2005, 6:34 p.m.,

in response to message #1 by Valentin Albillo

Without looking at Larry's or Arnaud's responses (I swear!) , in case they gave the answers, I'll go out on a limb and present the following:

Quote:

1. Find a 10-digit square which consists of just two different numbers, from 1 to 9 (0's not allowed)...The solution is unique.

According to my hp-42S program, that number would be 6661661161, with a square root of 81619.

Quote:

2. Find all 10-digit squares which consist of at most three different numbers, from 1 to 9 (0's not allowed).

According to my hp-42S program, the 10 digit squares that consist of **just** three different numbers are as follows:

COUNT	NUMBER	SQUARE
1	33334	1111155556
2	33335	1111222225
3	33338	1111422244
4	33359	1112822881
5	33989	1155252121
6	34641	1199988881
7	34827	1212919929
8	34965	1222551225
9	36361	1322122321
10	37962	1441113444
11	39388	1551414544
12	39581	1566655561
13	40204	1616361616
14	43923	1929229929
15	44623	1991212129
16	47162	2224254244
17	50235	2523555225
18	54123	2929299129
19	54335	2952292225
20	57665	3325252225
21	58167	3383399889
22	62156	3863368336
23	66515	4424245225

24	66665	4444222225
25	66667	4444488889
26	66668	4444622224
27	67485	4554225225
28	68187	4649466969
29	68962	4755757444
30	70107	4914991449
31	70173	4924249929
32	72285	5225121225
33	72335	5232352225
34	72475	5252625625
35	76478	5848884484
36	78196	6114614416
37	78541	6168688681
38	78881	6222212161
39	79162	626622244
40	80408	6465446464
41	81346	6617171716
42	81404	6626611216
43	81813	6693366969
44	83666	6999999556
45	86478	7478444484
46	97773	9559559529
47	98336	9669968896

I was a bit distressed that I found only 47 of these, as apposed to the 48 that V. stated exist. However, upon reading his challenge, it does say *at most*. The group should therefore include those that have *just* 3 numbers listed above, plus the solution that has just 2 numbers, plus any that consist of just one number. A quick check of 11111111, 22222222, etc. with my calculator revealed that none of these are perfect squares. So the list of 47 above plus the unique solution to the first problem yields the 48 solutions to the second problem. (I hope.)

I'm quite sure that I used a very non-elegant, non-optimized, brute-force method. The program I wrote consisted of several basic parts.

1. create the 10 digit numbers by squaring all possible numbers that could create them. The maximum 10 digit number that meets the first criterion is 999999998, whose square root is 99999.99999. So no need to check anything above 99999. The minimum that satisfies the first criterion is 111111112, whose square root is 33333.33333, so no need to check anything lower than 33334.
2. Break the 10 digit number up into 10 single digits stored in registers 1 through 10 (checking for numbers with zeroes and throwing them out along the way).
3. sort those digits in ascending order in registers 1 through 10.
4. run through registers 1 through 10 and count the number of times that two adjacent digits are not equal to each other.
5. See if the above is either 2, to solve the first problem or 3 to solve the second. If so, print out the number and the square.
6. Move on to the next number.

The program I wrote based on the above methodology was an HP-42S program. However, I developed and ran it on Free42. On my PC, my first version solved each of the problems in about 14 seconds. I then optimized the sorting and checking routines somewhat, and it solved each problem in about 9 seconds. I entered that version into a real 42S. After about 3 hours and 45 minutes I stopped it to make sure it was working correctly. It had found the first 8 solutions, and was about 2.4% of the way through the sequence of numbers. Based on that, it would have taken nearly 6 days to complete. Obviously, I chose a quite inefficient solution scheme. A different approach would be required to make it realistically soluable by a real 42S.

*Edited: 25 Apr 2005, 7:16 a.m. after one or more responses were posted*

### Re: Short & Sweet Math Challenges #7: Squares !

Message #12 Posted by **Jonathan Puvis (New Zealand)** on 21 Apr 2005, 9:57 p.m.,  
in response to message #11 by Jeff O.

I came up with pretty much the same algorithm (before you posted your message). I initially wrote it in Python (attached below), but my User-RPL version on the 48G ran for more than an hour after which i stopped it as the low battery warning had come one.

I could write the digit counting function in Saturn assembly to speed it up, but i'm sure there is a better algorithm for counting the digits, i just can't think of it.

```
#!/usr/bin/python
def count_digits(x):
    x = int(x)
    digits = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
    while x:
        digits[x % 10] = 1
        x = x / 10
    if digits[0]:
        return 99
    return sum(digits)

for x in range(33333, 99999):
    sq = x * x
    if count_digits(sq) == 2:
        print '2:', x, sq

print

for x in range(33333, 99999):
    sq = x * x
    if count_digits(sq) <= 3:
        print '<=3', x, sq
```

### HP-32S solution

Message #13 Posted by [Eamonn](#) on 21 Apr 2005, 11:32 p.m.,  
in response to message #1 by Valentin Albillo

Valentin,

It's been a while since you posted a S&SMC! As ever, they are quite thought provoking.

I had to have a go at writing a program for the HP-32S to solve the first problem.

The basic idea was to generate all the possible ten-digit numbers that consist of two different numbers, take their square root and test to see if the fractional part is non-zero. There are  $C(9,8) * 2^{10} = 36864$  such numbers.

My first attempt would have taken about 10 hours to generate and test all such numbers (It generated and tested about 1 number per second). I won't bother posting it.

After thinking about it a bit, I took a different approach to generating the numbers that generates and tests all of them in about 27 minutes (almost 23 numbers per second).

This second approach uses almost all the 390 bytes of memory on the HP-32S. I'm guessing that it will also run on the HP-33S - not sure if it will be much faster or slower. Perhaps someone would be willing to try it out.

A different approach is needed for the second part of the problem - if my math is correct, there are almost 5 million possibilities for a 10-digit number made up of 3 unique digits.

Anyway, here is the program that I wrote. The program can be run by hitting XEQ V. It stops and displays each solution it finds. The user needs to hit the R/S key to continue after each solution. The number of solutions found is stored in the E register. It finds one solution for the first problem ( $6661661161 = 81619^2$ ).

Eamonn

```
LBL T ; Test the value that is in C
RCL C
SQRT
FP
X<>0?
RTN
1
STO+ E ; Count of how many solutions were found
RCL C
STO D ; Store most recent solution
R/S ; Stops to display latest solution - User needs to hit R/S to continue
RTN
[CHKSUM = A889]
```

```
LBL A
XEQ T
RCL J
STO+ C
XEQ T
RCL K
STO+ C
XEQ T
RCL J
STO+ C
XEQ T
RTN
[CHKSUM = 5BA1]
```

```
LBL B
XEQ A
RCL L
STO+ C
XEQ A
RCL M
STO+ C
XEQ A
RCL L
STO+ C
XEQ A
RTN
[CHKSUM = 3AA5]
```

```
LBL C
XEQ B
RCL N
STO+ C
XEQ B
RCL O
STO+ C
XEQ B
RCL N
STO+ C
XEQ B
```



```
RTN
[CHKSUM = D82D]

LBL D
XEQ C
RCL P
STO+ C
XEQ C
RCL Q
STO+ C
XEQ C
RCL P
STO+ C
XEQ C
RTN
[CHKSUM = 1A38]

LBL E
XEQ D
RCL R
STO+ C
XEQ D
RCL S
STO+ C
XEQ D
RCL R
STO+ C
XEQ D
RTN
[CHKSUM = EEA7]

LBL V ; <---- Program Entry Point
0
STO E ; Initialize number of answers
9
STO A ; First Digit
[CHKSUM = 8B6D]

LBL F
RCL A
1
-
x=0? ; If B will be zero, then we're done
RTN ; <---- Program Exit Point
STO B ; Second Digit
[CHKSUM = 60AB]

LBL G
11.019
STO i
1
STO J
[CHKSUM = 44BD]
```

```

LBL H ; Setting up constants used by the program
10
*
1
-
STO (i) ; K(i) = 10 * K(i-1) - 1
ISG i
GTO H
10.019
STO i
RCL A
RCL- B
[CHKSUM = AF37]

```

```

LBL I ; Loop to multiply constants by (A-B)
STO* (i)
ISG i
GTO I
1111111111
RCL *B
STO C ; C is the first value to test
XEQ E ; Call recursive generation of test values
DSE B ; Loop on second digit
GTO G
DSE A ; Loop on first digit
GTO F
[CHKSUM = 62F8]

```

### Re: HP-32S solution

Message #14 Posted by [Arnaud Amiel](#) on 24 Apr 2005, 3:14 a.m.,  
in response to message #13 by Eamonn

Quote:

\_\_\_\_\_

This second approach uses almost all the 390 bytes of memory on the HP-32S. I'm guessing that it will also run on the HP-33S - not sure if it will be much faster or slower. Perhaps someone would be willing to try it out.

\_\_\_\_\_

It takes about 19min on the 33s that is about 33% faster.

Arnaud

### Re: Short & Sweet Math Challenges #7: Squares !

Message #15 Posted by [Bram](#) on 22 Apr 2005, 7:03 a.m.,  
in response to message #1 by Valentin Albillo

Hi V,

1. Thanks for the challenge

2. Did you loose track? Think there already has been a #7

<http://www.hpmuseum.org/cgi-sys/cgiwrap/hpmuseum/archv014.cgi?read=59918>

3. My program is for the 32SII. After a short analysis it appeared that "only" the numbers between 31622 and 99999 are eligible for an answer. I check them all for the criteria and I break out the moment a criterion fails. That's the only speed up, so finding the answer 81619 takes several hours.

The program is for the 3 dupl. problem; if you remove the marked instructions, you get the program for the single solution problem.

```

LBL A ; CK=1BB7 012.5
31622
STO N
LBL B ; CK=35F8 024.5
1
STO+ N
RCL N
x2
XEQ C
RCL N
99999
x>=y?
GTO B
STOP

LBL C ; CK=BA5C 013.5
FS? 0
PSE
CF 1
CF 2
CF 3 ; <=
STO R
10 ; squared number consists of 10 digits
STO I
LBL J ; CK=C11A 021.0
RCL R ; go strip off a single digit
IP
10
/
STO R
FP
x=0?
RTN ; no zeros allowed, next number
FS? 1 ; digit already encountered?
GTO K ; yes
STO X ; no, store it for comparison with more digits
SF 1 ; digit now has encountered
GTO M
LBL K ; CK=CDA4 015.0
RCL X ; get first unique digit
x=y? ; equal to the one in question?
GTO M
x<>y
FS? 2 ; like before, but now for the second digit
GTO L

```

```

STO Y
SF 2
GTO M
LBL L ; CK=B38A 015.0
RCL Y
x=y?
GTO M
x<>y ; <=
FS? 3 ; <= like before, but now for the third digit
GTO N ; <=
STO Z ; <=
SF 3 ; <=
GTO M ; <=
LBL N ; CK=2764 007.5
RCL Z ; <=
x=y? ; <=
GTO M ; <=
RTN
LBL M ; CK=102A 010.5
DSE I
GTO J ; go to the next digit to examine
RCL N ; all digits examined, so now we have an answer
x2
STOP
RTN

```

total length: 119.5

```

; storage registers
; N number to examine
; R number2 being split into digits
; X Y (Z) the digit to occur
;
; labels
; A loop through all N values
; B inner loop
; C check max diff digits in square of number N
; JKL(N) place holders
; M increment to the next digit to examine
;
; flags
; 0 set => show progression
; 12(3) keep track of new occurrence of digit

```

### Thanks a lot, Bram !

Message #16 Posted by [Valentin Albillo](#) on 22 Apr 2005, 7:25 a.m.,  
in response to message #15 by Bram

Hi, Bram:

Bram posted:

"2. Did you loose track? Think there already has been a #7 <http://www.hpmuseum.org/cgi-sys/cgiwrap/hpmuseum/archv014.cgi?read=59918> "

Yes, I certainly did. Though I searched my files to see what the current number should be, it seems I didn't keep a record of that particular S&SMC (#7), so I thought that #6 was the very last to date.

I've duly corrected the numbering, so that it won't be kept in the MoHP Archives section with the wrong, duplicated number.

Thanks a lot, see my post with the solutions and comments within 12 hours.

Best regards from V.

### Re: Thanks a lot, Bram !

Message #17 Posted by **Bram** on 22 Apr 2005, 10:28 a.m.,  
in response to message #16 by Valentin Albillo

Anytime Valentin, and in the meantime I rewrote my program to a more versatile one based on indexing. The marked line defines the criterion for the number of digits. Change the 3 into 2 for the first problem. Changing the comparison in the next line is for adoption to exactly x different digits or a least y different digits a.s.o.

regards,  
Bram

```
LBL A ; CK=1BB7 012.5
31622
STO N
LBL B ; CK=35F8 024.5
1
STO+ N
RCL N
x2
XEQ C
RCL N
99999
x>y?
GTO B
STOP
```

```
LBL C ; CK=78F9 009.0
FS? 0
PSE
STO R
9
STO i
LBL I ; CK=DD00 010.5
0
STO (i)
DSE i
GTO I
10 ; squared number consists of 10 digits
STO J
LBL K ; CK=0B4E 027.0
```

```

RCL R ; go strip off a single digit
IP
10
/
STO R
FP
x=0?
RTN ; no zeros allowed, next number
10
*
STO i
1
STO+ (i)
DSE J
GTO K
10
STO i
LBL J ; CK=E621 022.5
RCL (i)
x#0?
1
STO+ J
DSE i
GTO J
3 ; <= max. number of different digits
RCL J
x>y?
RTN
RCL N
x2
STOP
RTN

```

total length: 106.0

```

; storage registers
; N number to examine
; R number2 being split into digits
; A-I&J for counting occurrences
;
; labels
; A loop through all N values
; B inner loop
; C check max diff digits in square of number N
; IJK place holders
;
; flags

```

### Re: Third version

Message #18 Posted by [Bram](#) on 26 Apr 2005, 4:44 p.m.,  
in response to message #17 by Bram

Hi Valentin, once more....

My second program was more versatile, but much slower than my first one. I tried to speed up my original program by squeezing the inner part to a minimum number of instructions. In only 15 lines (starting at ENTER) I test all criteria and you may be interested in how I use "conditional comparison" to get this done.

I hope things are clear enough by my comments in the program.

```

LBL A ; CK=1BB7 012.5
31622 ; smallest root of ten digits number minus 1
STO N
LBL B ; CK=35F8 024.5
1
STO+ N
RCL N
x2
XEQ C ; examine all ten digits squares
RCL N
99999
x>=y?
GTO B
STOP

LBL C ; CK=AB00 015.0
FS? 0 ; wish to see progression?
PSE ; yes, show square that's going to be examined
STO R
0
STO X ; clear x Y Z on behalf of the three different digits
STO Y
STO Z ; <=
10 ; squared number consists of ten digits
STO I
LBL J ; CK=9B2F 037.5
RCL R ; go strip off a single digit
IP
10
/
STO R
FP
x=0?
RTN ; no zeros allowed => next number
ENTER ; put the digit to examine into Y for comparisons
x<>X ; save it in X and get former value from X
x#0? ; if former value was 0, we have a new digit; goto M for next digit
x=y? ; if equal, we have a duplicate digit; goto M for next digit
GTO M
x<>X ; digit is a new, second one; restore X with original contents
; and get back the digit to examine into X
x<>Y ; same as before; check against 2nd distinct digit that has occurred
x#0?
x=y?
GTO M
x<>Y ; <= C
x<>Z ; <= L
x#0? ; <= E
x=y? ; <= A
GTO M ; <= R these lines when checking for just two different digits
RTN ; we get to this point when a fourth digit is reached => next number
LBL M ; CK=102A 010.5

```

```

DSE I
GTO J ; go to the next digit to examine
RCL N ; all digits examined, so now we have an answer
x2 ; proudly show the result
STOP ; and allow the user to see it
RTN ; continue with next number

```

total length: 100.0

```

; storage registers
; I loop counter for ten digits
; N number to examine
; R number2 being split into digits
; X Y (Z) the digit to occur
;
; labels
; A loop through all N values
; B inner loop
; C check max diff digits in square of number N
; J place holder
; M increment to the next digit to examine
;
; flags
; 0 set => show progression

```

regards,  
Bram

## Re: Short & Sweet Math Challenges #8: Squares !

Message #19 Posted by [Olivier De Smet](#) on 22 Apr 2005, 9:51 a.m.,  
in response to message #1 by Valentin Albillo

Here is a quick hack for an HP86B (with advance prog module)

```

10 l=0 @ o=3 @ t=TIME @ FOR n=33333 TO 99999 @ IF o=10 THEN o=0 @ GOTO 60
20 k=1 @ SFLAG "" @ m=n*n @ SFLAG m MOD 10 @ m=m\10 @ FOR j=2 TO 10 @ b=m MOD 10 @ IF b=0 THEN 60
30 IF FLAG (b) THEN 50
40 IF k=3 THEN 60 ELSE SFLAG b @ k=k+1
50 m=m\10 @ NEXT j @ l=l+1 @ DISP l,n,n*n
60 o=o+1 @ NEXT n @ DISP "Fini: ";TIME -t @ END

```

```

run
1          33334          1111155556
2          33335          1111222225
3          33338          1111422244
4          33359          1112822881
5          33989          1155252121
6          34641          1199988881

```



7	34827	1212919929
8	34965	1222551225
9	36361	1322122321
10	37962	1441113444
11	39388	1551414544
12	39581	1566655561
13	40204	1616361616
14	43923	1929229929
15	44623	1991212129
16	47162	2224254244
17	50235	2523555225
18	54123	2929299129
19	54335	2952292225
20	57665	3325252225
21	58167	3383399889
22	62156	3863368336
23	66515	4424245225
24	66665	4444222225
25	66667	4444488889
26	66668	4444622224
27	67485	4554225225
28	68187	4649466969
29	68962	4755757444
30	70107	4914991449
31	70173	4924249929
32	72285	5225121225
33	72335	5232352225
34	72475	5252625625
35	76478	5848884484
36	78196	6114614416
37	78541	6168688681
38	78881	6222212161
39	79162	6266622244
40	80408	6465446464
41	81346	6617171716
42	81404	6626611216
43	81619	6661661161
44	81813	6693366969
45	83666	6999999556
46	86478	7478444484
47	97773	9559559529
48	98336	9669968896

Fini: 108.019

In fact as 0 isn't allowed only integer from 33333 to 99999 are tested and not ending with 0.

The given program is for the second problem (which include the first). Need 1h50 on real hardware to find all solutions. Change line 40 with '40 IF k=2 ...' for the first problem.

Thanks again for the challenge.

Olivier

*Edited: 22 Apr 2005, 9:57 a.m.*

**Re: Short & Sweet Math Challenges #8: Squares !**

Message #20 Posted by [Olivier De Smet](#) on 22 Apr 2005, 11:16 a.m.,  
in response to message #19 by Olivier De Smet

A better solution for the first problem:

```
list
10 t=TIME
20 FOR i=0 TO 511 @ a=100000000+VAL (DTB$ (i)) @ b=VAL (DTB$ (511-i))
30 FOR j=1 TO 9 @ FOR k=1 TO 9 @ m=j*a+k*b @ IF FP (SQR (m))=0 THEN DISP SQR (m),m
40 NEXT k @ NEXT j @ NEXT i @ DISP "Fini: ";TIME -t @ END
517068

run
81619                6661661161
Fini: 19.118
```

Just 20 mins for finding the value, but need the I/O module.

Olivier

**Re: Short & Sweet Math Challenges #8: Squares !**

Message #21 Posted by [J-F Garnier](#) on 22 Apr 2005, 11:26 a.m.,  
in response to message #20 by Olivier De Smet

Hi Olivier,

I was just about to post a similar solution for the 71B. You can improve this solution by noticing that only numbers ending by 1, 4, 5, 6 or 9 can be a square number. But I didn't find how to solve the 3-digit problem with this approach.

J-F

**Re: Short & Sweet Math Challenges #8: Squares !**

Message #22 Posted by [Olivier De Smet](#) on 22 Apr 2005, 11:35 a.m.,  
in response to message #21 by J-F Garnier

Yes I know but searching to eliminate such non solution is too costly in time :)

A better one :

```
list
10 t=TIME
20 FOR i=0 TO 511 @ a=100000000+VAL (DTB$ (i)) @ b=VAL (DTB$ (511-i)) @ FOR j=1 TO 9 @ m=j*a @ FOR k=1 TO 9 @ m=m+b @ IF FP (SQR (m))=0 THEN DISP SQR (m),m
30 NEXT k @ NEXT j @ NEXT i @ DISP "Fini: ";TIME -t @ END
517065

run
81619                6661661161
Fini: 15.199
```

Only 16 mins :)

Olivier

### Re: Short & Sweet Math Challenges #8: Squares !

Message #23 Posted by **J-F Garnier** on 24 Apr 2005, 11:00 a.m.,  
in response to message #22 by Olivier De Smet

A slightly improved version of Olivier's solution, for the HP-71B and using the BSTR\$ function from the Math module:

```
10 T=TIME
15 OPTION BASE 1 @ DIM F(5)
16 DATA 1,4,5,6,9
17 READ F()
20 FOR I=0 TO 511 @ A=VAL(BSTR$(I,2))*10+1 @ B=VAL(BSTR$(511-I,2))*10
30 FOR J=1 TO 5 @ M=F(J)*A @ FOR K=1 TO 9 @ M=M+K*B @ IF FP(SQRT(M))=0 THEN DISP SQRT(M),M
40 NEXT K @ NEXT J @ NEXT I @ DISP "Fini: ";TIME-T @ END
      81619          6661661161
Fini: 45.77
```

Running on Emu71 using a 1.7GHz processor.

J-F

### Re: Short & Sweet Math Challenges #8: Squares !

Message #24 Posted by **Olivier De Smet** on 25 Apr 2005, 3:02 a.m.,  
in response to message #23 by J-F Garnier

Hi,

As I told you I don't need to optimize, my program need only 15.2 sec running on an HP86B emulator on a 1.8Ghz pentium M :)

But you're right it's faster like this :

```
list
10 t=TIME @ OPTION BASE 0 @ DIM f(4)
20 f(0)=1 @ f(1)=4 @ f(2)=5 @ f(3)=6 @ f(4)=9
30 FOR i=0 TO 511 @ a=VAL (DTB$ (i))*10+1 @ b=VAL (DTB$ (511-i))*10 @ FOR j=0 TO 4 @ m=a*f(j) @ FOR k=1 TO 9 @ m=m+b @ IF FP (SQR (m))=0 THEN DISP SQR (m),m
40 NEXT k @ NEXT j @ NEXT i @ DISP "Fini: ";TIME -t @ END
      516894
run
      81619          6661661161
Fini: 8.486
```

Only 8.846 sec :)

Olivier

*Edited: 25 Apr 2005, 6:02 a.m.*

**Re: Short & Sweet Math Challenges #8: Squares !**

Message #25 Posted by [Olivier De Smet](#) on 22 Apr 2005, 11:39 a.m.,  
in response to message #21 by J-F Garnier

For 3 digit as someone said it there is too much candidates, so the reverse approach should be better I think because you only have to test (99999-33333) candidates.

For 2 digits it's the cost of the test which is important as you have either 66666 or  $512*9*9$  candidates.

Olivier

**Hi, Jean-François !**

Message #26 Posted by [Valentin Albillo](#) on 22 Apr 2005, 11:46 a.m.,  
in response to message #21 by J-F Garnier

Hi, J-F:

Your wonderful [Emu71 HP-71B's emulator](#) runs my solution program for the HP-71B in under 40 seconds !

By the way, did you get to see my latest HP-71B articles published in Datafile ? All of the featured programs were created with the help of your Emu71, and it runs them like a charm. I do include footnotes recommending both your emulator and Hrastprogrammer's one for the 48/49, so that readers can know that they can try the programs and fully enjoy them themselves.

The ones featured in the latest Datafile issue are an extremely good test of your emulator's performance, as they make use not only of the emulated basic 71B itself, but also of the emulated Math and HP-IL ROMs as well, and it does certainly pass the taxing ordeal with flying colors, to say the least !

Best regards from V.

**Re: Hi, Valentin**

Message #27 Posted by [J-F Garnier](#) on 24 Apr 2005, 9:52 a.m.,  
in response to message #26 by Valentin Albillo

>> By the way, did you get to see my latest HP-71B articles published in Datafile ?

Unfortunately no, I (still) didn't subscribe to Datafile, shame on me ...

J-F

**Re: Short & Sweet Math Challenges #8: Squares !**

Message #28 Posted by [Marcus von Cube](#) on 22 Apr 2005, 1:36 p.m.,  
in response to message #1 by Valentin Albillo

This is a solution for the HP 42S and HP 41C. [Edited]

Usage:

Enter the number of allowed different digits (2 or 3) in X and XEQ "SQ10". After the 33333 is displayed you have to press R/S (You can correct the starting number, if you like.)

If you enter a number >9, XEQ "SQ10" will just run the comparison engine (LBL 00) and show "OK" if the criterion is met. You need to store the number of digits manually in R02 before you can use the program this way.

```

LBL "SQ10"   Main entry point
0
STO 09
Rv
9
X<>Y
X<=Y?
GTO 10
XEQ 00
RCL 00
FC? 00
RTN
"OK"
AVIEW
RTN

```

Now comes the checking engine. It works on flags 0-9 which are all set in the beginning and cleared indirectly for each corresponding digit. The reset operations are counted in R00. If the count reaches the limit in R02, the check has failed and flag 0 will be cleared. (This is done at no cost, if a "0" digit is found.) A cleared flag 0 stops the loop.

R09 counts the total number of calls for statistical purposes.

```

LBL 00
VIEW ST X
SF 00
SF 01
SF 02
SF 03
SF 04
SF 05
SF 06
SF 07
SF 08
SF 09
0
STO 00
1
STO+ 09
Rv           RDN for HP 41
Rv

LBL 01       digit loop
STO 01
10
MOD
FC?C IND ST X  this does the flag testing and clearing!
GTO 02       this digit is already counted
RCL 00       get count
RCL 02       max # of different digits allowed
X<=Y?

```

```

CF 00
FC? 00
RTN          too many different digits
1
STO+ 00     count
LBL 02      get next digit
RCL 01
10
/
IP          INT for HP 41
X!=0?      any digits left?
GTO 01
RTN

LBL 05      find smallest digit from flags
           works only after a passed check!
RCL 00
RCL 02
-
X=0?       maximum # of digits reached?
GTO 06     look for smallest
Rv
1          return 1, because less than the
RTN       allowed number of digits were present

LBL 06
1
+
FS? IND ST X
GTO 06
RTN

```

This is the controlling loop. It starts at 33333+1 but it does not try each and every number up to 99999. When the left most 4 digits do not pass the check, there is no need to try any number that would result in the same 4 digits. I simply construct a number with the next higher value and get the SQRT from it:

If  $\text{guess}^2 == p,\text{qrs},\text{tuv},\text{wxy}$ . then new guess =  $\text{SQRT}((p,\text{qrs}+X)*10e6 + Y11,111. )$

$p,\text{qrs}+X$  is determined by adding 1 in a loop and checking the resulting 4 digit number until the check passes. Y is the smallest digit found in  $p,\text{qrs}+X$  by the check routine.

A similar trick is used to skip guesses when the first 3 digits fail the check.

(The following is awful spaghetti code and I'm willing to streamline it when I have time.)

```

LBL 10
STO 02      store # of allowed digits
0
STO 05
STO 06      scratch registers for first 3 to 4 digits
33333      the loop starts with this initial guess
STOP       correct the initial guess at will
           (The loop starts with this number + 1!)

LBL 11

```

```
STO 03          store first guess - 1

LBL 12
1
STO+ 03        increment guess
999999        final value
RCL 03
X>Y?
GTO 20        no more numbers to try
X^2
STO 04        store result to check (for later)
1E6
/
IP           first 4 digits (INT for HP 41)
x<>Y
RCL 05        first 4 digits from last loop
X!=Y?        changed?
GTO 13        check and find best next guess
RCL 04        squared guess
XEQ 00        check it
FC? 00
GTO 12        failed, try next
CLD          OK, show result
RCL 03
RCL 04
BEEP
STOP
GTO 12        back to main loop

LBL 13        find the next guess quicker by checking
1            the first 3 or 4 digits only
-
STO 05

LBL 14
1
STO+ 05        new value for first 4 digits
RCL 06        first 3 digits (saved)
RCL 05        first 4
10
/
IP
X=Y?
GTO 19        the first 3 are the same as before

LBL 15
STO 06        new value for first 3
XEQ 00        check
FS? 00
GTO 16        first 3 digits are OK, must try first 4
RCL 06
1
+
GTO 15
```

```

LBL 16      new first 4 digits from first 3 digits
RCL 06
10
*
XEQ 05      find smallest digit from flags
+
STO 05

LBL 19      check the first 4 digits
RCL 05
XEQ 00      check
FC? 00
GTO 14      check failed, try next
RCL 05      compute new guess
10
x
XEQ 05      find smallest digit
+
1E5
x
11110
+
SQRT
IP
GTO 11      restart with new guess

LBL 20      finis
CLD
CLX
END

```

See next post for some statistics.

*Edited: 23 Apr 2005, 2:33 p.m. after one or more responses were posted*

### Some Statistics for the above

Message #29 Posted by [Marcus von Cube](#) on 23 Apr 2005, 11:09 a.m.,  
in response to message #28 by Marcus von Cube

Performance:

The 42S took about 90 minutes to find the 2 digit solution. My 41CX arrived after 4 hours and 10 minutes.

The number of checks performed was 4487 until the first solution (81619) and 6625 total. (These numbers were computed before I've made my last changes, but the results should be similar.)

Anybody for a faster check routine? It looks like the loop over the digits takes most of the computing time.

### Re: Some Statistics for the above

Message #30 Posted by [Marcus von Cube](#) on 24 Apr 2005, 8:28 a.m.,



*in response to message #29 by Marcus von Cube*

With the current version I could reduce the time and number of calls further:

3771 calls to the check routine and 75 minutes to solve the 2 digit problem on a real unmodified 42S. The 41C took 3:45 minutes. (It's funny to watch the flags toggle in the display :-))

The total number of calls to the checking routine was 5140. This will be significantly higher for the three digit problem, because less numbers can be ruled out beforehand based on the first 4 digits.

## Re: Short & Sweet Math Challenges #8: Squares !

*Message #31 Posted by [Arnaud Amiel](#) on 23 Apr 2005, 6:47 p.m.,*

*in response to message #1 by Valentin Albillo*

As stated above, I believed that assembly was more suited to this problem so I gave it a try. The 49G and 49g+ are the only calcs to have assembly available out of the box so I tried on the 49g+. The program below (non optimised) solves question 2 in 43s. A 49g would take 1min 43s. It is easy to port it on the 48 where it should run as fast as on a 49 (or half speed on the SX).

Here is the program

```
SAVE
SETDEC
LC 33332
R1=C.A
```

\*NextNumber

```
C=R1.A C+1.A SKIPNC { P=0 SETHEX LOADRPL }
R1=C.A
C=0.W C=R1.A
CSL.W CSL.W CSL.W
A=C.W
GOSBVL =SPLITA
C=B.W D=C.W C=A.W
GOSBVL =MULTF
GOSBVL =PACK
C=0.W C=A.M
CSR.W CSR.W CSR.W CSR.W CSR.W
```

```
P=10 A=0.S B=0.S D=0.S
```

\*TestNumber

```
P-1 GOC GoodNumber
```

```
CSRC
```

```
?C=0.S GOYES NextNumber
```

```
?A#0.S { A=C.S GOTO TestNumber }
```

```
?A=C.S GOYES TestNumber
```

```
?B#0.S { B=C.S GOTO TestNumber }
```

```
?B=C.S GOYES TestNumber
```

```
?D#0.S { D=C.S GOTO TestNumber } %remove for question 1
```

```
?D=C.S GOYES TestNumber %remove for question 1
```

```
GOTO NextNumber
```

\*GoodNumber

```
A=0.W A=R1.A ASRC ASRC ASRC ASRC ASRC ASRC A+4.A
GOSBVL =PUSH%
SAVE
SETDEC
GOTO NextNumber
```

As usual archive before playing with assembly

Arnaud

PS: If you want I can send you the compiled program for 48 or 49 by email

## [LONG] S&SMC #8: My original solutions, comments and curiosities !

Message #32 Posted by [Valentin Albillo](#) on 25 Apr 2005, 5:13 a.m.,  
in response to message #1 by Valentin Albillo

Hi all,

Thanks to all of you interested in my S&SMC #8 and most specially to those who provided such ingenious and elegant solutions for such a variety of HP calcs and computers, we really saw a lot of interesting techniques being used here for the one and only purpose: find the solutions and find them fast. Congratulations to all of you, I \*really\* feel most satisfied with your contributions !

As promised, this is my original, 4-line solution for the HP-71B:

```
10 C=0 @ FOR N=33334 TO 99999 @ M$=STR$(N*N) @ IF POS(M$,"0") THEN 40 ELSE T$=M$[1,1]
20 FOR I=2 TO 10 @ IF POS(T$,M$[I,I]) THEN 30 ELSE IF LEN(T$)=3 THEN 40 ELSE T$=T$&M$[I,I]
30 NEXT I @ C=C+1 @ DISP N;M$,T$
40 NEXT N @ DISP "OK:";C;"found"
```

Calling it produces the 48 solutions, displaying the number, its square, and the distinct digits it consists of:

>CALL

```
33334 1111155556 156
33335 1111222225 125
33338 1111422244 142
33359 1112822881 128
33989 1155252121 152
34641 1199998881 198
...
81404 6626611216 621
81619 6661661161 61  <- only *two* different digits, the solution to the first part
81813 6693366969 693
83666 6999999556 695
86478 7478444484 748
97773 9559559529 952
98336 9669968896 968
```

OK: 48 found

It takes 3 min 28 sec when run under Emu71 in a 1.4 Ghz PC. The program searches all numbers which generate 10-digit squares fulfilling the conditions. As no 0's are allowed, the smaller possible square would be 111111111, so we begin the search at SQR(111111111) rounded to the nearest integer, which is 33334, and go on searching till SQR(999999999), which comes out to 99999. We

square each number in turn and convert the result to a string. The first POS then immediately discards numbers having a "0" anywhere. Then we keep on extracting individual digits which are added to an auxiliary string, using POS to detect repetitions (in which case we don't add the digit again) and LEN to see if there are more than 3 distinct digits already collected, in which case we forfeit further processing of this number and go instead to check the next candidate.

However, this is not the fastest way to proceed because the HP-71B is optimized for numerical computations and string processing is relatively slow. So I originally came out instead with this other version, which uses flags to register which digits have been used:

```
10 C=0 @ FOR N=33334 TO 99999 @ M=N*N @ CFLAG ALL @ P=0
20 FOR I=1 TO 10 @ D=MOD(M,10) @ IF NOT D THEN 50 ELSE M=M DIV 10
30 IF FLAG(D) THEN 40 ELSE IF P=3 THEN 50 ELSE SFLAG D @ P=P+1
40 NEXT I @ C=C+1 @ DISP N;N*N
50 NEXT N @ DISP "OK:";C;"found"
```

It works the same, only using flags instead of strings. Each candidate number is decomposed into its constituent digits. If the digit is a 0, we skip to the next candidate, else a flag is set corresponding to the digit. If more than 3 flags have already been set, no further digits are isolated but we go instead to test the next candidate. This version finds the exact same solutions but it takes just 72 seconds (under Emu71), i.e., it's 3 times faster than the string version. A real HP-71B takes much longer of course, but it's a real beauty to see the display's flag indicators making a frantic dance while the program runs !

In both versions, changing the "3" to any other single digit N (1-9) would find squares made up of N distinct digits. For instance, with N=2, it produces 6661661161, the only 10-digit square consisting of just two different digits, 1 and 6 in this case. Also, you can search for 12-digit squares instead, say, by simply changing the search limits to 333334 and 999999 respectively, and by changing the limit of the I loop to 12. This can be automated by simply creating a new, generalized version that asks the user for the number of digits and maximum different values, like this:

```
1 INPUT "# Digits (1-12) = ";U @ INPUT "# Diff. val. (1-9) = ";V
10 C=0 @ FOR N=INT(SQR((10^U-1)/9)) TO INT(SQR(10^U-1)) @ M=N*N @ CFLAG ALL
20 P=0 @ FOR I=1 TO U @ D=MOD(M,10) @ IF NOT D THEN 50 ELSE M=M DIV 10
30 IF FLAG(D) THEN 40 ELSE IF P=V THEN 50 ELSE SFLAG D @ P=P+1
40 NEXT I @ C=C+1 @ DISP N;N*N
50 NEXT N @ DISP "OK:";C;"found"
```

Let's try it ! To find the five 5-digit squares consisting of at most 2 different values:

>RUN

```
# Digits (1-12)    = 5 [ENTER]
# Diff. val. (1-9) = 2 [ENTER]
```

```
109 11881
173 29929
212 44944
235 55225
264 69696
```

OK: 5 found

While to find the thirty 12-digit squares consisting of at most 3 different values:

>RUN

```
# Digits (1-12)    = 12 [ENTER]
# Diff. val. (1-9) = 3 [ENTER]
```

```

333334 111111555556
333335 111112222225
333338 111114222244
333359 111128222881
333858 111461164164
363639 132233322321
387288 149991994944
461761 213223221121
492162 242223434244
505525 255555525625
515415 265652622225
586893 344443393449
649788 42224444944
658357 433433939449
664388 441411414544
665908 443433464464
666515 444242245225
666665 444442222225
666667 44444888889
666668 444446222224
682908 466363336464
782104 611686666816
805262 648446888644
818333 669668898889
828667 686688996889
834386 696199996996
869924 756767765776
939938 883483443844
974417 949488489889
994836 989698666896

```

OK: 30 found

The 'flags' version also has the added advantage that it translates very easily to RPN machines with little or no alpha capabilities, such as the HP-41C or HP-15C. For instance, here's the translated version for an HP-41CX, a meager 46 steps:

```

01 *LBL "SQ3" 16 STO 04      31 GTO 04
02 CLX      17 10          32 *LBL 05
03 X<>F     18 STO 05      33 1
04 CF 08    19 *LBL 01     34 ST+ 02
05 CF 09    20 RCL 03      35 1E5
06 RCLFLAG  21 INT         36 RCL 02
07 STO 01   22 10          37 X#Y?
08 33334    23 ST/ 03       38 GTO 00
09 STO 02   24 MOD         39 STOP
10 *LBL 00   25 X=0?        40 *LBL 04
11 X^2      26 GTO 05       41 DSE 05
12 STO 03   27 FS? IND X   42 GTO 01
13 RCL 01   28 GTO 04      43 RCL 02
14 STOF LAG 29 SF IND X   44 X^2
15 .003     30 ISG 04       45 VIEW X
                                46 GTO 05

```

It runs under the V41 emulator in 72 min (a real HP-41CX would take a lot longer) to produce all 48 solutions, and again, it's a joy to see all those flag indicators frantically turning on and off.

[R/S]

1111155556  
 1111222225  
 ...  
 9559559529  
 9669968896

For practical reasons and in order to to make it accessible to most HP calculators, this challenge specifically addressed 10-digit squares, but if you feel like it there are marvels out there waiting to be discovered, here are some examples:

$$10099510939154979751^2 = 102000121210111101102120011101220022001 \text{ (39 digits, all 0,1,2)}$$

$$557963558954625926861^2 = 311323333121312322332133323111223321313321 \text{ (42 digits, all 1,2,3)}$$

$$675754811056988742949784^2 = 45664456466666655544556545564444555565545646656 \text{ (48 digits, all 4,5,6)}$$

Actually, even the 48-item list of solutions for the original challenge holds a number of very interesting results upon close inspection, such as these beautifully-patterned squares:

1111155556 ( 11111-5555-6 )  
 1111222225 ( 1111-22222-5 )  
 1616361616 ( 16-16-36-16-16 )  
 2929299129 ( 29-29-29-91-29 )  
 4444222225 ( 4444-22222-5 )  
 4444488889 ( 44444-8888-9 )  
 5252625625 ( 52-52-625-625 )  
 6617171716 ( 66-17-17-17-16 )

Thanks for your interest in this humble S&SMC #8, hope you enjoyed it, and

Best regards from V.

### Re: Short & Sweet Math Challenges #8: Squares !

Message #33 Posted by [Tizedes Csaba \[Hungary\]](#) on 27 Apr 2005, 7:33 a.m.,  
 in response to message #1 by [Valentin Albillo](#)

:) Err, I missed it...! Thanks for the challenge! Csaba

### You're welcome, Csaba ! [N.T.]

Message #34 Posted by [Valentin Albillo](#) on 27 Apr 2005, 9:30 a.m.,  
 in response to message #33 by [Tizedes Csaba \[Hungary\]](#)

Best regards from V.

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